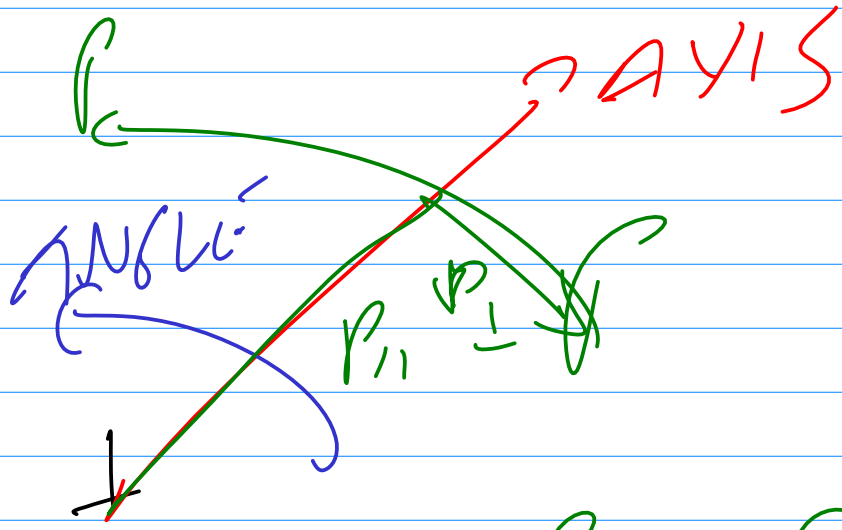
9/18/24 - 1

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 3\times 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

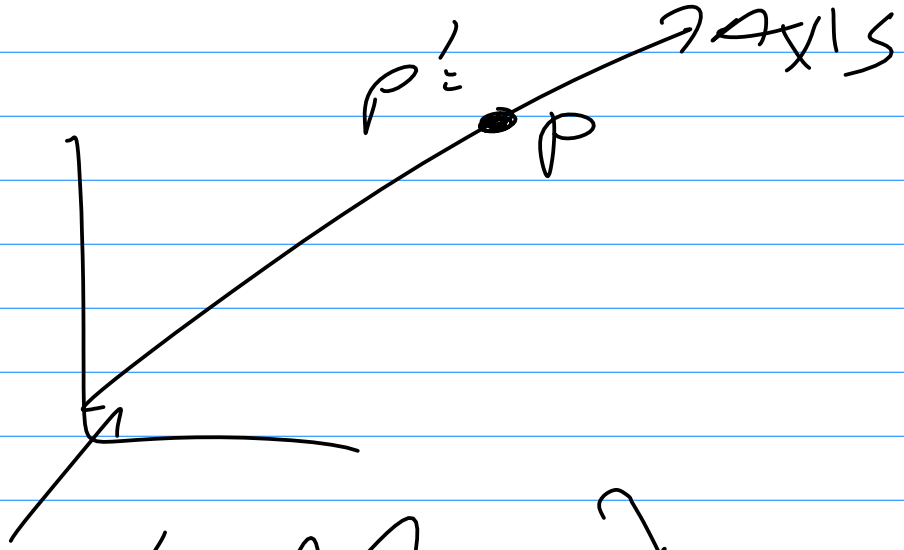
ROTATION MATRIX

$$p' = M_3 M_2 M_1 p$$



$$p = p_{\parallel} + p_{\perp}$$

2

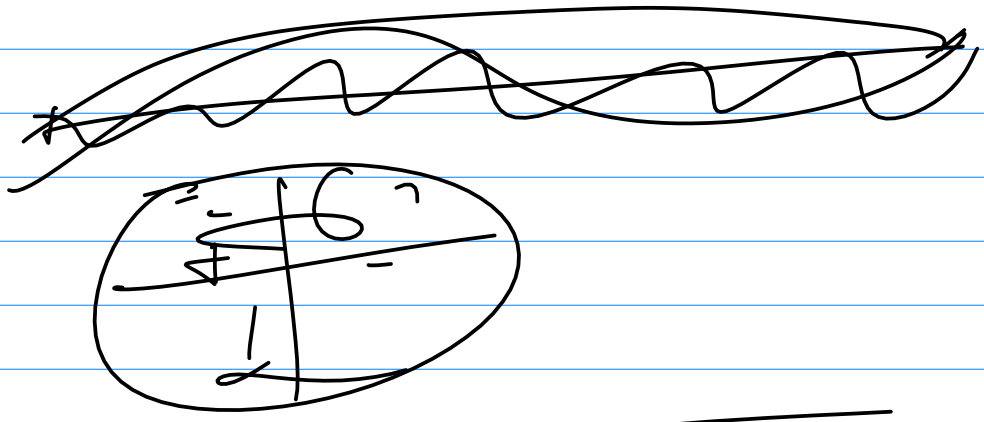


SOMETIMES $Mv = \lambda v$ ← EIGENVECTOR

EIGENVALUE

VIOLIN

DRUM



$.07 + .29 + .64 = 1$

$1 + 2\cos\theta = 1$

$\cos\theta = 0$

$\theta = \pm \pi/2$ $= \pm 90°$

QUAT

2D GEOM

(x, y)

TRANSLATÉ BY

$D_x + D_y$

$(x + D_x, y + D_y)$

ROTATE BY θ

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

COMPLEX ~~#~~

$z = x + iy$

ADD $D_x + iD_y$

$x + D_x + (y + D_y)i$

$z' = z e^{i\theta}$

if $\theta = \frac{\pi}{2}$ $(90°)$

$e^{i\theta} = i$

$(x + iy)i = -y + ix$

$(2, 3) \to (-3, 2)$

ISOMORPHISM

$$a_2 + r_2(a_1 r_2) = a_2 + r_2 a_1 + r_1 r_2$$

QUATERNIONS

$$q = A + Bi + Cj + Dk$$

i, j, k : INDETERMINATES

$$i^2 = j^2 = k^2 = -1$$

$$ij = k \quad jk = i \quad ki = j$$
$$ji = -k \quad kj = -i \quad ik = -j$$

$$q_1 = 1 + 2i \qquad q_2 = 3i + 4j$$

$$q_1 + q_2 = 1 + 5i + 4j$$

$$q_1 q_2 = (1 + 2i)(3i + 4j)$$
$$= 3i + 4j + 6i^2 + 8ij$$
$$= 3i + 4j + -6 - 8k$$
$$= -6 + 3i + 4j + 8k$$
$$q_2 q_1 = -6 + 3i + 4j - 8k$$

COMPLEX
$z = x + iy$
$x, y \in \mathbb{R}$
i INDETERMINATE
$i^2 = -1$

QUAT

POINT P

(1, 2, 3)

$P_x i + P_y j + P_z k$

$i + 2j \to 3k$

ROTATION OF θ ABOUT (A_x, A_y, A_z)

$$Q = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(A_x i + A_y j + A_z k\right)$$

$$P' = qPq^*$$