

12/5/11

CURVES

- MATH - WHAT FORM OF EQUATIONS?

- AP1

WHAT FORM OF EQUATION?

1. EXPLICIT $y = x^2$
2. IMPLICIT $x^2 + y^2 = 1$
3. PARAMETRIC

$$x = \frac{2t}{t^2 + 1}$$

$$y = \frac{t^2 - 1}{t^2 + 1}$$

IN: t OUT: (x, y)

ex. $0 \leq t \leq 1$

HOMOGENEOUS

AKA RATIONAL

$$x_H = 2t$$

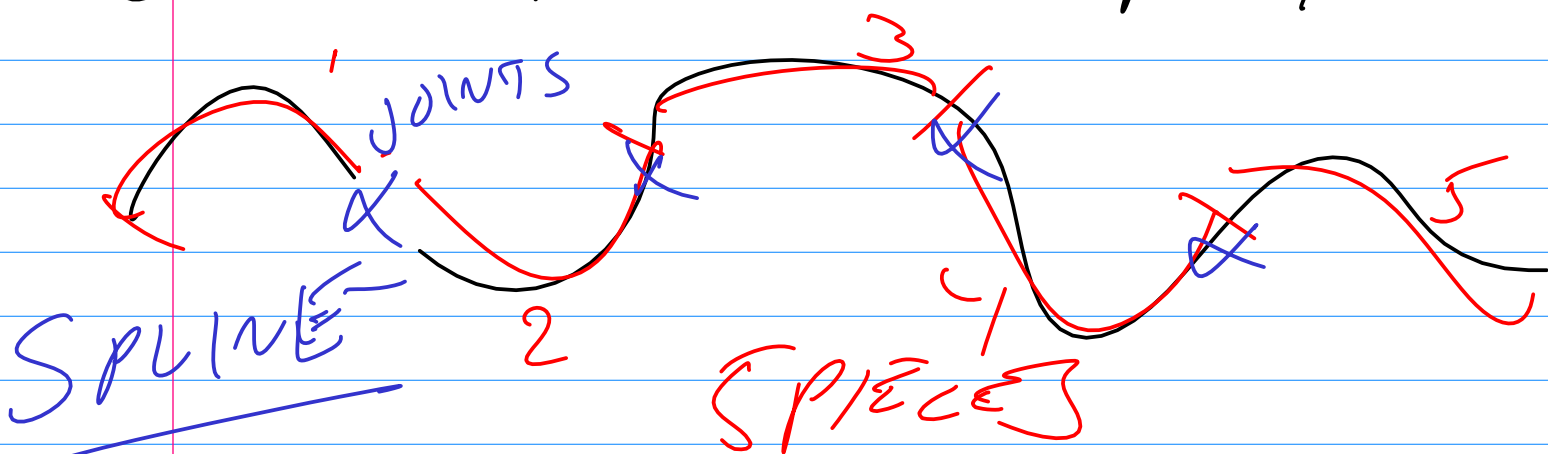
$$y_H = t^2 - 1$$

$$w = t^2 + 1$$

$$x = \frac{2t}{t^2 + 1}$$

$$y = \frac{t^2 - 1}{t^2 + 1}$$

USUAL: PARAMETRIC POLYNOMIALS
FOR COMPLICATED CURVES -
CONNECT PIECES OF POLYNOMIALS

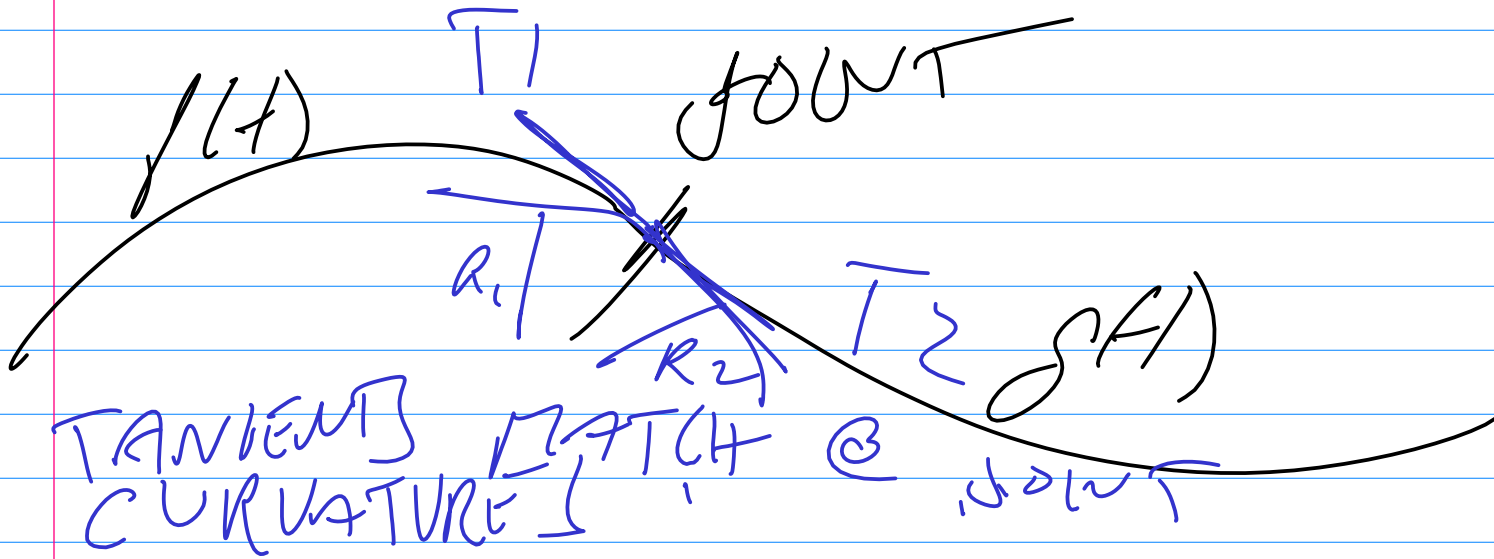


EACH PIECE IS PERHAPS CUBIC
VERY HIGH DEGREE IS BAD.

3



USER DOESN'T SEE JOINTS



SIMILAR TO MATCHING 1st 2 DERIVATIVES.

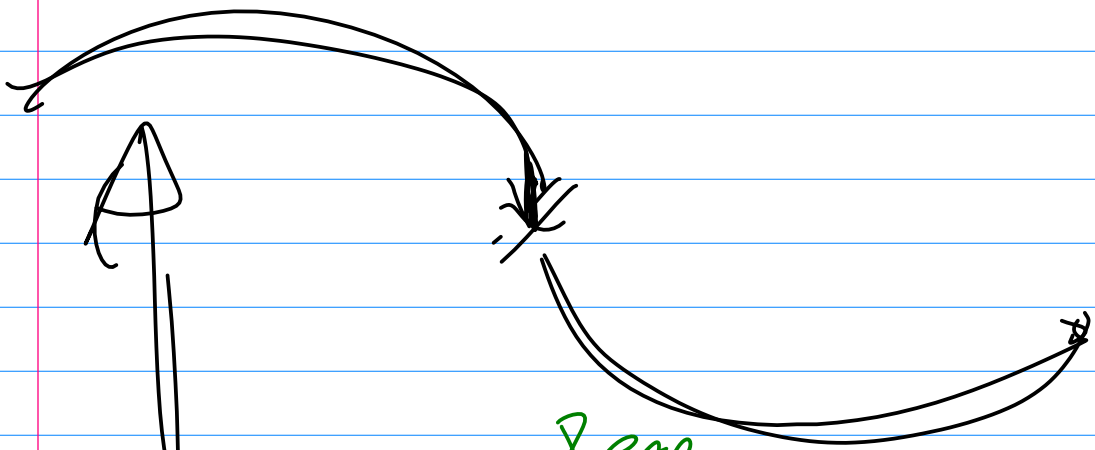
$$f(1) = g(1)$$

$$f'(1) = g'(1)$$

$$f''(1) = g''(1)$$

REQUIRES AT LEAST A CUBIC

4



3 PARAMETERS

$$x = \sum_{n=0}^3 a_n t^n$$

$$y = \sum_{n=0}^3 b_n t^n$$

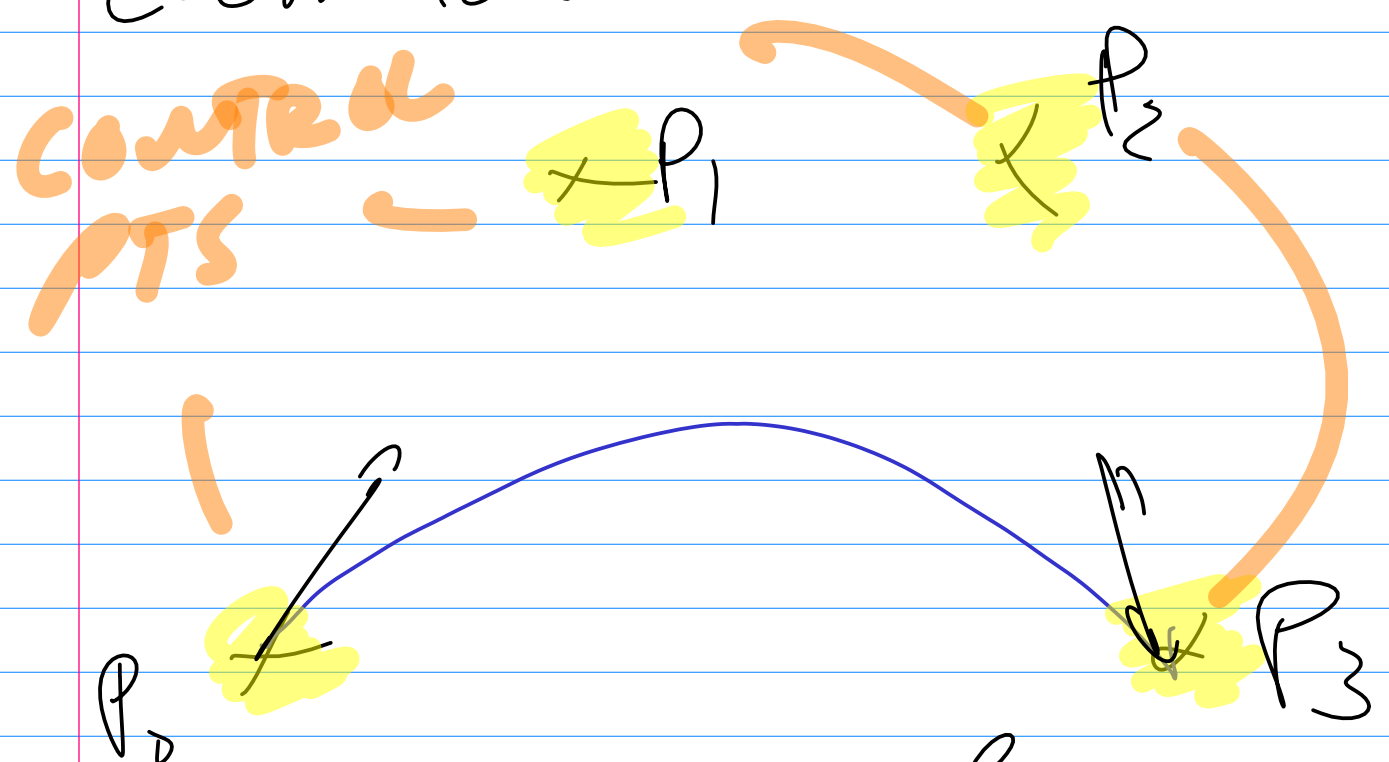
$$0 \leq t \leq 1$$

NOT RATIONAL
HERE.

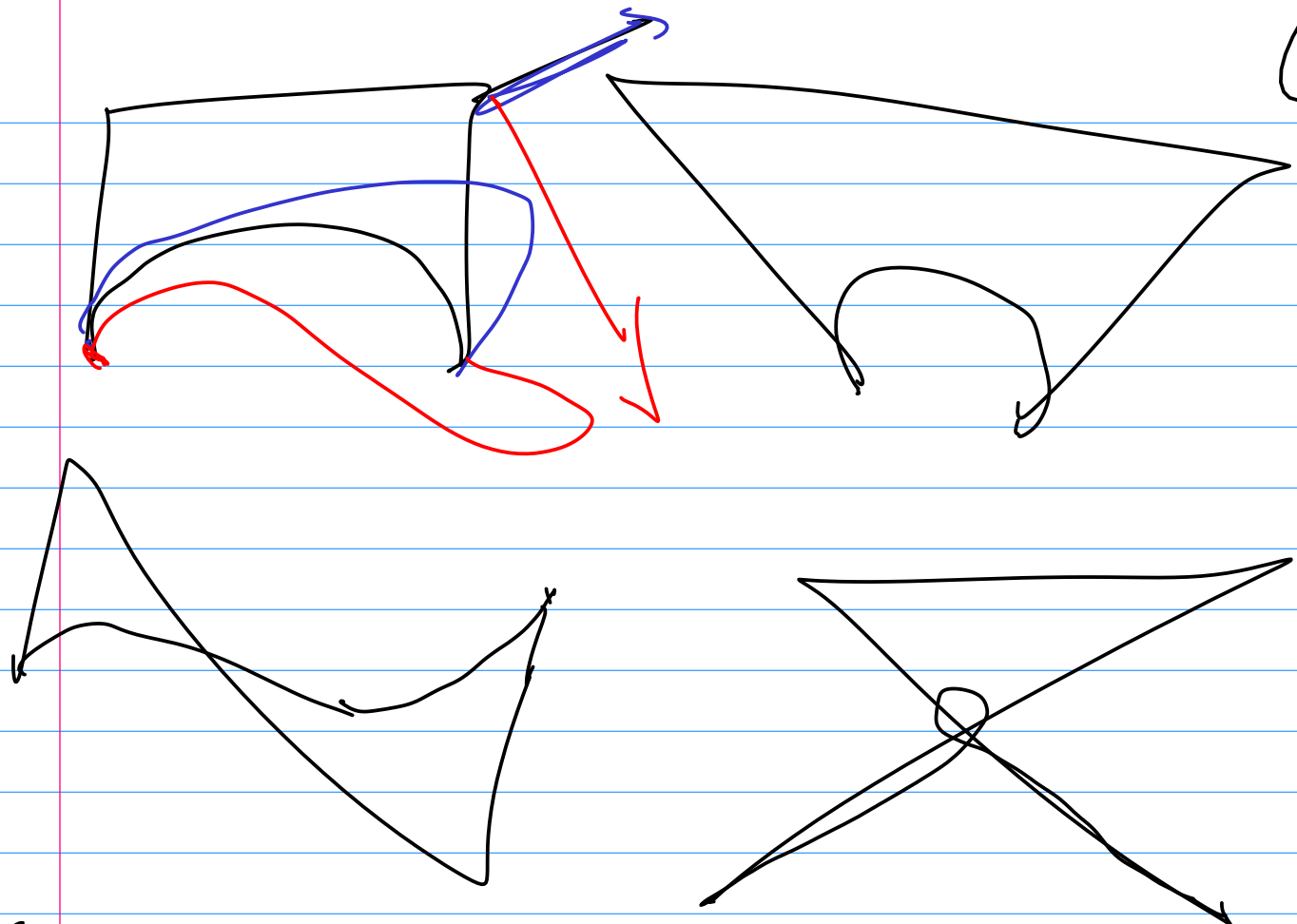
BEZIER
CURVE

DESIGNER DOES ENTER 8 NUMBERS FOR EACH SPLINE SECTION.

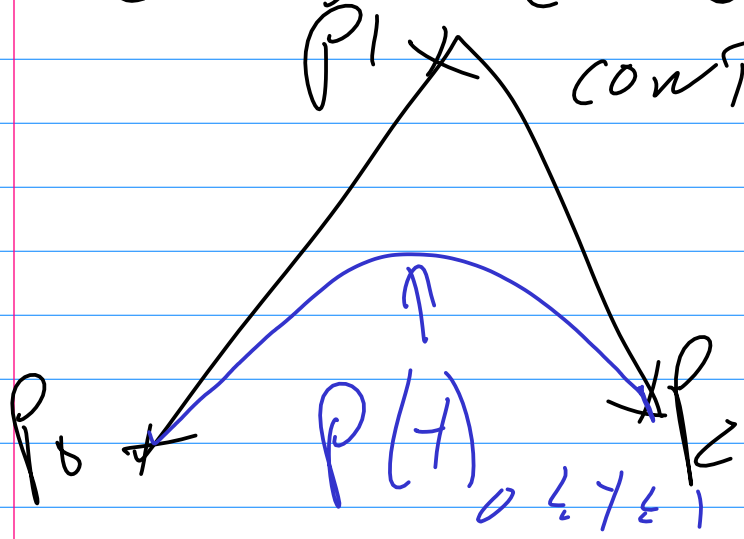
DESIGNER ENTERS 4 CONTROL POINTS
COMPUTER COMPUTES POLYNOMIAL COEFFICIENTS.



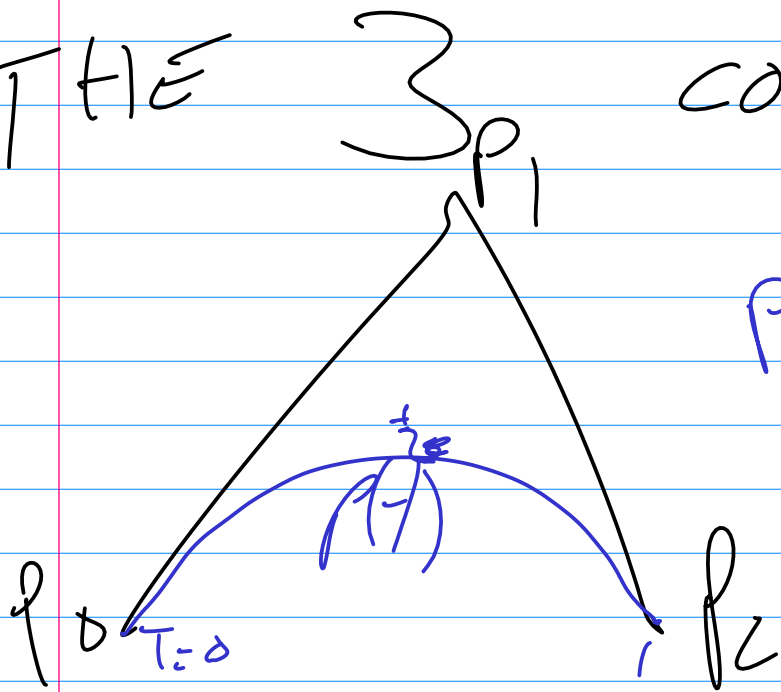
CURVE STARTS AT P_0
ENDS AT P_3
W/ TANGENT IS P_0P_1



4 control points \rightarrow 3 d.f.
 How to compute curve from control points?
 I'll use QUADRATIC.
 Control polygon



EACH POINT ON CURVE IS
 A WEIGHTED SUM OF
 THE 3 CONTROL POINTS

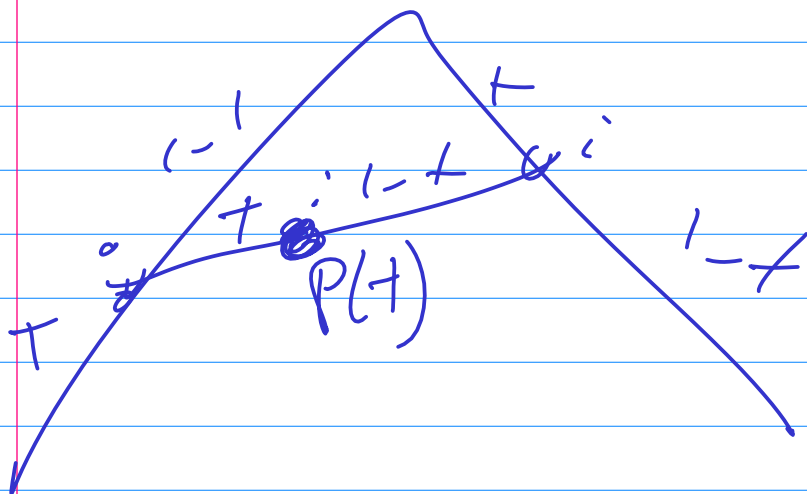


$$P(0) = P_0$$

$$P\left(\frac{1}{2}\right) = \frac{1}{4}P_0 + \frac{1}{2}P_1 + \frac{1}{4}P_2$$

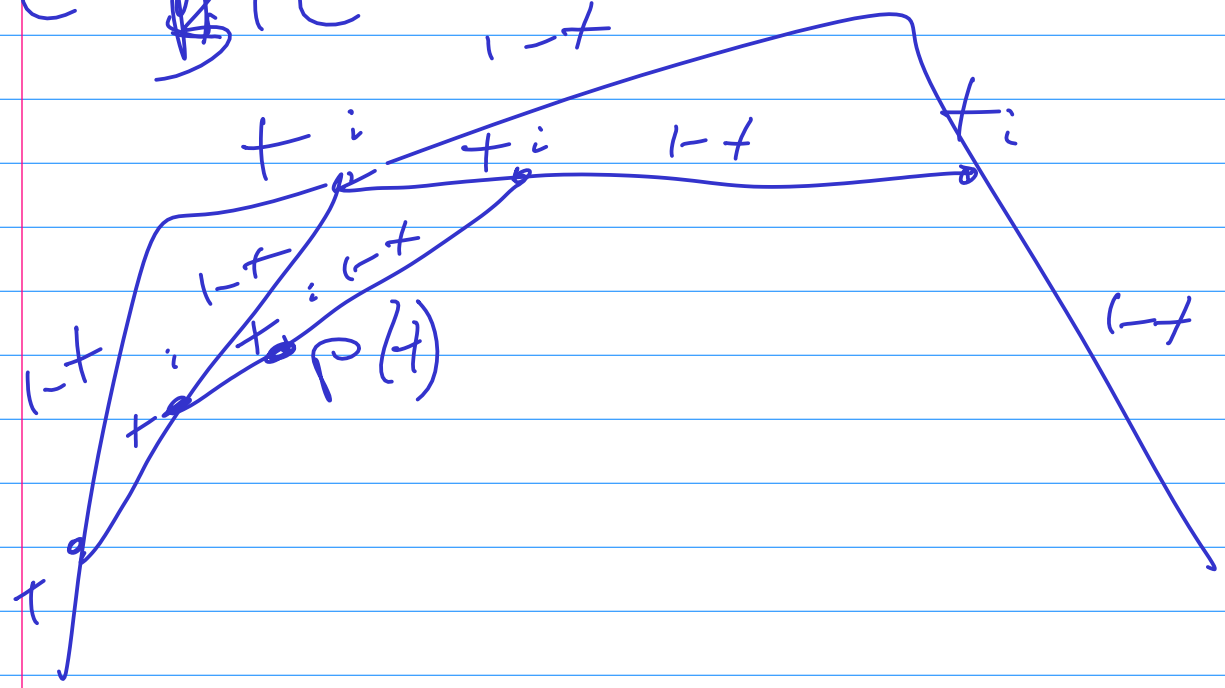
$$P(1) = P_2$$

HAND METHOD FOR QUADRATIC

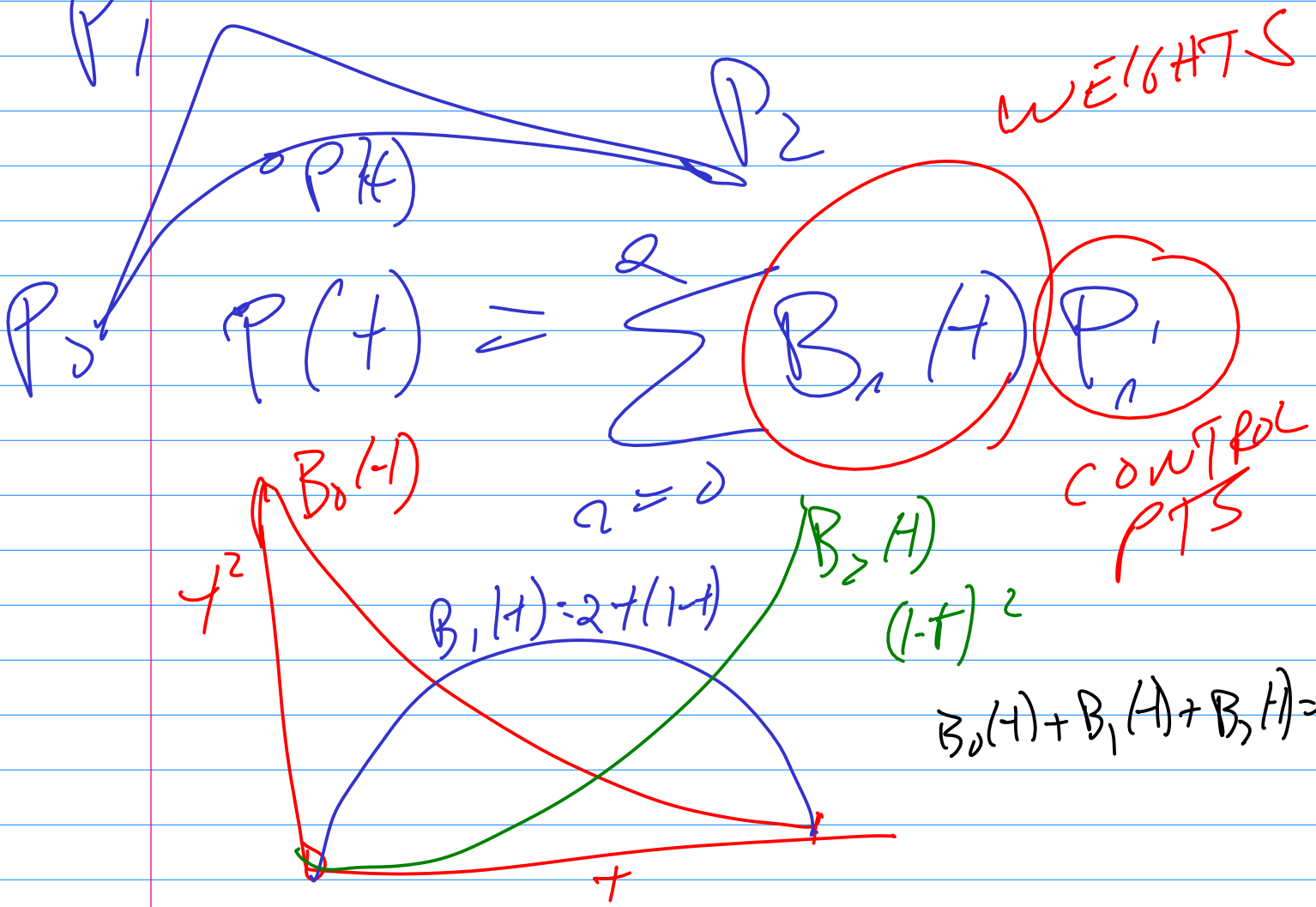


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CUBIC



QUADRATIC

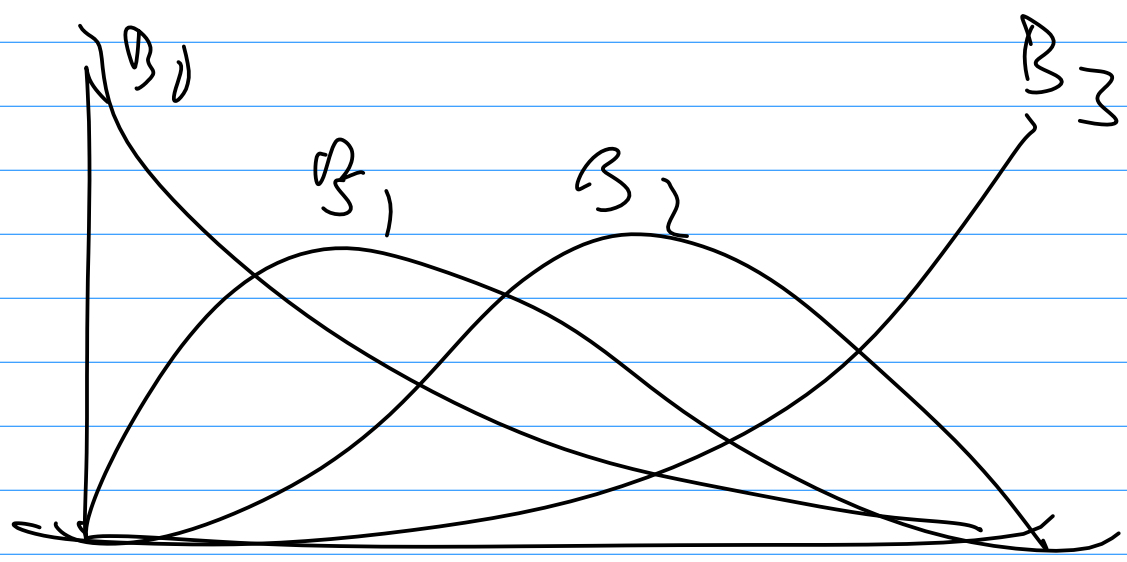


CUBIC

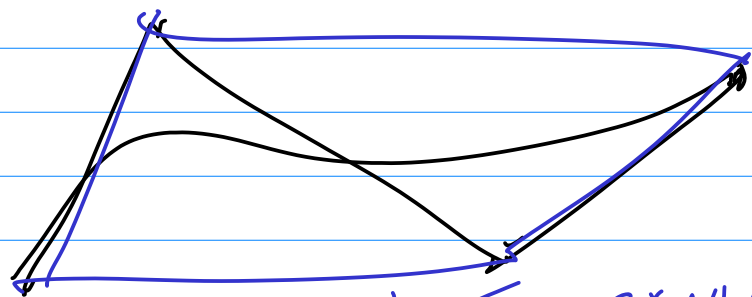
$$P(t) = \sum_{i=0}^3 B_i(t) P_i$$

$$B_i(t) = \binom{3}{i} t^i (1-t)^{3-i}$$

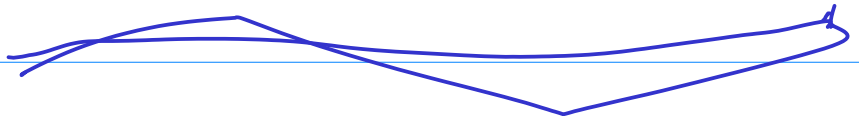
$$B_0(t) = t^3 \quad B_1(t) = 3t^2(1-t)$$



CONVEX HULL



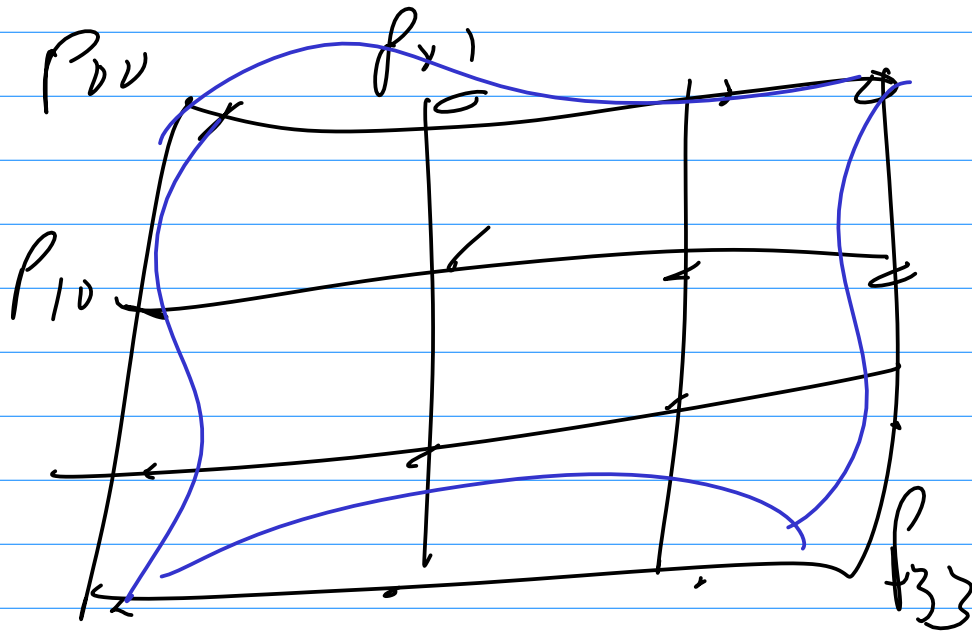
CURVE IS INSIDE CONVEX HULL OF CONTROL POLYGON.



SURFACE

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- YOU CAN SPECIFY A 4×4
GRID OF CONTROL POINTS

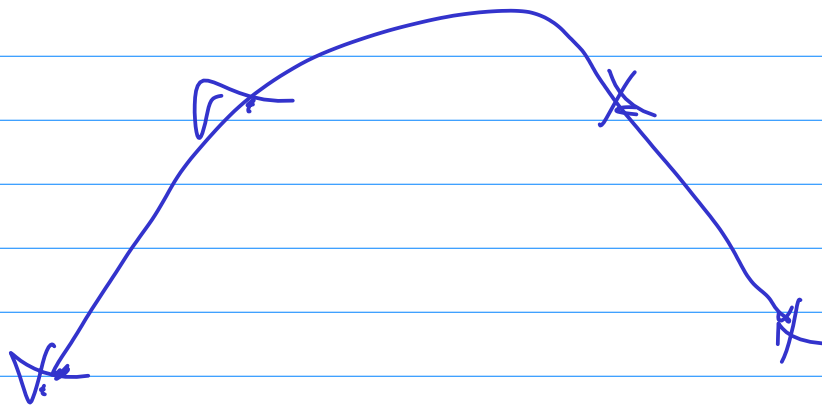


16
3-D
POINTS
48 DF

COMPUTES A 3D BICUBIC PATCH
FROM CONTROL GRID.

$$P(u, v) = \sum_{i=0}^3 \sum_{j=0}^3 B_i(u) B_j(v) \vec{P}_{ij}$$

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Q: WHY NOT HAVE CURVE
GO THROUGH ALL 4
CONTROL POINTS?



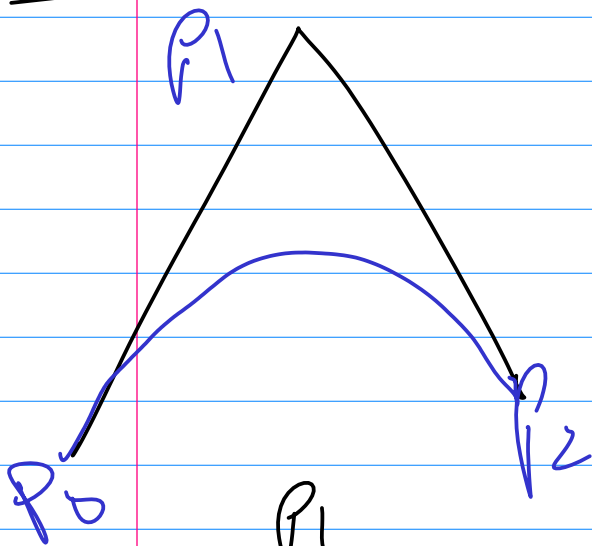
YOU COULD DO IT, EASILY
BUT IT HAS UNDESIRABLE
PROPERTIES

- EVEN IF CONTROL POLYGON IS
FLATISH, CURVE SWINGS WIDE

SPLINES

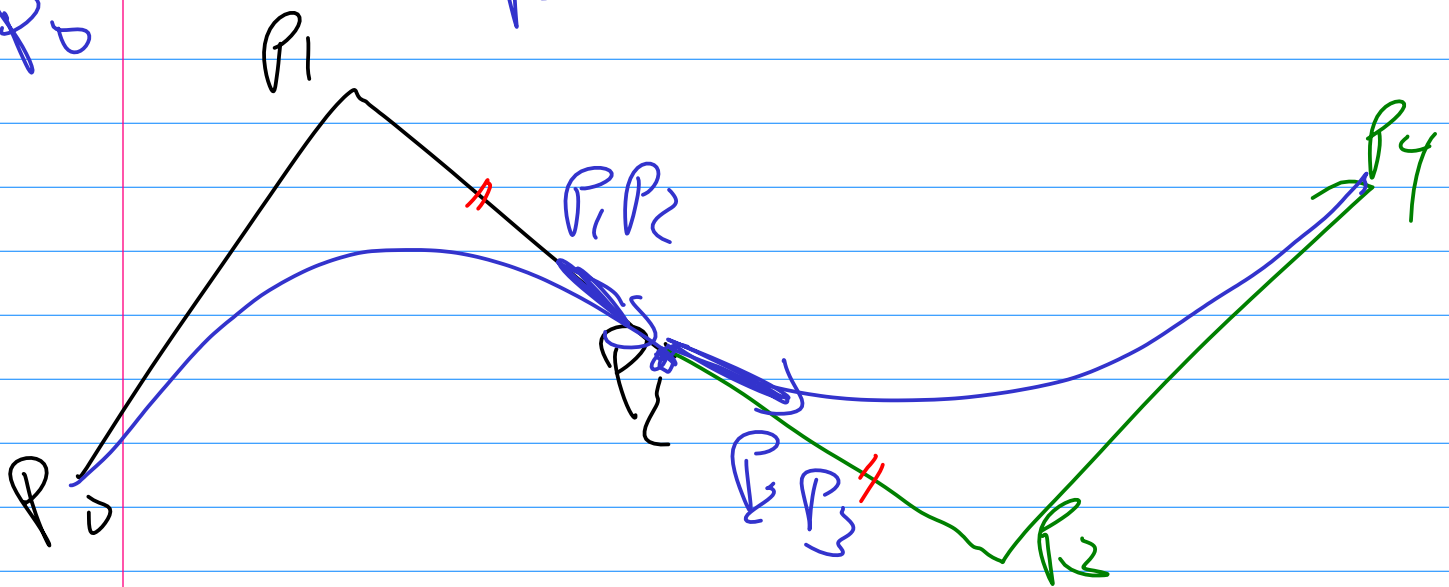
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QUADRATIC



TANGENT AT P_0
IS P_0P_1

TANGENT AT
 P_2 IS P_1P_2

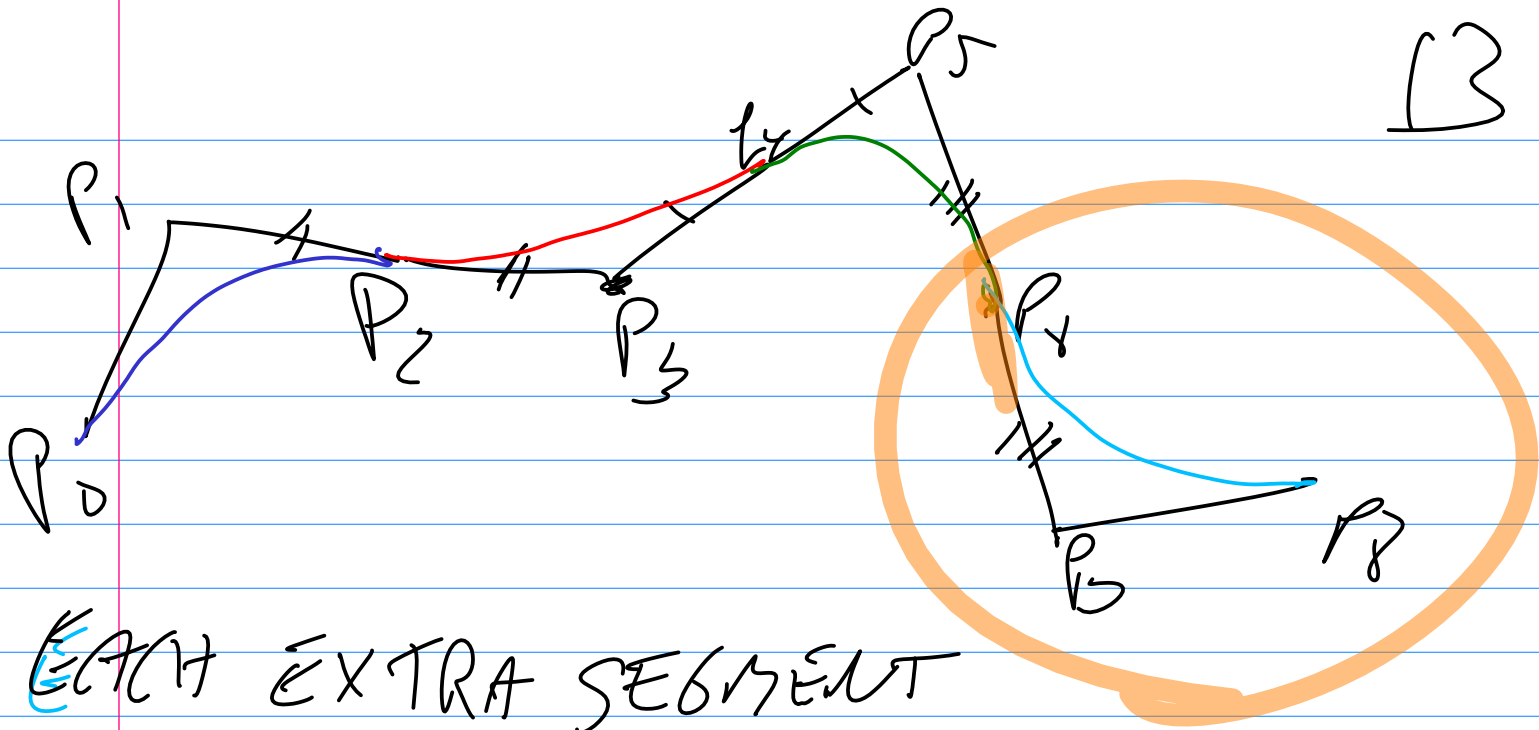


2 BEZIER'S $P_0 P_1 P_2$

+ $P_2 P_3 P_4$

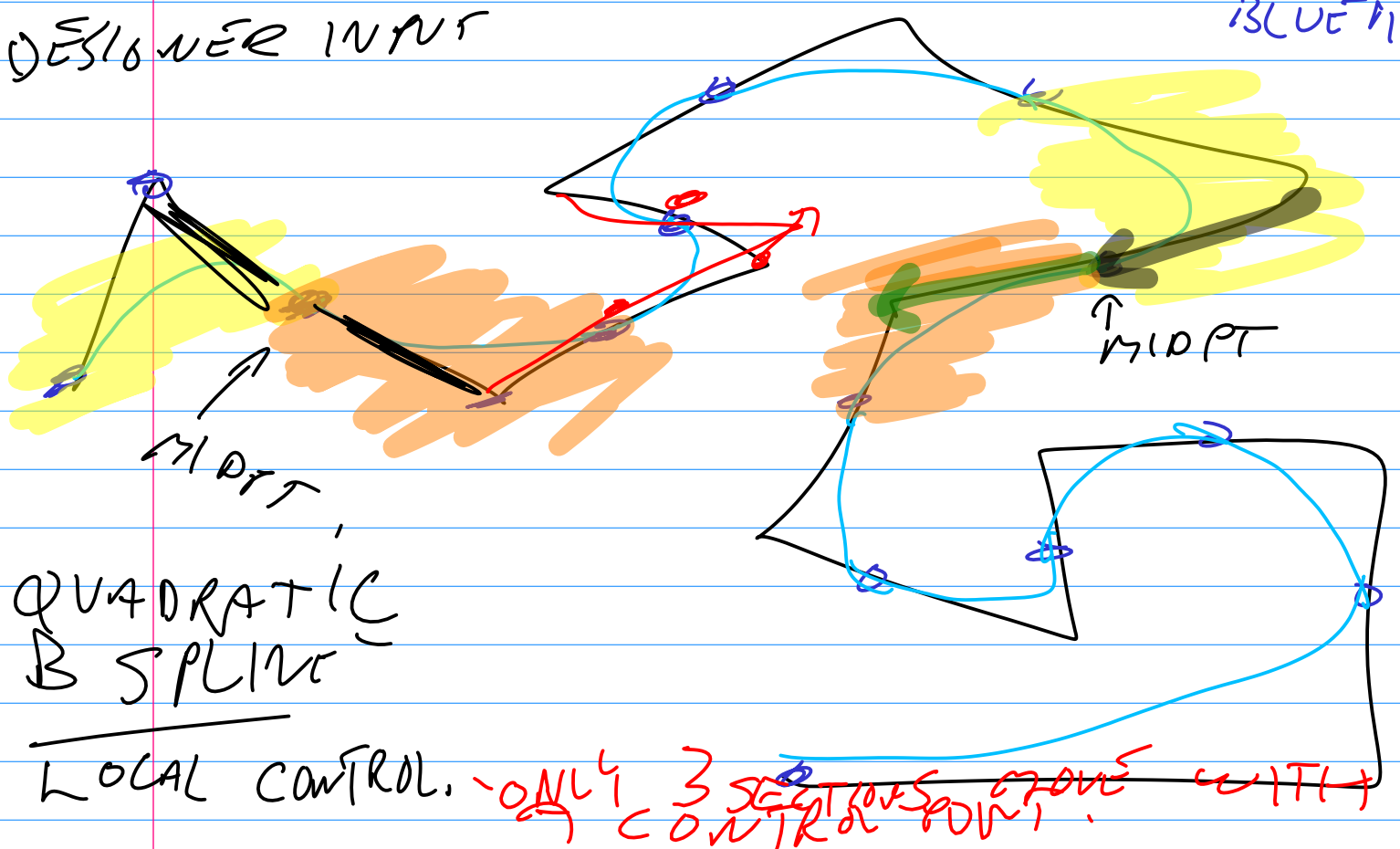
IF $P_1 P_2 = P_2 P_3$

TANGENTS
MATCH



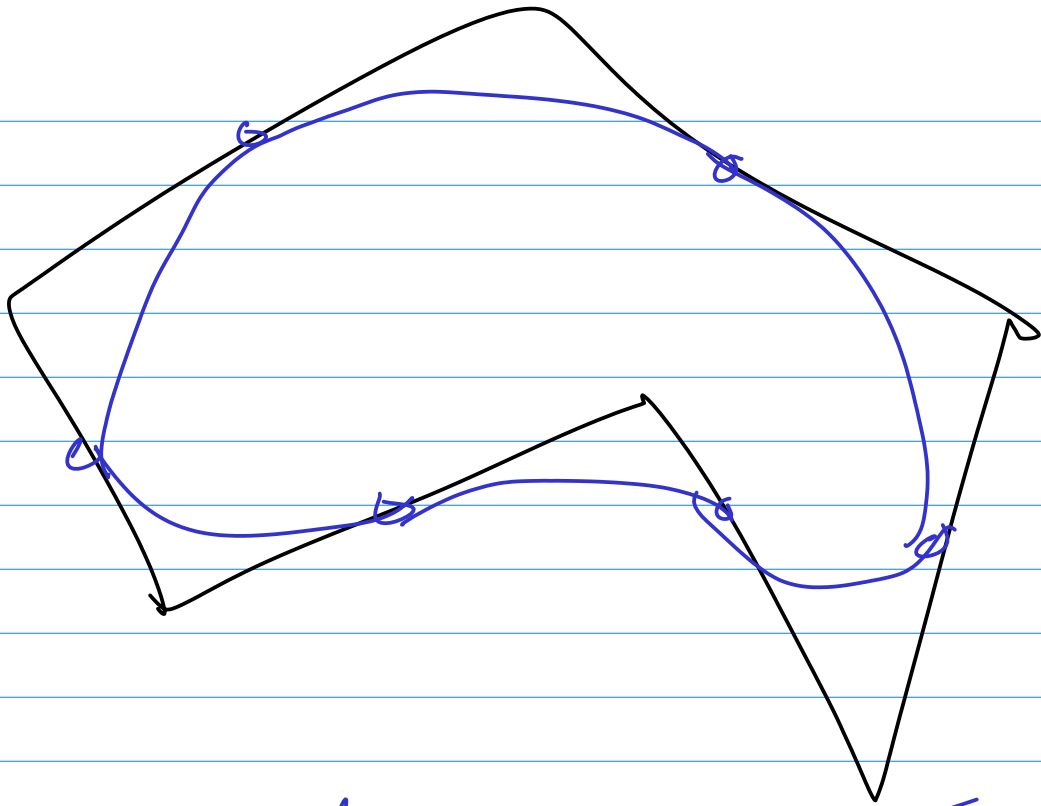
EACH EXTRA SEGMENT HAS ONLY 1 FREE POINT THAT'S NOT CONVENIENT FOR DESIGN RATHER DO THIS: DESIGNER INPUT

YOU CALCULATE BLUE

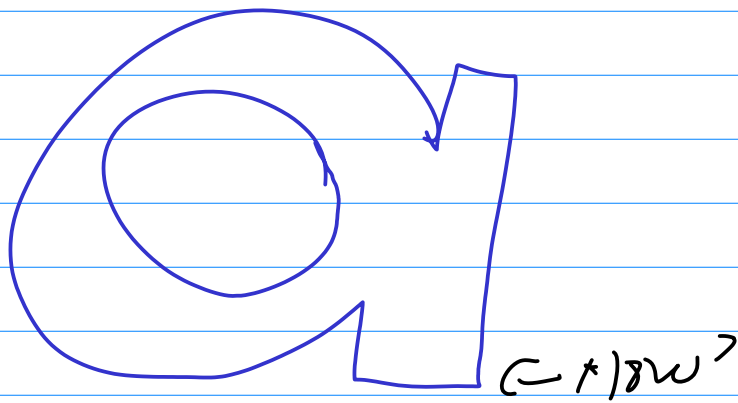


QUADRATIC B SPLINE

LOCAL CONTROL. ONLY 3 SECTIONS MOVE WITH CONTROL POINT

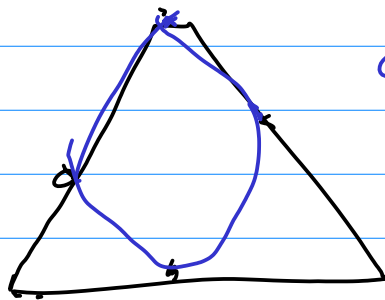
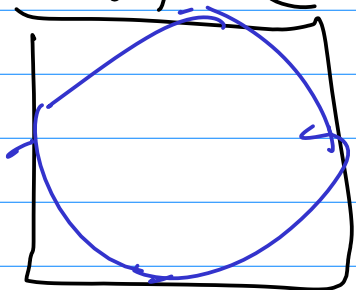


APPLICATION - OUTLINE FONTS



POSTSCRIPT
+ TT
FONTS

DO CORNER BY MAKING EDGE OF
CONTROL POLYGON REALLY
SMALL



TANGENT
NOT
CONTINUOUS

