

10/27/11

H.C.

DIFFERENT REP FOR  
POINTS

CART 3D

$(X, Y, Z)$

H.C.

$(X_H, Y_H, Z_H, W)$

$$X = \frac{X_H}{W}, \quad Y = \frac{Y_H}{W}, \quad Z = \frac{Z_H}{W}$$

$(\frac{1}{4}, \frac{2}{4}, \frac{3}{4})$

$(1, 2, 3, 4)$

$(10, 20, 30, 40)$   
H.C.

CARE

3x3 MAT

M

$\longleftrightarrow$

4x4

$(\begin{array}{ccc|c} M & & & 0 \\ & & & 0 \\ & & & 0 \\ \hline 0 & 0 & 0 & 1 \end{array})$

CART

SCALE BY  
(2, 3, 4)

HOM

2

$$\begin{pmatrix} 4 \\ 12 \\ 20 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 5 \\ 1 \end{pmatrix}$$

$\begin{pmatrix} 8 \\ 24 \\ 40 \\ 2 \end{pmatrix} =$

$\begin{pmatrix} 4 \\ 8 \\ 10 \\ 2 \end{pmatrix}$

HC UPWARD COMPATIBLE WITH CART

ADV 1 TRANSLATION.

$$\begin{pmatrix} X+4 \\ Y+5 \\ Z+6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

YOU CAN SCALE HC POINT

$$\begin{matrix}
 \text{e.g.} \\
 \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \\ 30 \\ 10 \end{pmatrix} \quad \text{etc}
 \end{matrix}$$

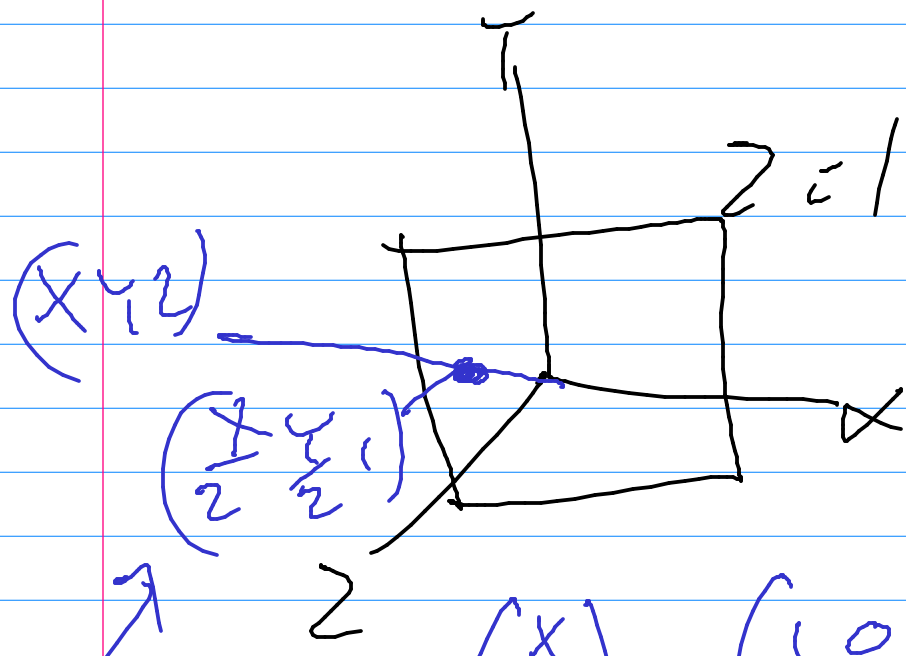
YOU CAN SCALE HC MAT

(4,5,6) TRANS

$$\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & 8 \\ 0 & 2 & 0 & 10 \\ 0 & 0 & 2 & 12 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x+8 \\ 2y+10 \\ 2z+12 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} x+4 \\ y+5 \\ z+6 \\ 1 \end{pmatrix}$$

# PROJECTION IS A MATRIX



$$\begin{pmatrix} x \\ y \\ z \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ z \end{pmatrix}$$

$\swarrow$  CART  $\begin{pmatrix} x/2 \\ y/2 \\ z \\ 1 \end{pmatrix}$

$$\mathbb{R}^4 = \left( P, T, R, S, T, R, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right)$$

VIEW NORMALIZATION IS

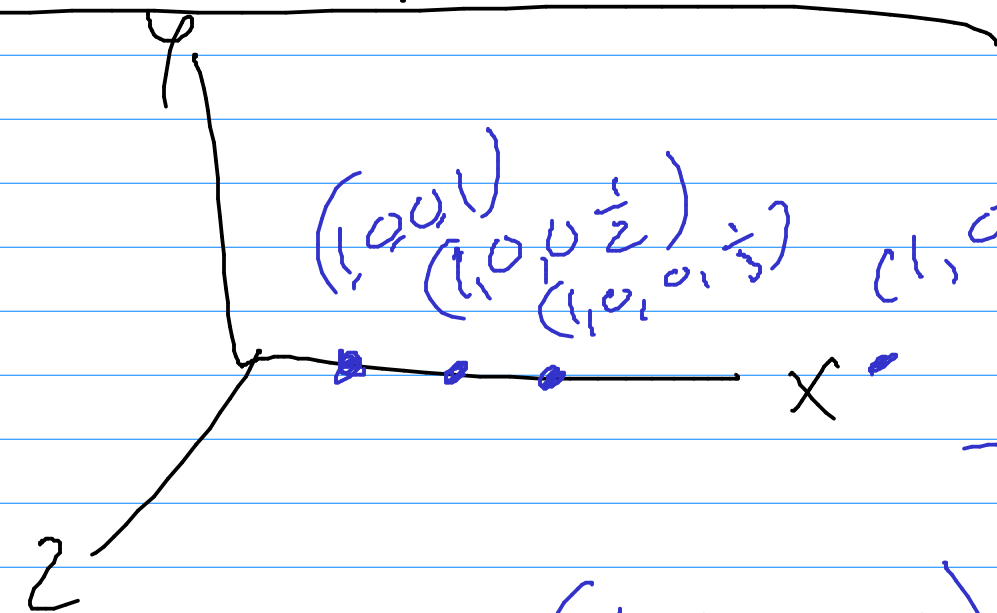
ALSO A HC MATRIX

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \rightarrow \begin{pmatrix} X/Z \\ Y/Z \\ 1/Z \end{pmatrix}$$

VIEW NORM

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

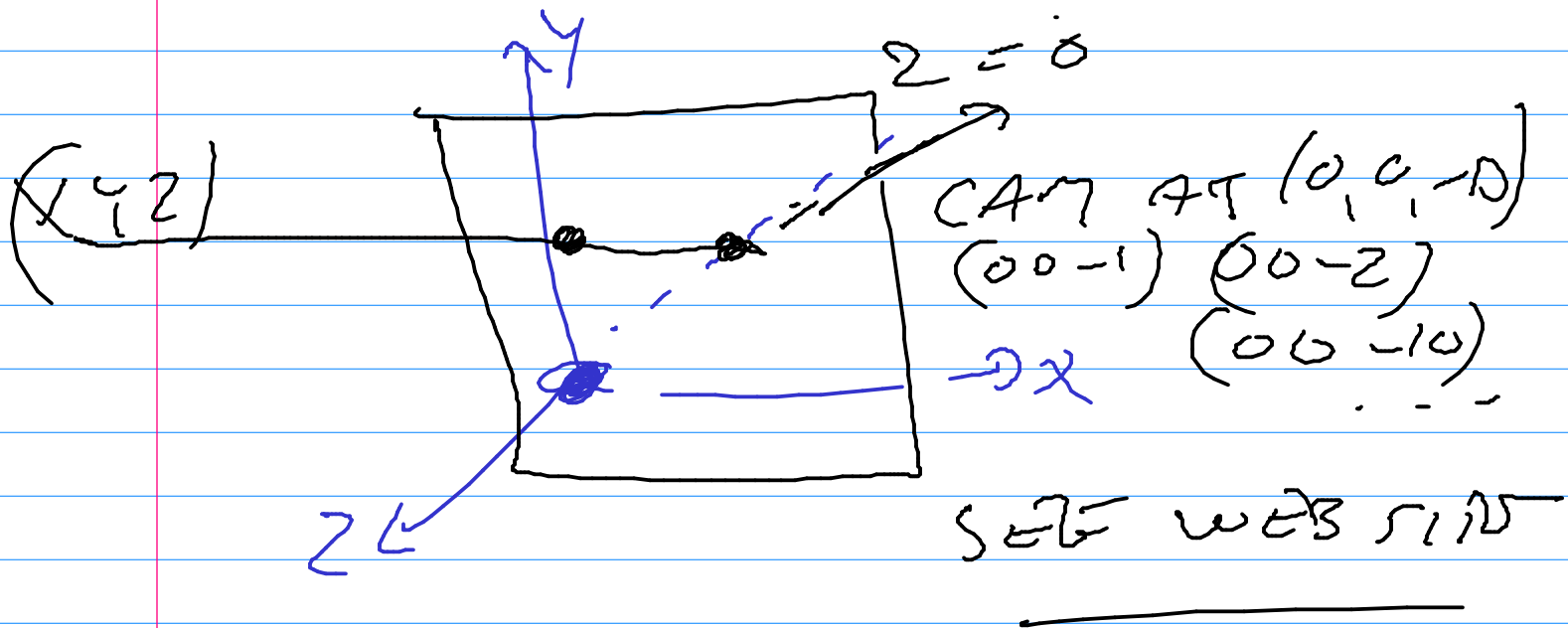
POINTS AT  $\infty$



LIMIT FOR  $(1, 0, 0, w)$   
 AS  $w \rightarrow 0$  IS POINT AT  
 AT END OF X AXIS.

USE? A PARALLEL PROJ  
IS A PERSPECTIVE PROJ  
WITH CAMERA AT  $\infty$ .

ONE LESS SPECIAL CASE



$$X' = X \left( \frac{D}{Z+D} \right) = X \left( \frac{1}{\frac{Z}{D} + 1} \right)$$

$$Y' = Y \left( \frac{D}{Z+D} \right)$$

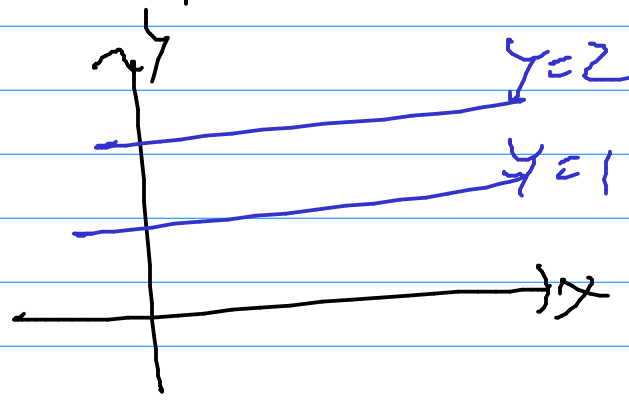
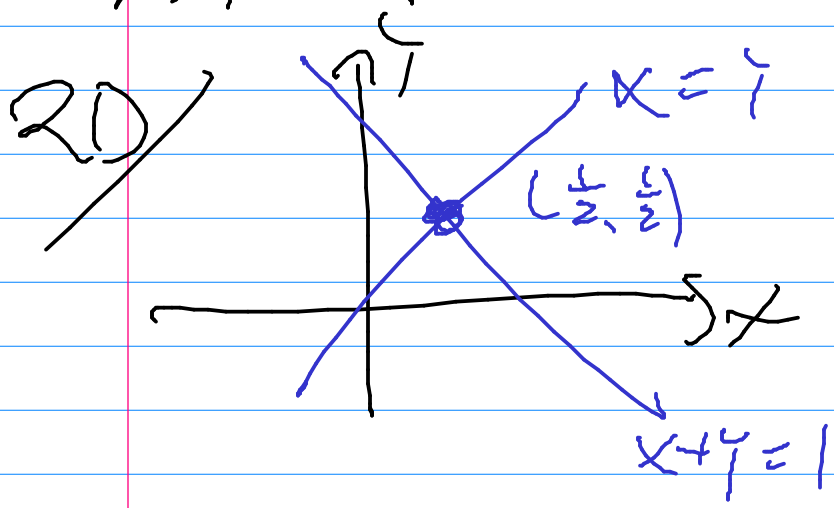
$$Z' = 0$$

$$\text{IF } D = \infty$$

$$\begin{aligned} X' &= X \\ Y' &= Y \\ Z' &= 0 \end{aligned}$$

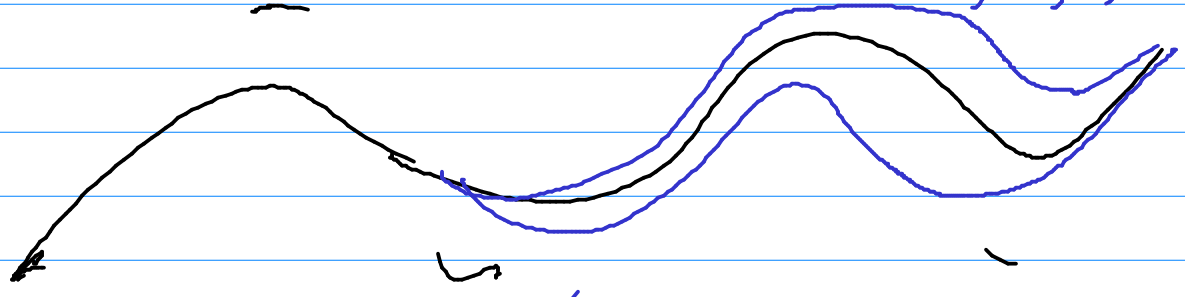
# MORE ADVANTAGES

PARALLEL LINES INTERSECT AT AN INFINITE POINT.



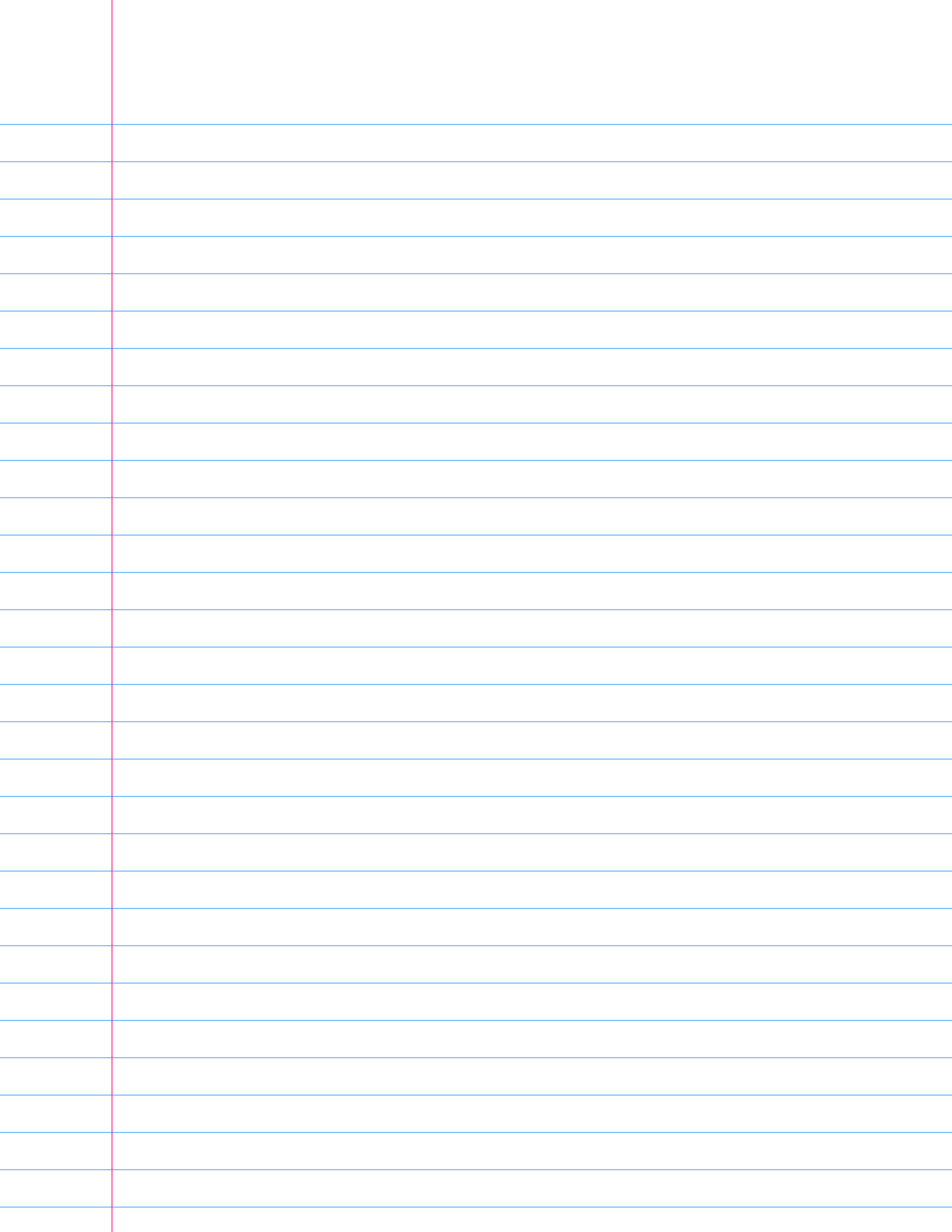
CAD: FIT A CURVE NEAR A SEQUENCE OF POINTS

$(2, 2)$   
 $(3, 1) \rightarrow (3, 1, 1)$   
 $(3, -1, 1)$



INCREASING  $w$  INCREASES THAT POINT'S WEIGHT.





ADU / 10  
CIRCLE IS EXACT.

PARAMETRIC EQNS ARE COMMON

$$X = X(T)$$

$$Y = Y(T)$$

$$X_H = T^2 - 1$$

$$Y_H = 2T$$

$$W = T^2 + 1$$

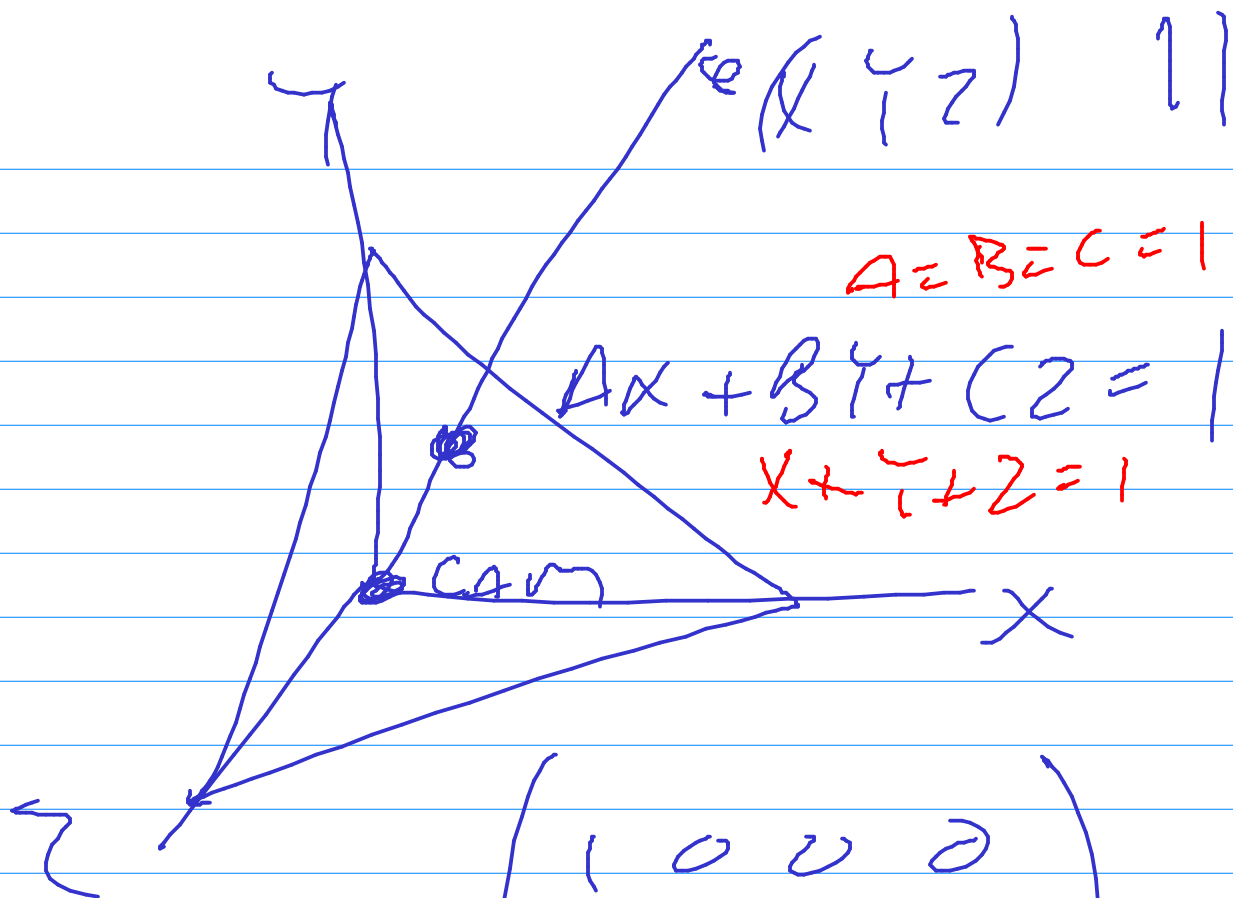
CIRCLE

CART

$$X = \frac{T^2 - 1}{T^2 + 1}$$

$$Y = \frac{2T}{T^2 + 1}$$

$$X^2 + Y^2 = \frac{T^4 - 2T^2 + 1 + 4T^2}{T^4 + 2T^2 + 1} = 1$$



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ A & B & C & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{9} & \frac{3}{19} & \frac{5}{19} \\ \frac{1}{7} & \frac{3}{7} & \frac{5}{7} \end{pmatrix}$$

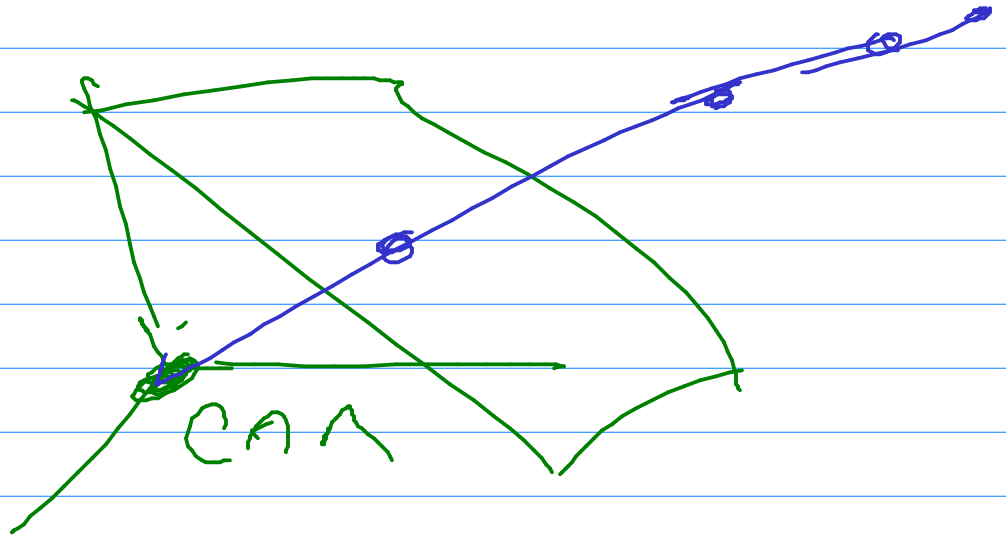
IS THIS GOOD?

1. PROJ POINT IS THE ORIG POINT SCALED (SINCE CAM IS 4 (000))
2. PROJ POINT IS ON  $x+y+z=1$

R?

Q: WHAT HAPPENED TO W?  
THE W OF THE ORIGINAL POINT DOES NOT AFFECT THE PROJECTED POINT.

A: YES. ~~SO~~ THE CAMERA IS AT  $(0,0,0)$ , SO  $(X, Y, Z)$ ,  $(2X, 2Y, 2Z)$ ,  $(5X, 5Y, 5Z)$  ETC ALL PROJECT TO SAME POINT.



# COLOR + LIGHT

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GVHA CH 11 p 405

VISIBL LIGHT

V B C G Y O R

400nm

700nm

4000 $\text{\AA}$

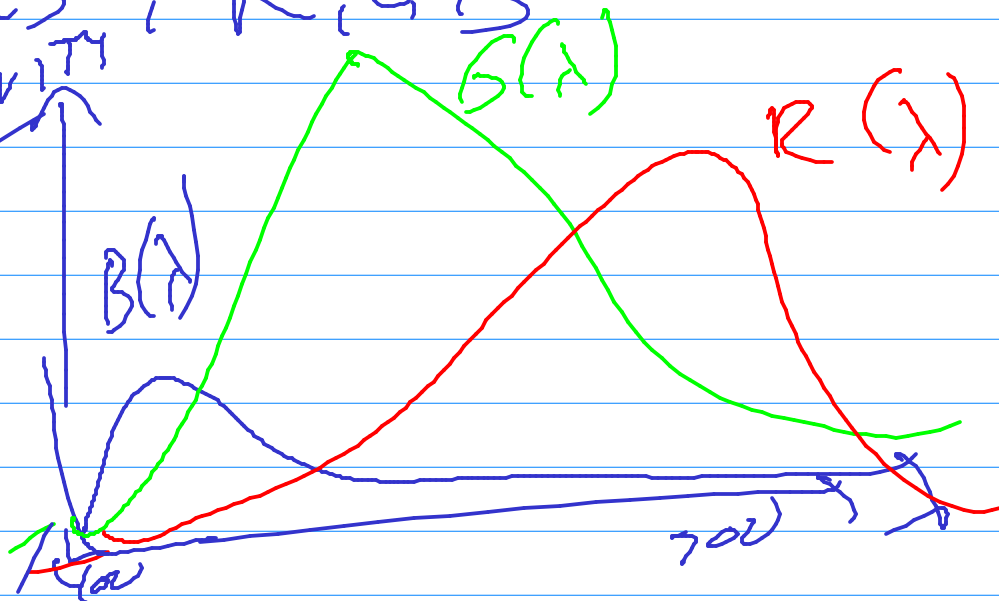
INCOMING LIGHT IS A CONTINUUM

$I(\lambda)$

MAGENTA

COLOR SENSORS IN EYE  
3 TYPES: R, G, B

SENSITIVITY



APPARENT REDNESS 14  
OF LIGHT  $R = \int R(\lambda) I(\lambda) d\lambda$

$$G = \int G(\lambda) I(\lambda) d\lambda$$

$$B = \int B(\lambda) I(\lambda) d\lambda$$

PERCEIVED COLOR IS  $(R, G, B)$

THERE ARE AN  $\infty$  NUMBER OF  
DIFFERENT COLORS THAT ARE  
PERCEIVED THE SAME.

3D  
METAMORPHIC

WE TALK ABOUT  $(R, G, B)$  COLORS.



# ADDITIVE COLORS

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OVERLAPPING SPOTLIGHTS  
ONTO DARK SCREEN

# SUBTRACTIVE

OVERLAPPING INKS ONTO  
WHITE PAPER



ALSO USE BLACK INK  
CMYK

# HSV

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HUE SATURATION VALUE

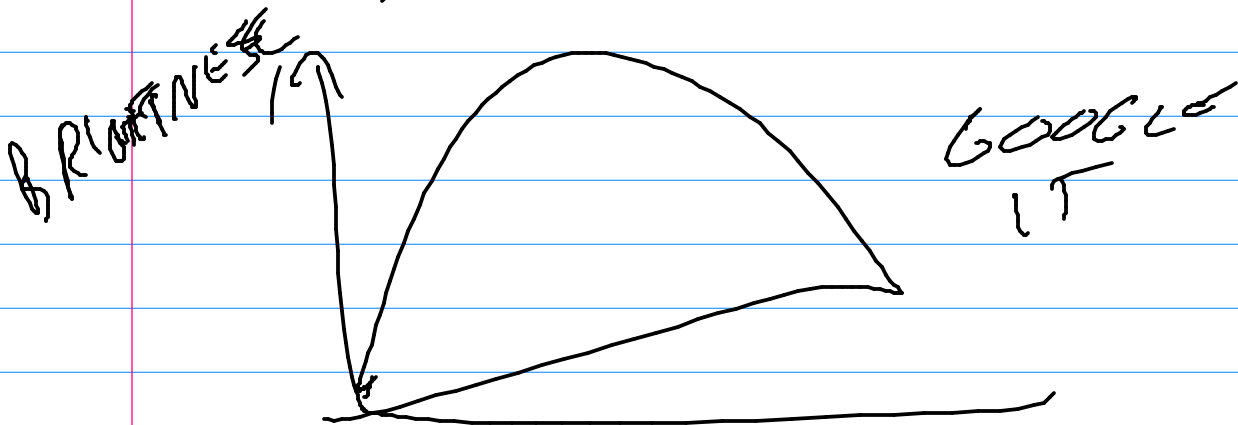
(RGB) 3D VECTOR

YOU CAN CHANGE BASIS.

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## CIE CHROMATICITY DIAGRAM

Q SUPPOSE A LIGHT AT  $\lambda = 500\text{nm}$   
& A LIGHT  $\lambda = 700\text{nm}$  OVERLAP.  
YOU WILL SEE ONE COLOR.  
WHAT IS IT?



THIS MAPS COLORS INTO A  
PERCEPTUAL COORDINATE  
SYSTEM.



ABOVE WAS GENERAL STUFF ABOUT PEOPLE + COLOR

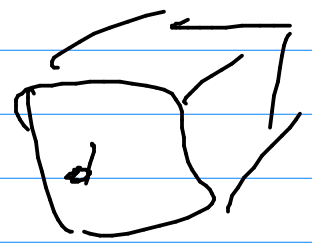
NOW WE'LL SEE COLOR + OPENGL / DIRECTX

P 413

PHONG'S LIGHTING MODEL

≠ PHONG SHADING OF A PATCH

KEY CONCERN IN GRAPHICS IS APPARENT COLOR OF AN OBJECT



P413

18

SPECULAR - SHININESS  
+ DIFFUSE  
+ AMBIENT

= TOTAL LIGHT AT  
EACH POINT OF  
SURFACE OF OBJECT.