ECE 477 10/24/11

QUATERNION REVIEW

$z = a + bi$

$i^2 = -1$

$q = a + bi + cj + dk$

$i^2 = j^2 = k^2 = -1$

$ij = -ji = k \quad$ etc.

NOT COMMUTATIVE

$q_1 = 2 + 3i + 4j \qquad q_2 = i - k$

$q_1 q_2 = (2 + 3i + 4j)(i - k)$

$= 2i + 3\underbrace{i^2}_{-1} + 4\underbrace{ji}_{-k} - 2k - \underbrace{3ik}_{-j} - 4\underbrace{jk}_{i}$

$= 2i - 3 - 4k - 2k + 3j - 4i$

$= -3 - 2i + 3j - 6k$

FOR AXIS a, $|q| = 1$

ANGLE θ

$$q = \cos\frac{\omega}{2} + \sin\frac{\omega}{2}\left(a_x i + a_y j + a_z k\right)$$

q^* : —

$p = x i + y j + z k$

$p' = q\, p\, q^*$

ROTATE 90° ABOUT X AXIS

$q = .7 + .7i$

$q^* = .7 - .7i$

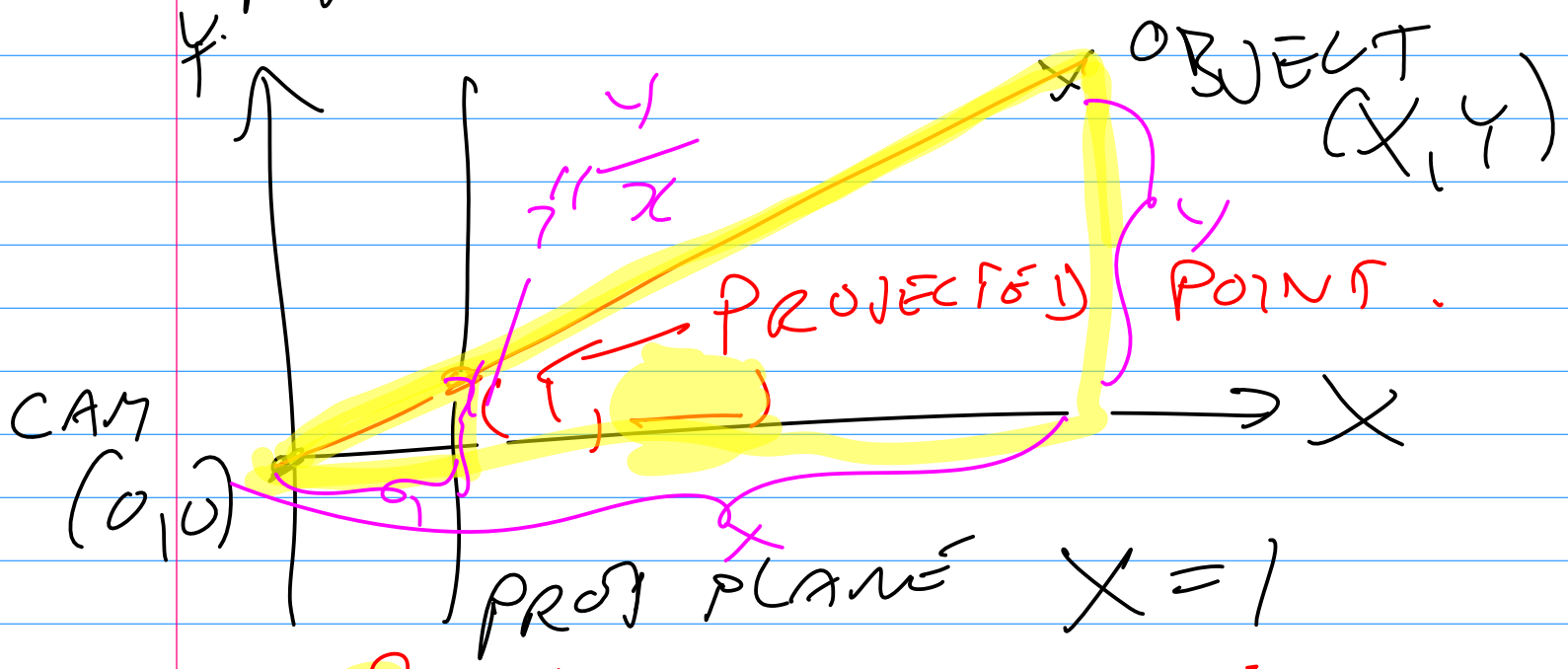
$q_2 q_1 p\, q_1^* q_2^*$

COMBO ROTATION

IT HAS AN AXIS & ANGLE.

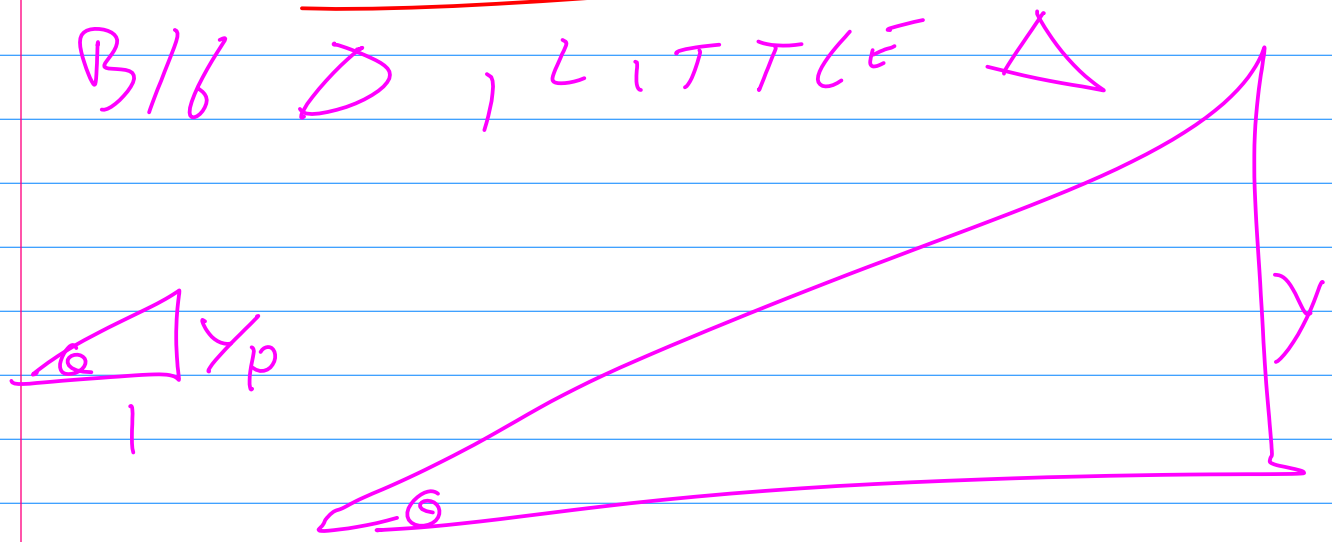
ANIMATION IS EASY.

PROJECTIONS



SIMILAR TRIANGLES

B/C D, LITTLE △



SLOPES ARE SAME $\frac{Y_p}{1} = \frac{y}{x}$

(x, y) PROJECTS TO $\left(1, \frac{y}{x}\right)$

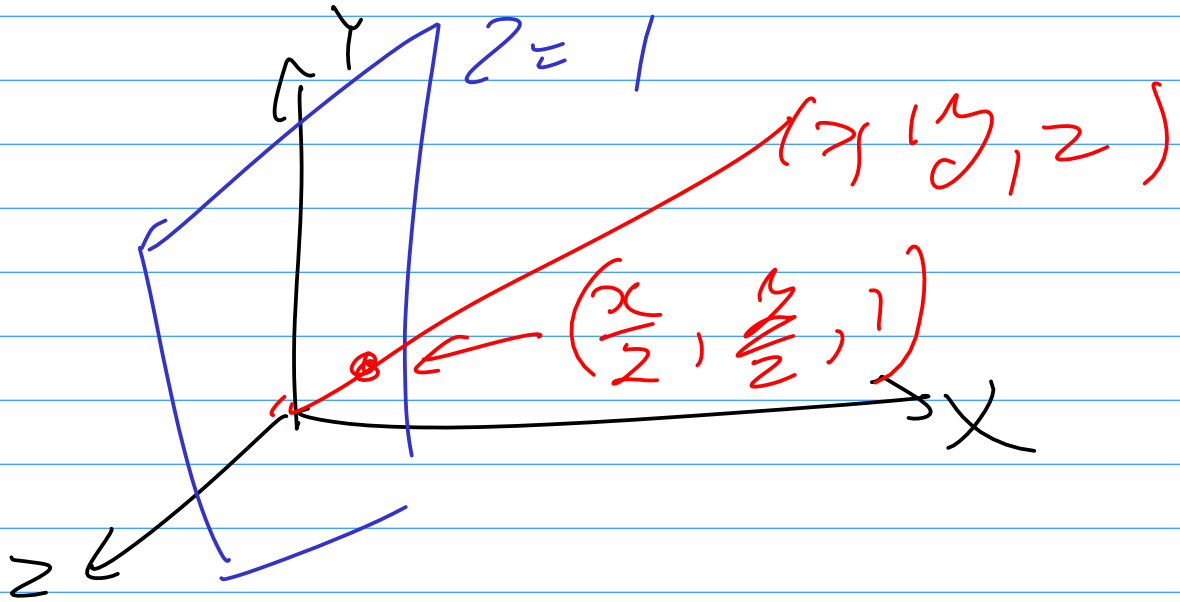
3D CAMERA AT $(0, 4, 0)$

PROJ PLANE $x = 1$

$$(x, y, z) \to \left(1, \frac{y}{x}, \frac{z}{x}\right)$$

SUPPOSE PROJ PLANE $z = 1$

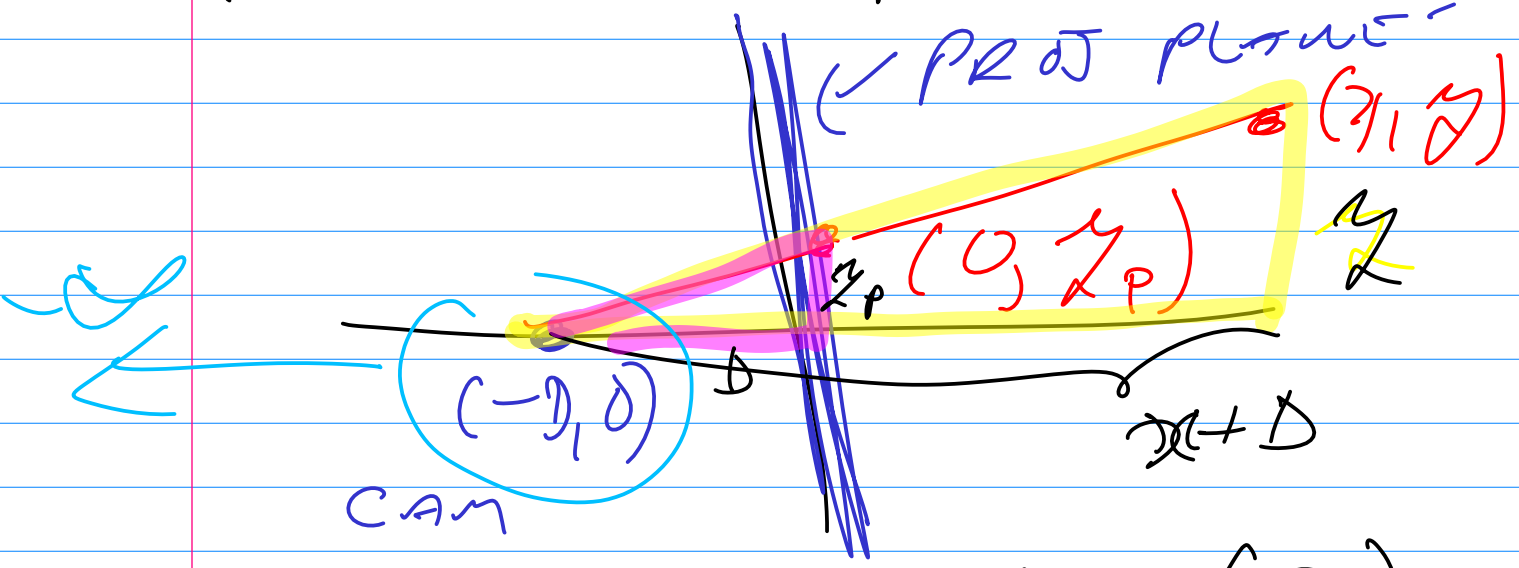
$$(x, y, z) \to \left(\frac{x}{z}, \frac{y}{z}, 1\right)$$



ANOTHER 2D CASE

CAM IS AT $(-D, 0)$

PROJECTION PLANE $x = 0$



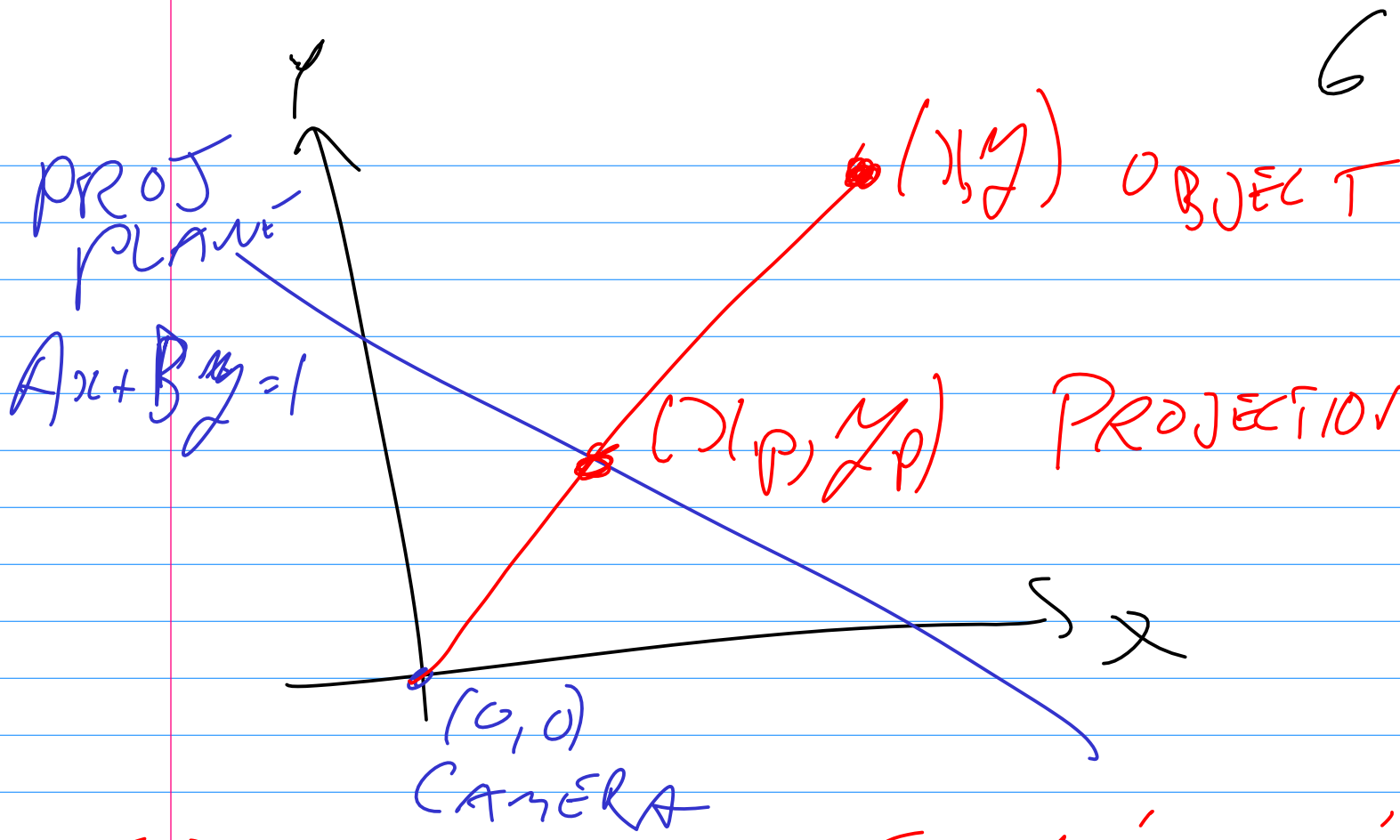
$$\frac{y_p}{D} = \frac{y}{x+D} \Rightarrow y_p = \left(\frac{D}{x+D}\right) y$$

$$= \left(\frac{1}{\frac{x}{D}+1}\right) y$$

IF $D = 1$ $\quad y_p = \left(\frac{1}{x+1}\right) y$

$D = 10$ $\quad y_p = \left(\frac{10}{x+10}\right) y = \left(\frac{1}{\frac{x}{10}+1}\right) y$

AS $D \to \infty$ $\quad y_p \to y$



EQUATION OF LINE: $y' = mx'$

$y' = \left(\frac{y}{x}\right) x'$ $\quad y_p = \frac{y}{x} x_p$

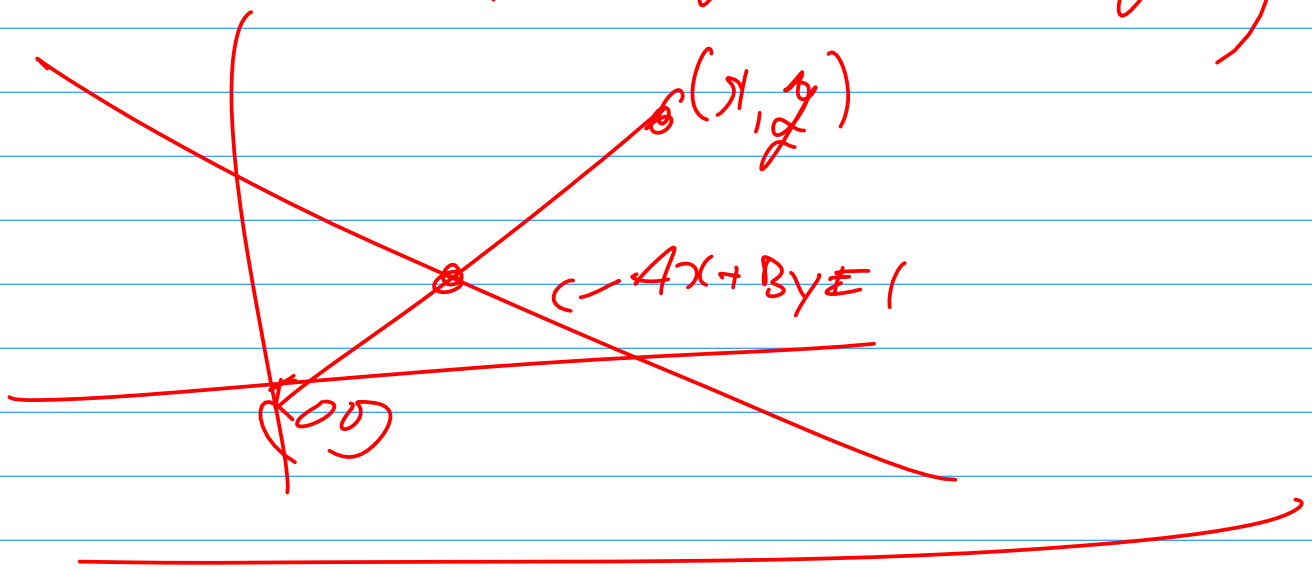
ALSO $Ax_p + By_p = 1$

$Ax_p + B\frac{y}{x} x_p = 1$

$x_p = \frac{1}{A + B\frac{y}{x}} = \frac{x}{Ax + By}$

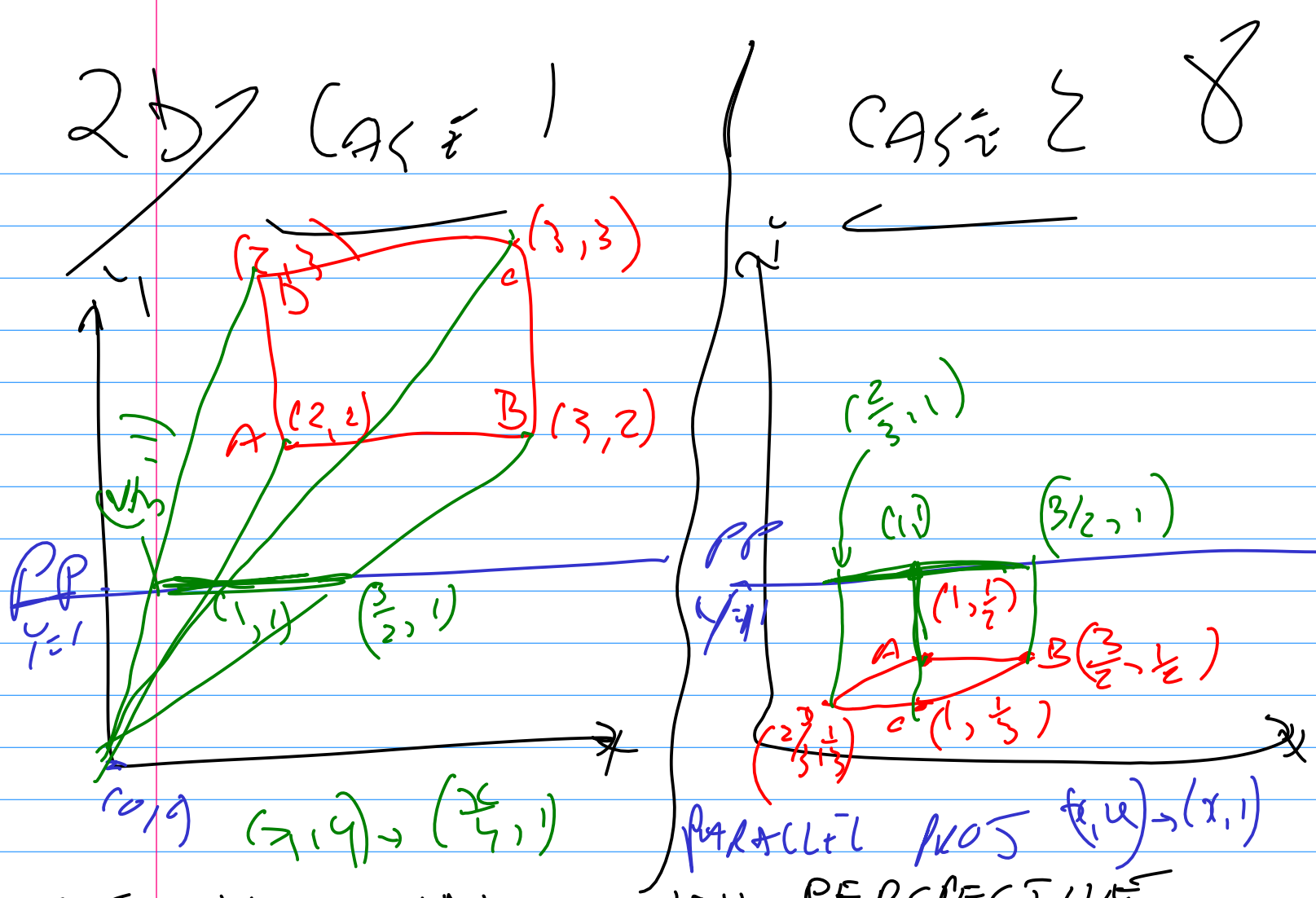
$y_p = \frac{y}{Ax + By}$

$$(x,y) \longrightarrow \left(\frac{x}{Ax+By}, \frac{y}{Ax+By}\right)$$



Q: HOW DOES OPENGL HANDLE PERSPECTIVE PROJECTIONS?

A: IT TRANSFORMS THE WHOLE SCENE TO MAKE IT A PARALLEL PROJECTION



THE ORIGINAL OBJECT WITH PERSPECTIVE PROJECTION PRODUCED THE SAME IMAGE AS THE DISTORTED OBJECT W. PARALLEL.

VIEW NORMALIZATION

TODAY'S BIG IDEA

WHAT IS THAT TRANSFORM?

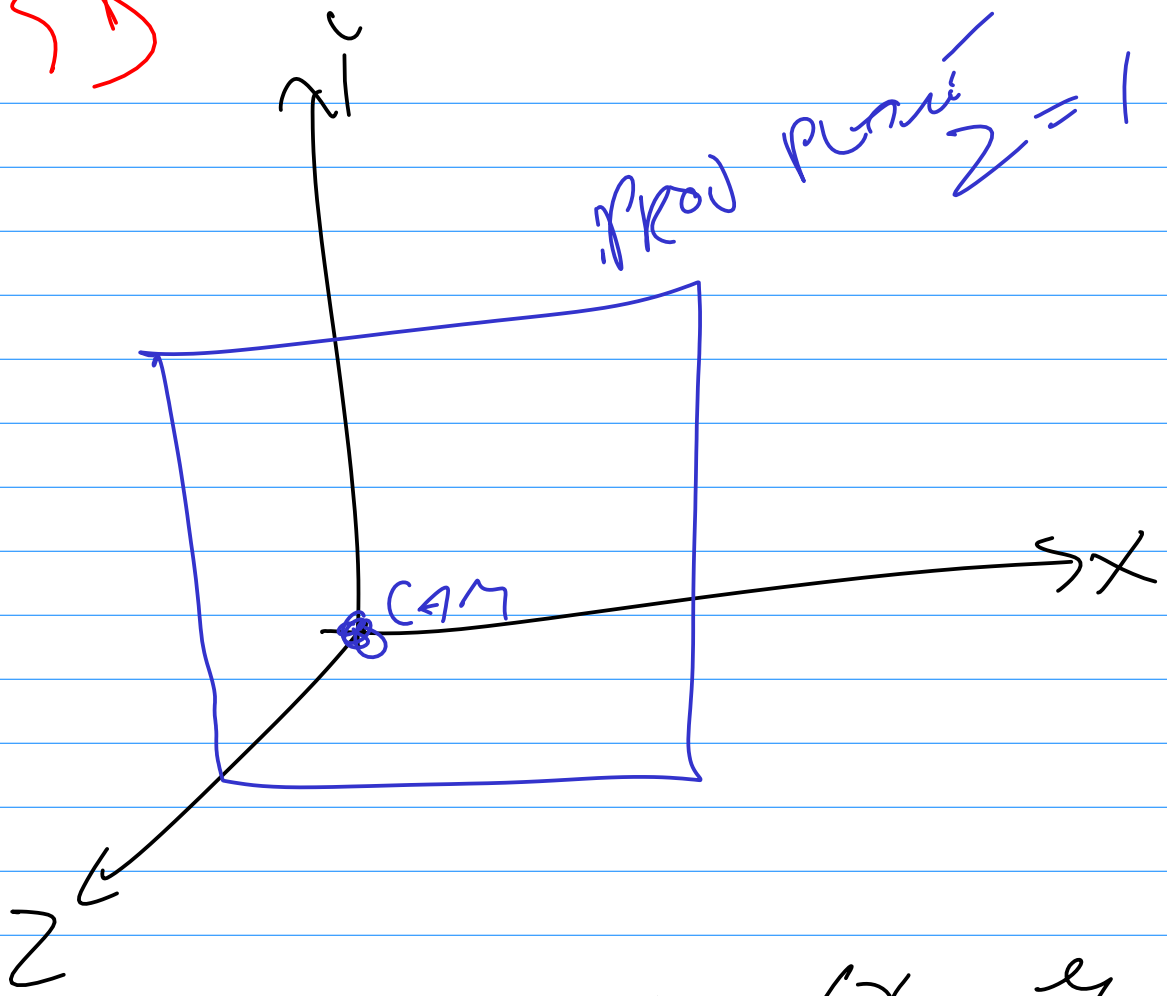
$$(x, y) \rightarrow \left(\frac{x}{y}, \frac{1}{y}\right)$$

IN THIS CASE

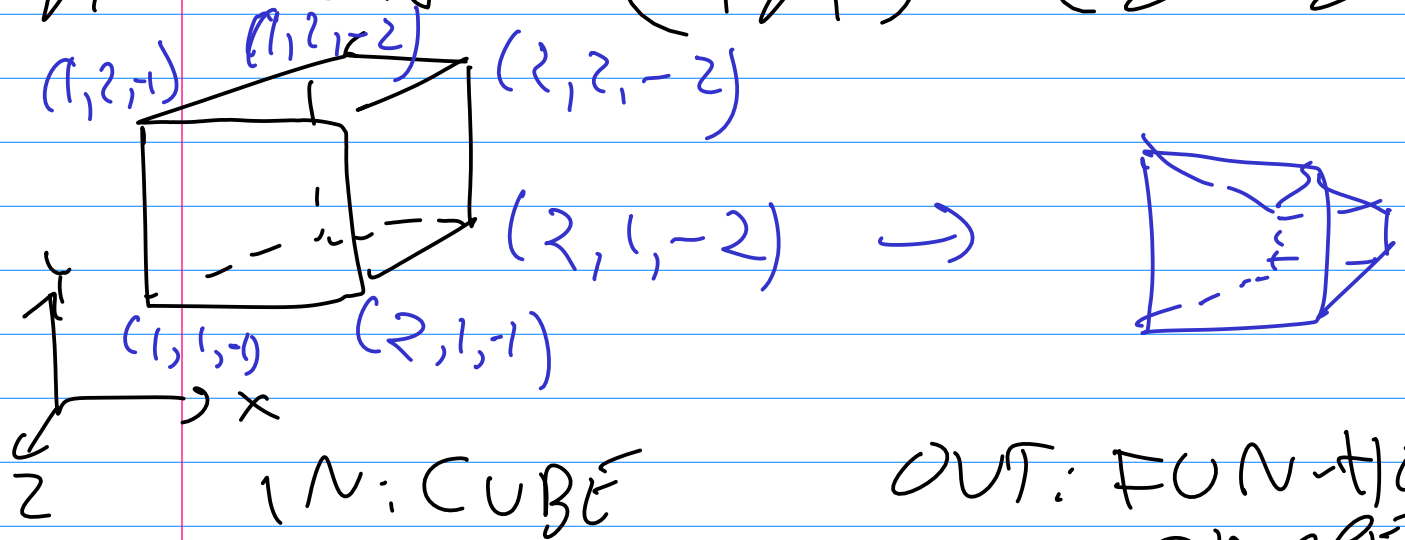
THAT CAN BE DONE WITH A 4x4 MATRIX MULTIPLY W/ HOMOGENEOUS COORDS

3D

9



VIEW NORM: $(x, y, z) \rightarrow \left(\frac{x}{z}, \frac{y}{z}, \frac{1}{z}\right)$



IN: CUBE

OUT: FUN-HOUSE SHAPE

PERSPECTIVE PROJECTION OF CUBE GIVES SAME IMAGE AS PARALLEL PROJECTION OF FUN HOUSE

WE COMBINE MATRIXES FOR OBJECT TRANSFORM, CAMERA SETUP PROJECTION

↓

ALL GOES INTO 1 MATRIX.

$$P' = MP$$

PUTTING POINTS INTO COLOR+ Z BUFFERS IS EASY.

$$(x, y, z) \rightarrow (x, y, z, 1)$$