10/17/2011

TRANSLATE (1, 2, 3)

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

ROTATE 45° ABOUT Z AXIS

$$\begin{pmatrix} .7 & -.7 & 0 & 0 \\ .7 & .7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

10/17/2011

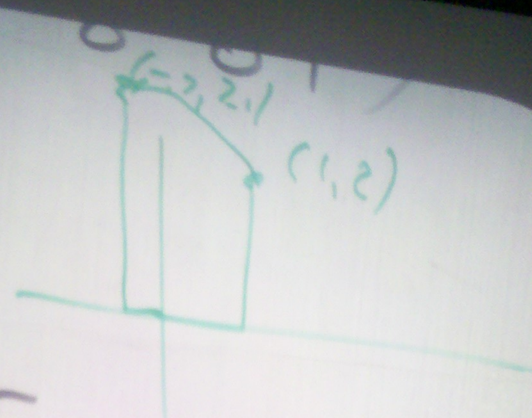
TRANSLATE (1, 2, 3)

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

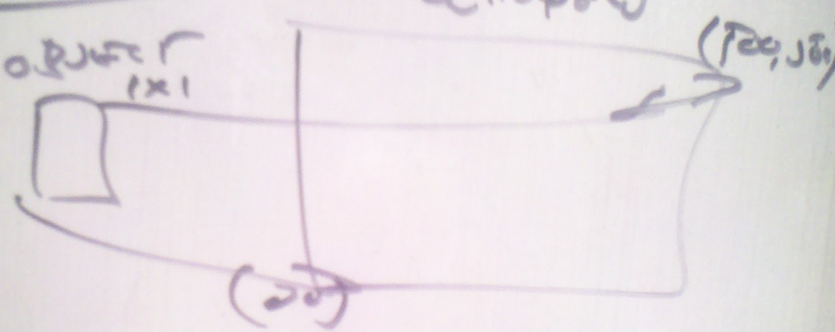
$$\begin{pmatrix} 2 \\ 4 \\ 4 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

ROTATE 45° ABOUT Z AXIS

$$\begin{pmatrix} -.7 \\ 2.1 \\ 1 \\ \end{pmatrix} = \begin{pmatrix} .7 & -.7 & 0 & 0 \\ .7 & .7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$



Scale



Scale by 500

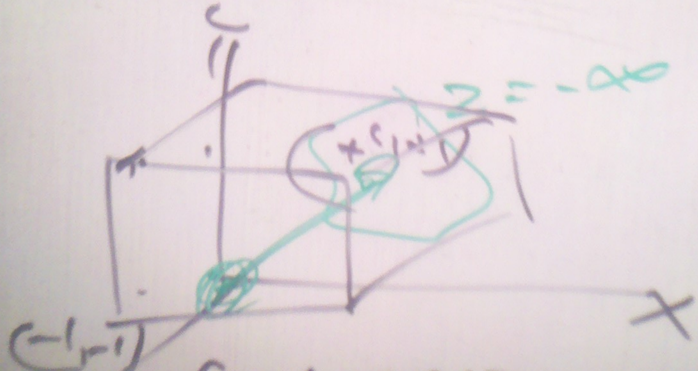
$$\begin{pmatrix} 500 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P' = T_4 S_3 R_2 T_1 P$$

OPENGL RIGHT

MODELVIEW

PROJECTION.



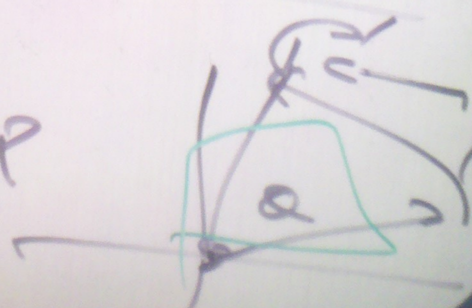
GLU LOOKAT

PROJECTION MATRIX TRANSFORMS OBJECT SO ORIGINAL CLIP VOLUME BECOMES 2×2×2 CUBE ON ORIGIN EVEN FOR PERSPECTIVE

ROTATION

2D EASY

$$p' = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} p$$



~~[illegible]~~ HOW TO ROTATE ABOUT ANOTHER POINT?

TO ROTATE ABOUT (C_x, C_y)

1. TRANSLATE BY $(-C_x, -C_y)$
2. ROTATE
3. TRANSLATE BY (C_x, C_y)

3D 1st DEFINE ROTATION.

A: 1. RIGID TRANSFORM
2. ABOUT ORIGIN.
3. PRESERVES PARITY
HANDEDNESS.

3. PRESERVES PARITY
HANDEDNESS.

ALL 3D ROTATIONS HAVE A LINE OF INVARIANT POINTS - "THE AXIS"

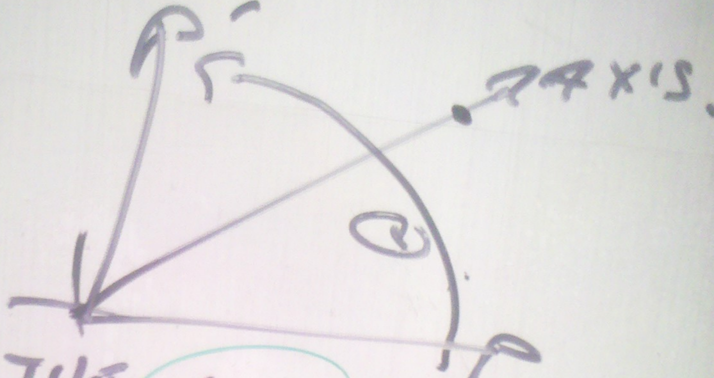
ALL PHYSICALLY POSSIBLE TRANSFORMATIONS ARE A ROTATION + TRANSLATION.

Q HOW TO SPECIFY A 3D ROTATION?

A1: EULER ANGLES.
A2 AXIS + ANGLE.

A1: EULER ANGLES.

A2. AXIS + ANGLE.



GIVE THE AXIS AND ANGLE.

3D POINT, NORMALIZED

ROTATIONS DON'T COMMUTE USUALLY

$$R_1 R_2 \neq R_2 R_1$$

HOW TO GO

How to go

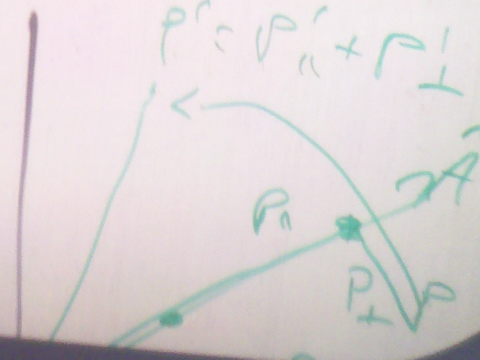
(AXIS, ANGLE) → VECTOR FORMULA

$\downarrow$

3D MATRIX

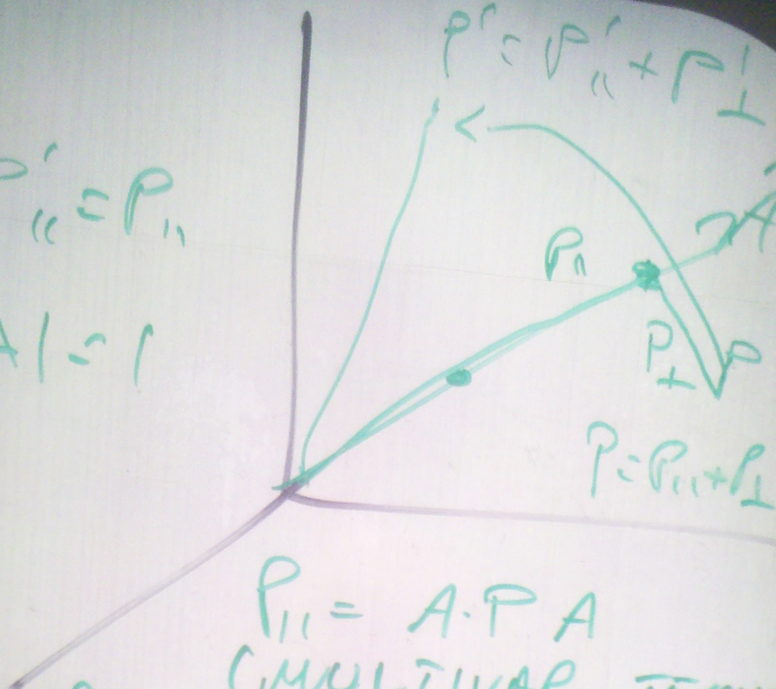
$$p' = p'_{\parallel} + p'_{\perp}$$

$$p'_{\parallel} = p_{\parallel}$$



$P'_{\parallel} = P_{\parallel}$

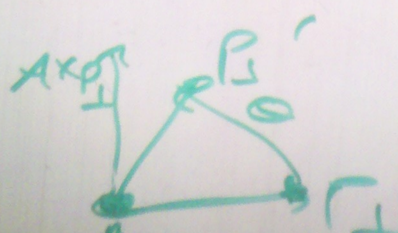
$|A| = 1$



$P_{\parallel} = A \cdot P\, A$

(MULTIVAR TEACHES)

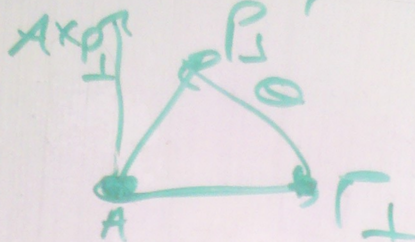
$P_{\perp} = P - P_{\parallel} = P - A \cdot P A$



$$P_{\parallel} = A\cdot P\, A$$

(MULTIVAR TEACHES)

$$P_{\perp} = P - P_{\parallel} = P - A\cdot P\, A$$



$$P_{\perp}' = \cos\theta\, P_{\perp} + \sin\theta\, A\times P_{\perp}$$

$$P' = P_{\parallel}' + P_{\perp}'$$

$$= \cancel{A\times P}$$

$$= A\cdot P\, A + \cos\theta\,(P - A\cdot P\, A)$$

$$+ \sin\theta\,(A\times P - \cancel{A\cdot P\, A\times A})$$

$$: \cancel{(A\cdot P)\,A}$$

$$\boxed{P' = \cos\theta\, P + (A\cdot P)(1-\cos\theta)\,A + \sin\theta\; A\times P}$$

$A = (0,0,1) \quad \theta = 45^\circ$

$\cos\theta = \sin\theta = .7$

$$P' = .7P + P_z(.3)(0,0,1) + (-.7P_y, .7P_x, 0)$$

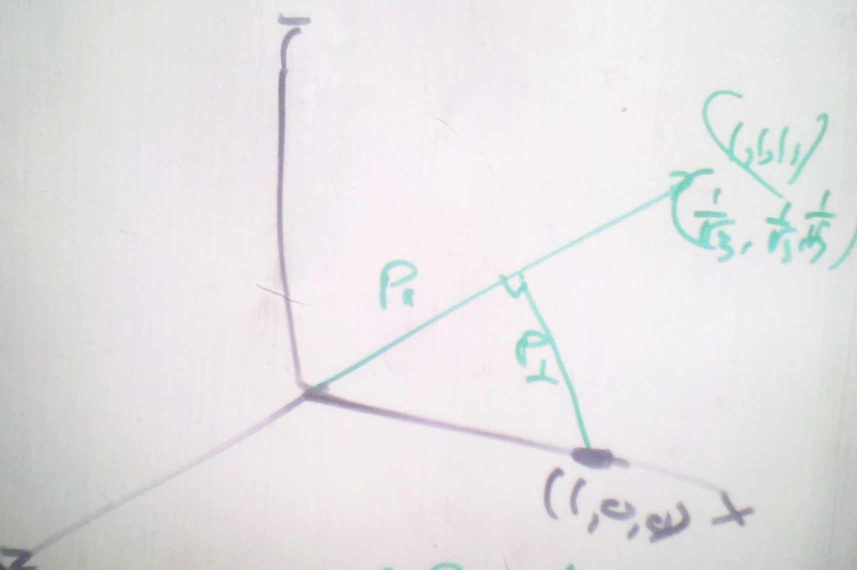
$$A\times P : \begin{pmatrix} i & j & k \\ \cancel{A_x}^{0} & \cancel{A_y}^{0} & \cancel{A_z}^{1} \\ P_x & P_y & P_z \end{pmatrix} = (-P_y, P_x, 0)$$

$$P' : (.7P_x - .7P_y,\ .7P_x + .7P_y,\ P_z)$$

THAT'S A 45° ROTATION

$$: (.7P_x - .7P_y,\ .7P_x + .7P_y,\ P_z)$$

THAT'S A 45° ROTATION IN 2D. IN (X,Y) PLANE



$$A \cdot P = \frac{1}{\sqrt{3}}$$

13

I WANT A MATRIX ROTATION FORMULA. THAT MEANS

I WANT TO TAKE A, θ (AXIS, ANGLE) & COMPUTE A MATRIX M.

APPLYING M ROTATES P.

$$P' = MP$$

eg. $A = (0,0,1)$, $\theta = 45°$

$$M = \begin{pmatrix} .7, & -.7, & 0 \\ -.7, & .7, & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rightarrow 4\}$ NOT 4x4

$\rho_1' = M\rho_1 \qquad \rho_2' = M\rho_2$

$p_1 = p\cos\theta$

$$\begin{pmatrix} p' \end{pmatrix} = \underbrace{\begin{pmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{pmatrix}}\begin{pmatrix} p \end{pmatrix}$$

PART 1 ✓ $M_1 = (\cos\theta) I$

PART 2 I WANT $M_2 \rightarrow$

$\Delta p = \boxed{(a \cdot p)\, a\, (1-\cos\theta)}$

$$M_2 p = \underbrace{\boxed{(a\cdot p)\,a}}_{\text{HARD.}}\underbrace{(1-\cos\theta)}_{\text{EASY}}$$

$$a\cdot p\,a = \begin{pmatrix} a_1^2 & a_1a_2 & a_1a_3 \\ & m_{13} = a_1a_3 & \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

EG $a = (0,0,1)$ $\theta = 45^\circ$

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \cancel{1}.3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ .3 \end{pmatrix}$$

$Mp = a\cdot pa$

$a\cdot p = 1$

$(1-\cos\theta) \to (0$

$Mp = a \cdot pa$

$a \cdot p = 1$

$(a \cdot p) a (1 - \cos\theta) \to \begin{pmatrix} 0 \\ 0 \\ \cdot 3 \end{pmatrix}$

$1 \quad (001) \quad \cdot 3$

I WANT M_3 $\ni$

$M_3 p = a \times p$

$a = (001)$ $\quad M = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$p = (111)$

$\begin{pmatrix} 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} \quad \begin{pmatrix} -1 \end{pmatrix}$

Courtesy

I WANT M_3 ∋

$$M_3 p = a \times p$$

$a = (0\,0\,1)$

$$M = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$p = (1\,1\,1)$

$$Mp = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$a \times p = \begin{pmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$