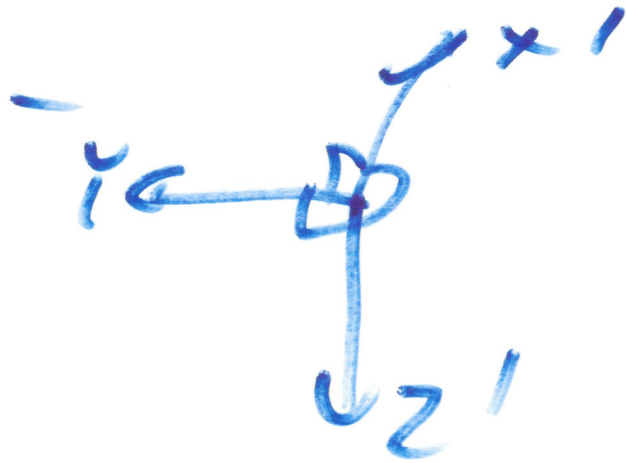
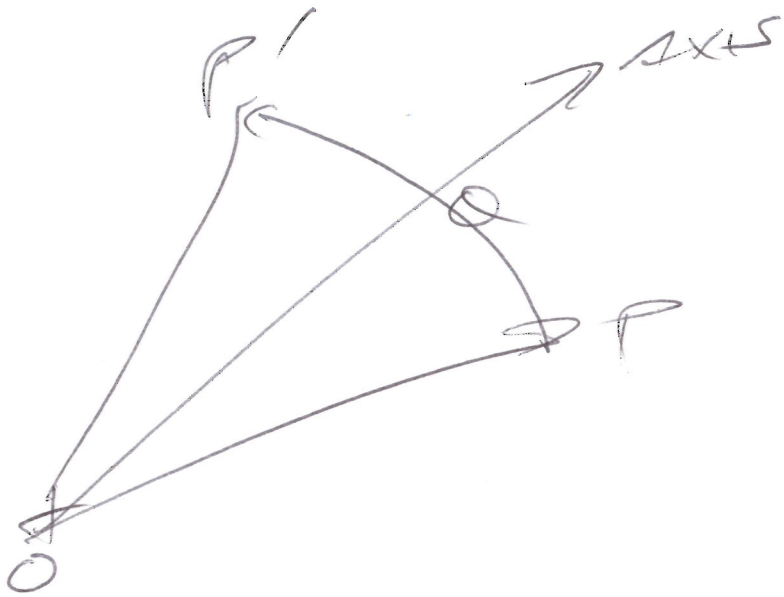


10/6/10 LECTURE #14 - 1



2

$$\begin{aligned} & c + (1-c)a_1^2 + c + (1-c)a_2^2 + c + (1-c)a_3^2 \\ &= 3c + (1-c)(\cancel{a_1^2 + a_2^2 + a_3^2}) \\ &= 3c + 1 - c = 1 + 2c \end{aligned}$$

$$.07 + .29 + .64 = 1$$

$$\cos \theta = 0$$

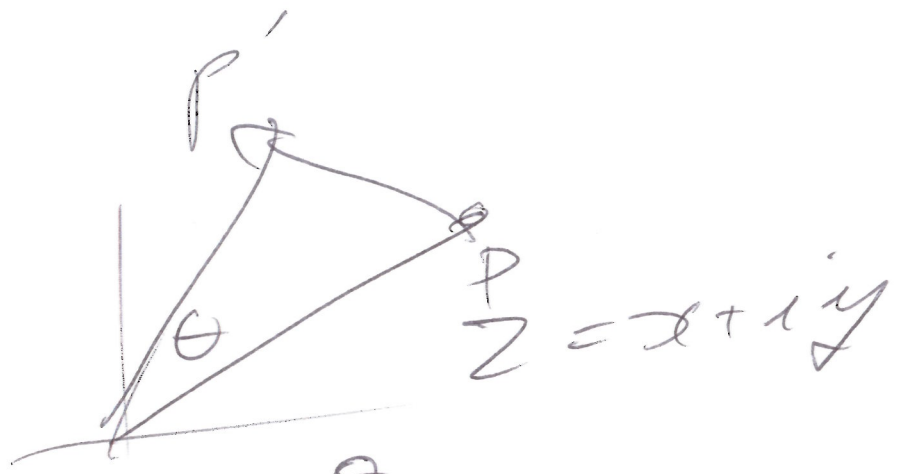
$$\theta = 90^\circ \text{ or } 270^\circ$$

M P

M⁺₁₀₀ P



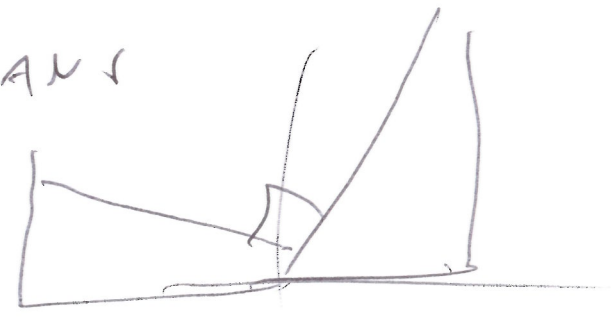
2D



$$z = x + iy$$

$$P' = P e^{i\theta}$$

θ - RADIANS



$$\theta = 90^\circ = \frac{\pi}{2} \quad e^{i\theta} = i$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

TO ROTATE $x+iy$ BY 90° , MULTIPLY BY i

$$(3, 4) \quad 3+4i \rightarrow 3i+4i^2 = -4+3i$$

$$\textcircled{C} \quad \underline{x+iy} \quad i^2 = -1$$

5-

$$H: \quad a + \underline{b}i + \underline{c}j + \underline{d}k \quad \{ = 1 + 2i + 3j + 4k$$

$$(1 + 2i + 3j + 4k) + (5 + 6i + 7j + 8k)$$

$$= (6 + 8i + 10j + 12k)$$

$\times 3$

$$\rightarrow \underline{(18 + 24i + 30j + 36k)}$$

$$i j k = k k = -1$$

$$q_1 = 1 + 2i$$

$$q_2 = 3i + 4j$$

$$q_1 q_2 = 3i + 4j + 6i^2 + 8ji$$

$$\underline{q_1 q_2} = 3i + 4j - 6 + 8k$$

$$= -6 + 3i + 4j + 8k$$

$$\underline{q_2 q_1} = 3i + 6i^2 + 4j + 8ji = -6 + 3i + 4j - 8k$$

ROT BY 180° ABOUT X AXIS

$$a = (1, 0, 0)$$

$$\frac{\theta}{2} = 90^\circ \quad c = 0 \quad s = 1$$

$$q = 0 + 1(1 + 0j + 0k) = i$$

$$q^* = -i$$

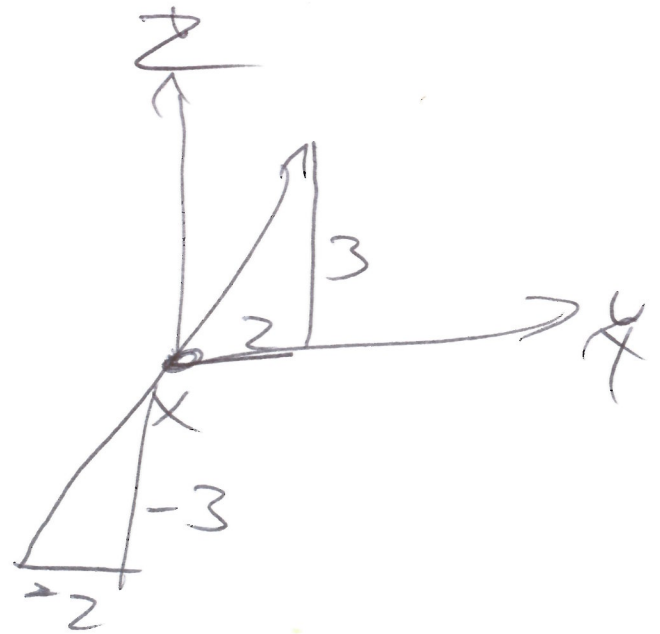
$$P' = r P (-r)$$

POINT IS $(1, 2, 3)$

$$P = 1 + 2j + 3k$$

$$\begin{aligned} P' &= r(1 + 2j + 3k)(-r) \\ &= (-1 + 2k - 3j)(-r) \\ &= (1 - 2j - 3k) \end{aligned}$$

$$P' = (1, -2, -3)$$



2 ROTATIONS

1st: 180° ABOUT Z AXIS $q_1 = k$
 2nd: " X $q_2 = i$

$$\underbrace{q_2 q_1}_{\text{}} P \underbrace{q_1^* q_2^*}_{\text{}}$$

$$q_2 q_1 = ik = -j$$

$$P' = -j P j = 180^\circ \text{ ABOUT } -Y$$

THIS SHOWS HOW TO COMBINE 2 ROTATIONS
 + FIND ONE ROTATION = COMBO

$$\hat{g} = -\hat{i} + -\hat{j} = \cos\frac{\theta}{2} + \sin\frac{\theta}{2} (\hat{e}_x + \hat{e}_y + \hat{e}_z) \quad \text{[D]}$$

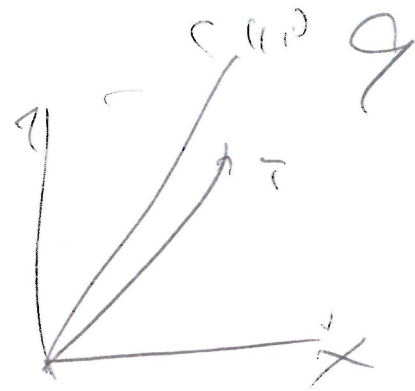
$$\cos\frac{\theta}{2} = -1 \quad \frac{\theta}{2} = \pm 45^\circ \quad \theta = \pm 90^\circ$$

$$\frac{\theta}{2} = 45^\circ \text{ or } 135^\circ \quad \theta = \pm 90^\circ \text{ or } 270^\circ$$

$$\theta = 90^\circ \quad \text{Axis is } (0, 1, 0)$$

120° ABOUT (1, 1, 1)

$$q = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$



$$\frac{\theta}{2} = 60^\circ \quad \cos 60^\circ = \frac{1}{2} \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$q = (\cos) \frac{\theta}{2} + \sin \frac{\theta}{2} (q_x i + q_y j + q_z k)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} \left(\frac{1}{\sqrt{3}} i + \frac{1}{\sqrt{3}} j + \frac{1}{\sqrt{3}} k \right) = \frac{1}{2} (1 + i + j + k)$$

$$q^* = \frac{1}{2} (1 - i - j - k)$$

WHAT DOES X AXIS ROTATE TO?

$$p = (100) = i$$

$$p' = q p q^* = \frac{1}{2} (1 + i + j + k) \frac{1}{2} (1 - i - j - k)$$
$$= \frac{1}{4} (1 - 1 - k + j) (1 - 1 - j - k)$$

$$= \frac{1}{4} (\cancel{1} + \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + \cancel{1} + \cancel{1} + \cancel{1} - \cancel{1} - \cancel{1} - \cancel{1} - \cancel{1} + \cancel{1} + \cancel{1} - \cancel{1} - \cancel{1})$$

(10)

$$= \frac{1}{4} (1 + 1 + 1 + 1)$$

$$= 1 \rightarrow (0, 1, 0)$$

(100) ROTATES TO (010)

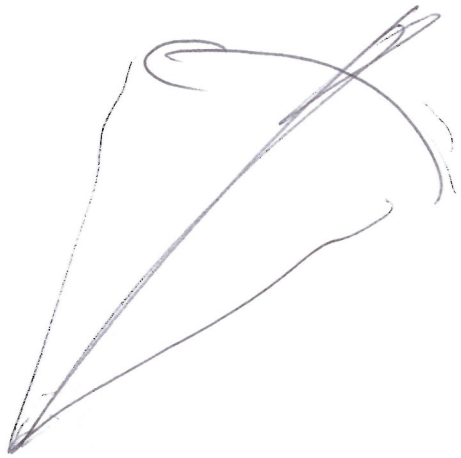
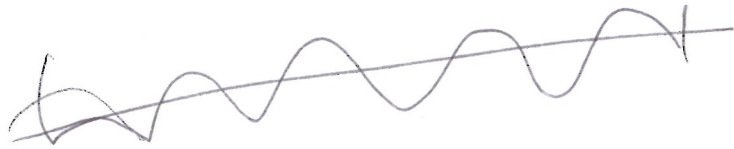
ANIMATE 120° ABOUT $(1, 1, 1)$ IN 120 STEPS

$$\theta = 1^\circ \quad 120 \text{ TIMES}$$

$$g = \cos \frac{1}{2}^\circ + \sin \frac{1}{2}^\circ \left(\frac{1 + j + k}{\sqrt{3}} \right)$$

EIGENVALUES ENRICHMENT

11



3x3 MATRIX

1) $\cos \theta \pm i \sin \theta$

$$Mv = \lambda v$$

CHANGES LEN
NOT DIR