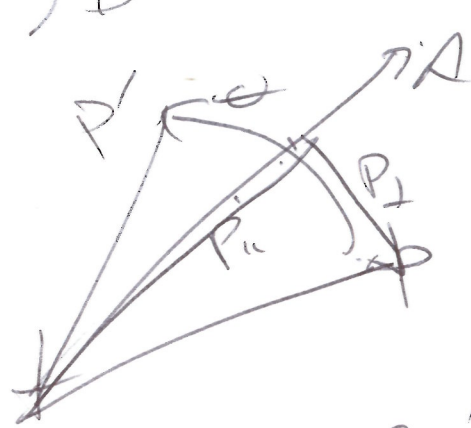


3D ROTATION

10/4/10 = 1



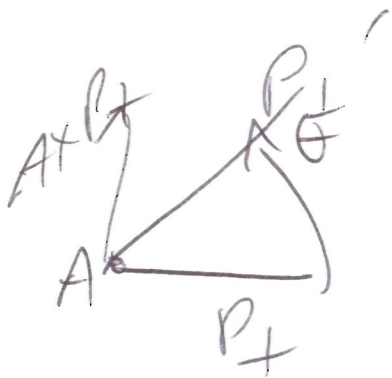
$$|A| = 1$$

$$P = P_{||} + P_{\perp}$$

$$P_{||} = A \cdot P \cdot A$$

$$P'_{||} = P_{||}$$

$$P_{\perp} = P - P_{||}$$



$$P'_{\perp} = \cos \theta P_{\perp} + \sin \theta A \times P_{\perp}$$

$$P = (3, 2, 1)$$

$$\theta = 45^\circ$$

$$a = (1, 0, 0)$$

$$|a| = 1$$

$$P_{||} = (3, 0, 0)$$

$a \cdot P \cdot a$
3

$$P_{\perp} = (0, 2, 1)$$

$$a \times P_{\perp}$$

$a \times P_{\perp}$

(100)

(021)

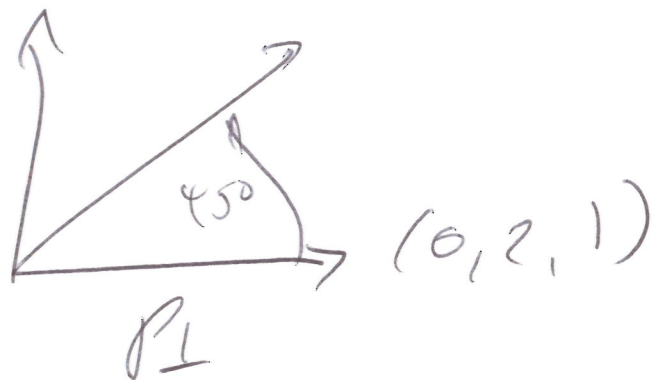
$(0, -1, 2)$

$= (0, -1, 2)$

$$\cos 45^\circ = .7$$

$$\sin 45^\circ = .7$$

10/4/10 2



$$P'_{\perp} = .7 P_{\perp} + .7 a \times P_{\perp} = (0, -1, 2.1)$$

$$P' = (3, 0, 0) + (0, -1, 2.1) = (3, -1, 2.1)$$

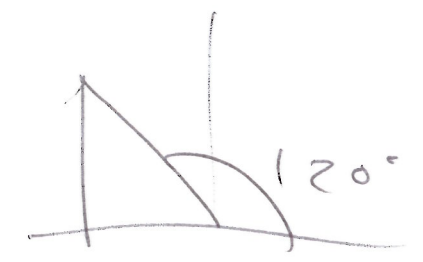
10/4/10-3

$A(1,1,1) \rightarrow (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

$|A| = \sqrt{3}$

$\cos 120^\circ = -\frac{1}{2}$

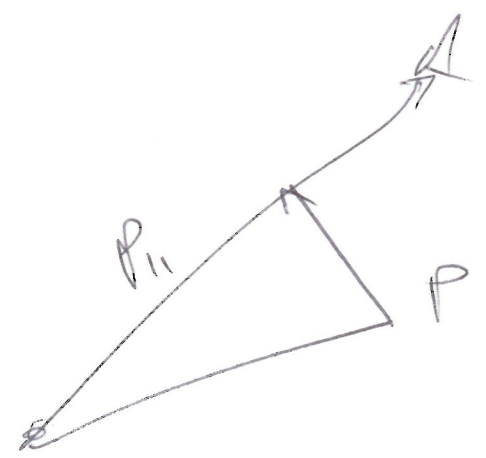
$\sin 120^\circ = \frac{\sqrt{3}}{2}$



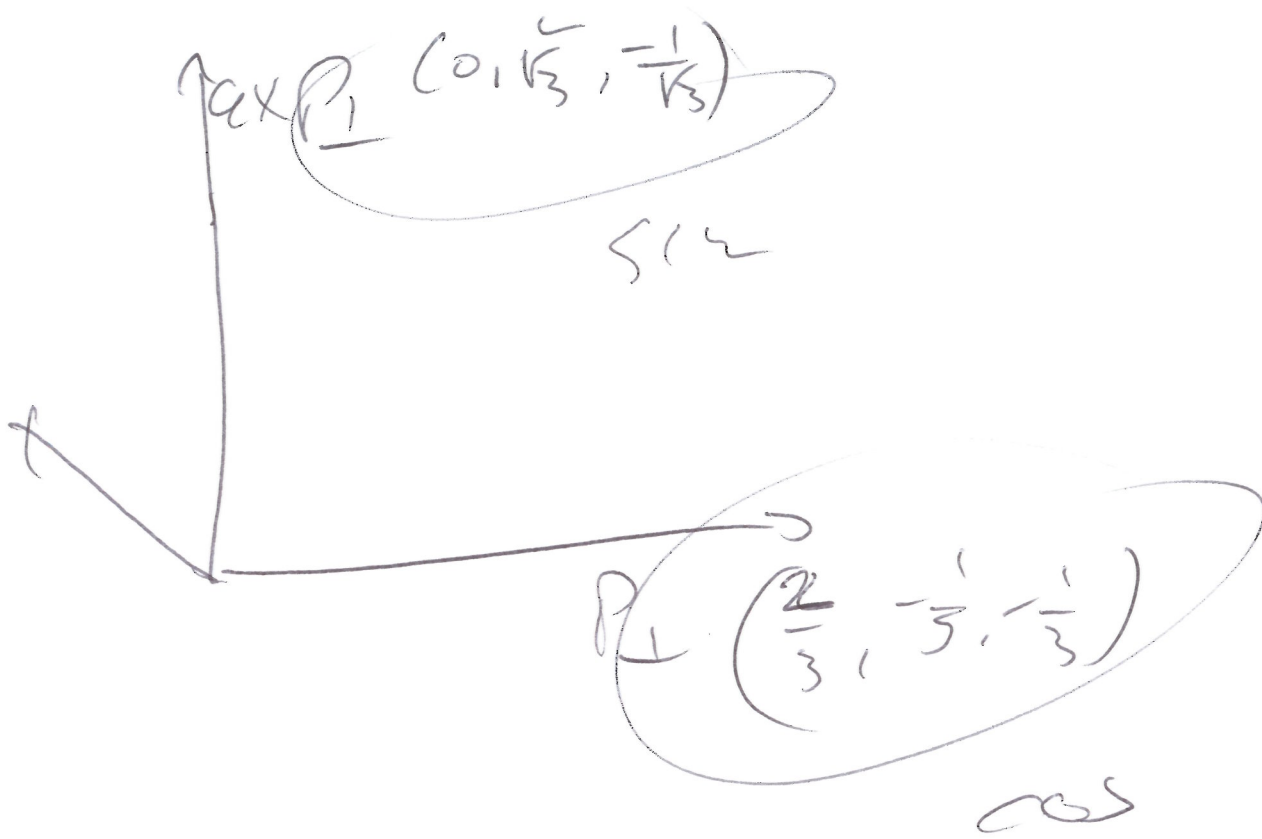
$P = (1, 0, 0)$

$P_{||} = \frac{1}{\sqrt{2}} a = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

$P_{\perp} = (\frac{2}{3}, \frac{1}{3}, -\frac{1}{3})$



10/4/10-4



WANT A MATRIX $M_{a,\theta}$

$$P' = M_{a,\theta} P$$

1) want $M_1 \Rightarrow$

10/4/10 - 5

$$M_1 P = P \cos \theta$$

$$M_1 = \begin{pmatrix} \cos \theta & 0 & 0 \\ 0 & \cos \theta & 0 \\ 0 & 0 & \cos \theta \end{pmatrix} = \cos \theta \mathbf{I}$$

2) want $M_2 \Rightarrow$

$$M_2 P = a - P a (1 - \cos \theta)$$

$$a = (1, 2, 3)$$

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\underline{M_p} = \underline{a \cdot p a}$$

$$= \begin{pmatrix} 27 \\ 54 \\ 81 \end{pmatrix}$$

$$p = (8, 8, 1)$$

$$a \cdot p = 8 + 16 + 3 = 27$$

$$a \cdot p a = (27, 54, 81)$$

$$p = (3, 5, 6)$$

$$a \cdot p = 3 + 10 + 18 = 31$$

$$a \cdot p a = (31, 62, 93)$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 31 \\ 62 \\ 93 \end{pmatrix} \swarrow$$

6

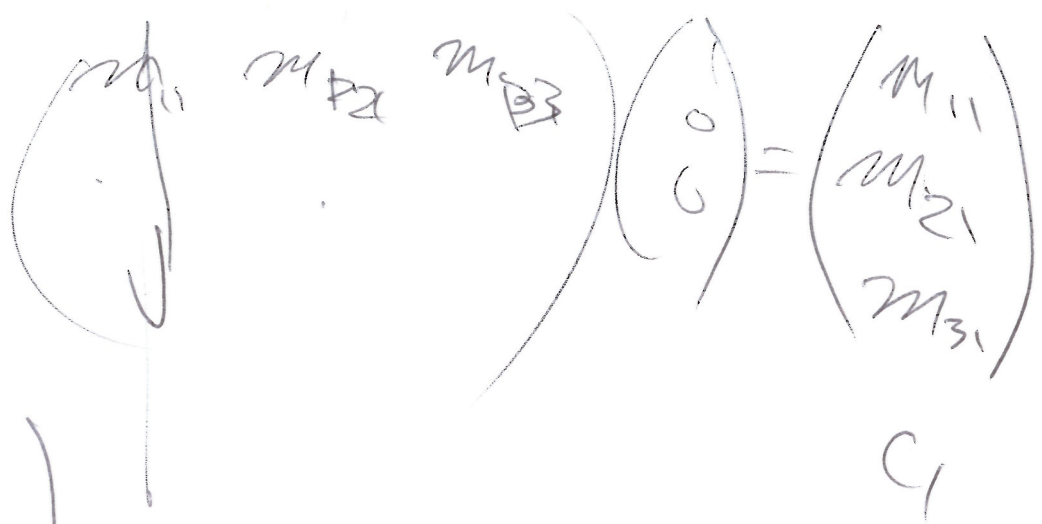
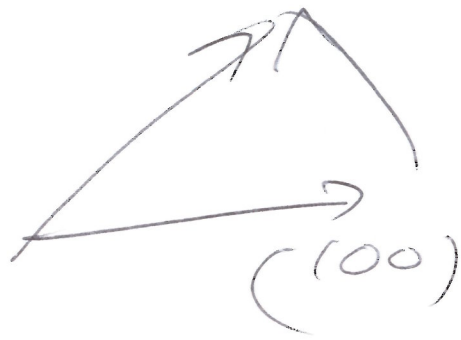
$$M = \begin{pmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 8 \\ 8 \\ 1 \end{pmatrix} = \begin{pmatrix} -22 \\ 23 \\ -8 \end{pmatrix}$$

7

$$9 \times P = \begin{matrix} 1 & 2 & 3 \\ \times & 8 & 8 & 1 \end{matrix} = (-22, 23, -8)$$

$$P = \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix} \begin{matrix} 1 & 2 & 3 \\ 3 & 5 & 6 \end{matrix} = (-3, 3, -1)$$

$$\begin{pmatrix} 6 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -1 \end{pmatrix}$$



$$|C_1| = 1$$

$$C_1 \cdot C_1 = 1$$



$$C_1 \cdot C_2 = 0$$

$$|C_2| = 1$$

$$C_1 \cdot C_3 = \delta_{13}$$

$$\begin{matrix} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{matrix}$$

$$|C_3| = 1$$

$$\det(M) = \neq 1$$