Constructing a DEM from Grid-based Data by Computing Intermediate Contours

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ABSTRACT

We present a technique for creating a digital elevation model (DEM) from grid-based contour data. The method computes new, intermediate contours in between existing isolines. These are found by finding the shortest line segment that connects points on two neighboring contours with differing elevations. The midpoint of the line segment becomes a point on the intermediate contour. The contours are completed by connecting individual points. The new contours are then used as data for successive iterations, until an initial surface is formed. Peaks are computed by Hermite splines that follow the slope trend. Gaussian smoothing is applied to the entire surface or only to newly computed elevations, yielding an approximated or interpolated surface, respectively. The DEMs are tested with quantitative methods, and are shown to compare favorably to well established algorithms.

Categories and Subject Descriptors

J.2 [**Computer Applications**]: Physical Sciences and Engineering

General Terms

Algorithms

Keywords

Contours, interpolation, approximation, DEM, grid, GIS

1. INTRODUCTION

Geographical Information Systems (GIS) are becoming increasingly popular for visualizing spatial data. Most systems layer patterns or colors, which depict data such as soil type, roads, and the like, over a two-dimensional map. As technology continues to improve, users increasingly expect to view such data in three-dimensions, as is now done in ArcView and MapInfo. The user can then view the desired data in the context of the surrounding topology.

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A Digital Elevation Model (DEM) is often used to store three-dimensional elevation data via a regular grid. DEMs are often generated from sparse data because they are storage intensive and/or it is difficult to obtain a desired area. We compute DEMs from isoline data because contour maps are readily available from many sources. One such source is the United States Geological Survey (USGS) which supplies contour maps from many areas in the form of Digital Line Graphs (DLG). We use a grid-based approach because such methods often produce DEMs that preserve terrain morphology better than others, such as those using a Triangulated Irregular Network (TIN)[17]. Examples of systems that generate DEMs from contours are TOPOGRID [16], available in ArcInfo, TAPES-C [5], and TOPOG [6].

In this paper, we describe a technique that computes intermediate contours in between the original isolines. A preliminary surface is formed by computing successive intermediate contours by using data from the previous iteration. Peak (and pit) areas, where intermediate contours can not be generated, are computed by applying Hermite splines; any remaining gaps are filled by inverse distance weighting using elevation points found in each of the four cardinal directions. The final surface is smoothed by one of two methods, yielding either an interpolated or an approximated surface. The final DEMs are shown to compare favorably with those computed with established methods.

2. PREVIOUS WORK

There are many ways to interpolate or approximate a surface. This paper describes a method that uses the information inherent in contour data. One such approach finds the average between two linearly interpolated profile lines, one oriented north-south, the other E-W; this often leads to overestimation [22]. Weighted averaging methods, such as inverse distance weighting, are discussed in [29] and [13]; these often use natural neighbors [24]. To reduce artifacts in these methods, one may first detect ridges and valleys, interpolate them, and then apply any other method on the enhanced data [14]. Another method is to compute flow lines between contours forming rectangular elements from which elevations are then interpolated [18]. The skeleton extraction technique forms new contours within the original data that can then be used to create a DEM [12]. This idea is taken one step further in [27], where new contours are created at the intersection of the dilation of adjacent contours; this is repeated until the entire surface is filled. Instead of using information along a contour, one can find the steepest slope perpendicular to a contour. Steepest slope chains are found in [1], which are then interpolated by a cubic Hermite function. A similar idea is described in [8], although the steepest slope computation is done differently.

Contours can also used as data for constructing a surface using smoothing splines [7]. There are many minimum curvature (thin plate) methods, based on the early work of Briggs [3], the best known implementation of which is TOPOGRID [15], available in ArcInfo. In addition to including a roughness penalty in the thin plate equations, the method first computes ridge lines and streams, creating a more accurate DEM. There are many other methods based on the thin plate spline, including [28], in which a smoothing term is added to the minimum curvature equations, resulting in an approximated, but smooth, surface; in [25], a two-stage method that computes an initial surface using median squares regression and then applies the minimum curvature equation; and [26], which incorporates a tension parameter to minimize discontinuities.

Another popular method for creating DEMs is through the use of the Triangulated Irregular Network (TIN), first implemented in cartography by Franklin [10] following the ideas of Peucker and Douglas [19]. A TIN can be computed from contours, using methods described in [11] and improved in [31].

3. MAXIMUM INTERMEDIATE CONTOURS METHOD

A property of contour lines is that, in general, successive isolines run approximately parallel to one another. The way a cartographer might create a surface is to iteratively generate new contours in between those already existing. This is done by drawing a new contour midway between successive contours. The Maximum Intermediate Contours (MIC) algorithm computes new, in between contours in much the same way. By finding data points in successive contour lines that are closest to one another, we can find a midpoint between the two contours. Repeating this process for all points along one contour will create a series of points that, when connected, form a new contour. Because the contour interval is constant and because elevations in between contours usually do not deviate from the slope defined by the contours, the new intermediate contour is assigned an elevation that is exactly midway between the elevations of the original contours on either side. The MIC method computes new contours in this manner until a surface is formed.

Figure 1: A portion of contour map.

The complete MIC algorithm is as follows, where $e =$ elevation:

Perform linear interpolation along boundaries
Repeat
    For all points P on contour lines
        Choose a point P_1 from one contour line A
        Find the closest point P_2 on contour line B
            such that $P_{2e} > P_{1e}$ *(a)*
        Determine the midpoint P_m between P_1 and P_2
        Calculate elevation: $P_{m\ e} = \frac{1}{2}(P_{1e} + P_{2e})$
        Connect the points to form a continuous contour
    End for
Until all intermediate contours are complete
Fill gaps with inverse distance weighting
Compute peaks/pits with Hermite curves
Apply Gaussian interpolation or approximation smoothing

The condition shown in line *(a)* that the elevations of neighboring contours differ assures that a new contour will have an elevation whose value is between the heights of its neighbors. This condition assures that peaks/pits will not be flat and it prevents the formation of a peak in a saddle area; these areas are computed in a later stage in the algorithm. Bresenham's circle algorithm [2], which finds the discrete grid points that are closest to the true circle, is employed to find the closest point P_2 from P_1. We generate circles with successively larger radii from P_1 until the circle contacts a point P_2, which has an elevation value higher than P_1's. The midpoint of the line that connects P_1 and P_2 is then found and assigned an elevation in between the elevations of the two contours.

Figure 2: One iteration of intermediate contours.

Consider Figure 1, which shows a portion of our test case. Notice the saddle area; the small contour due east has a lower elevation than its neighbors. The intermediate contour generated in the area between the small contour and its neighbors will not close, because a) the neighbors above (and below) are closer than the contour to the west, and b) the contour to the west has the same elevation, and so can not be considered a neighbor. Therefore, the algorithm closes such contours, albeit in a simple manner by connecting them with a line segment. This could be improved in the future. Figure 2 shows the final intermediate contours.

The above procedure for computing intermediate contours is repeated until an initial surface is created. The number of iterations necessary is approximately $\log_2 D$, where D is the average distance between contours. Computing only intermediate contours is not sufficient to produce a DEM; there may be small gaps in the computed surface because the method assumes that successive, increasing elevation contours have a convex shape, generally circling a local maxima. This assumption does not hold near the edges of the grid, where the next contour may be outside of the current

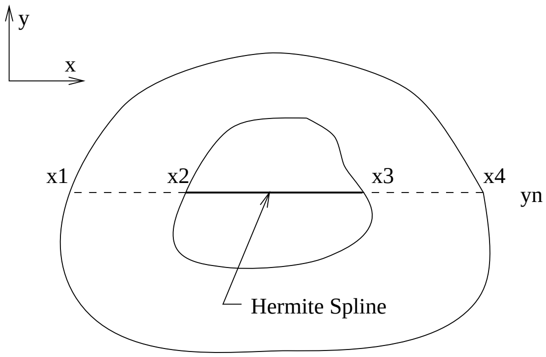

Figure 3: Computation of peak area using Hermite spline

computation area. To reduce the gaps along the edges of the grid, a one-dimensional boundary interpolation is first performed along each edge of the grid. Inverse distance weighting is used to fill in any remaining gaps. Additionally, peaks (or pits) are not computed in the intermediate contours stage at all, because there is only one contour encircling a local maxima (minima) and thus no way to compute an intermediate contour. These areas are interpolated by cubic Hermite curves [9] following the slope across the maximum (minimum) contour (Figure 3):

$$Q(t) = (2t^3 - 3t^2 + 1)x_2 + (-2t^3 + 3t^2)x_3 + (t^3 - 2t^2 + t)R_2 + (t^3 - t^2)R_3 \tag{1}$$

where

$$R_2 = \frac{z_{x_2,y_n} - z_{x_1,y_n}}{x_2 - x_1} \tag{2}$$

is the tangent vector on the left side, and

$$R_3 = \frac{z_{x_4,y_n} - z_{x_3,y_n}}{x_4 - x_3}. \tag{3}$$

is the tangent vector on the right side. The parameter $t = [0, 1]$ and is computed as $t = 1.0/(x_3 - x_2)$.

The Hermite spline follows the direction of the tangents formed by the contours on either end of a peak (or pit) ensuring that the connection between the peak (pit) area and the rest of the surface is smooth. Although this method has given us good results, peaks may be computed using a more complex method such as shown in [23].

The final surface is found by smoothing the surface. This is necessary because the intermediate contours may not be optimal or may produce small artifacts, as when a line segment closes a contour as shown in Figure 2. The elevation value of a point is the weighted average of its neighbors in each of the four cardinal directions, where the weight is based on a Gaussian distribution of distance. The smoothing can be done as an interpolation or approximation; in the former, the original contour data is left unchanged, whereas in the latter, the contour data may be altered in the smoothing operation. Also, in the case of the approximation, the smoothing function tends to planeness, so care must be taken to find a good compromise between smoothness and accuracy of the surface.

4. EVALUATION CRITERIA

The criteria used to assess the quality of a computed DEM are as follows:

1. In general, the surface should look reasonably realistic with minimal artifacts. A shaded relief map is the conventional way to assess the surface visually [30]. Although this is a qualitative measure, it is very useful and may show artifacts that are not discovered easily through quantitative tests.

2. The total squared curvature must be as low as possible, indicating a smooth surface. Although natural surfaces exhibit some curvature, artifacts such as "stepping" of neighboring contours, contribute greatly to the total curvature. For $N = n^2$ total points, this is found by comparing each computed elevation value to its four neighbors [3]:

$$C_{sq} = \sum_{i=2}^{n-1}\sum_{j=2}^{n-1}(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j})^2 \tag{4}$$

where each u represents the elevation at the grid location indexed by i and j.

3. Because small local imperfections may bias the total squared curvature, an average absolute curvature of the surface is computed as well:

$$C_{ave} = \frac{1}{(n-2)^2}\sum_{i=2}^{n-1}\sum_{j=2}^{n-1}|(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j})| \tag{5}$$

4. DEM elevations falling on the original contour lines must have values equal to (interpolation) or almost equal to (approximation) the contour labels [4], measure by the root mean square error (*RMSE*) [21]:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(u_i - w_i)^2} \tag{6}$$

where u_i = the interpolated DEM elevation of test point i
w_i = the true elevation of test point i

In this paper, the *RMSE* refers to the error of the surface compared to the original contour map. Following [4], an acceptable difference between a computed point and the contour elevation is five per cent of the contour interval. An *RMSE* of zero indicates a true interpolated surface.

5. Within an area bounded by a contour pair, the DEM elevations should vary almost linearly. Although this is not true in all cases, in general a linear fit between contours indicates a constant slope and thus the absence of terracing artifacts. Elevations are grouped into integer intervals between two contours, and then reclassified into relative elevations [4][20]. For example, if a contour pair were 100-120, then the relative elevations, or height classes, would be 0, 1, 2, ..., 19 corresponding to the actual elevations of 100, 101, 102, ..., 119. The height classes are computed for each elevation pair and then displayed as a histogram. A flat histogram indicates a smooth surface and a good linearity between the contours, while other patterns show various artifacts resulting from the particular interpolation or approximation method.

5. RESULTS

The MIC method was tested with an 800×800 grid of contours taken from a USGS DLG of Mt. Washington, NH, shown in Figure 4. The elevations are in meters and the contour interval is 20 meters. For comparison purposes, a DEM was also computed using ArcInfo's TOPOGRID. The surface computed by each method is shown using a shaded relief map. Quantitative results are shown in Table 1.

Figure 5 shows the result of one iteration of intermediate contours applied to the data in Figure 4. These contours are then used as data for subsequent iterations that produce all of the MIC results. Figure 6 shows the DEM created by the MIC method with five iterations of interpolation smoothing. Although this is a true interpolation, one can see the ghosting of the intermediate contours. The curvature is also rather high, as can be seen in the table. Figure 7 shows the approximated DEM created by the MIC method with one iteration of Gaussian smoothing. The surface is much smoother, as borne out by the curvature values, and has an acceptable *RMSE*, but there is still some contour ghosting. The DEM computed with five iterations of smoothing, shown in Figure 8 is very smooth, but the surface has a higher *RMSE*, which results in seven percent of the contour interval, slightly higher than suggested in [4]. Finally, Figure 9 shows the result from the TOPOGRID procedure. The surface is not very smooth, exhibits the highest *RMSE*, and there is a rather large artifact visible in the north-east corner.

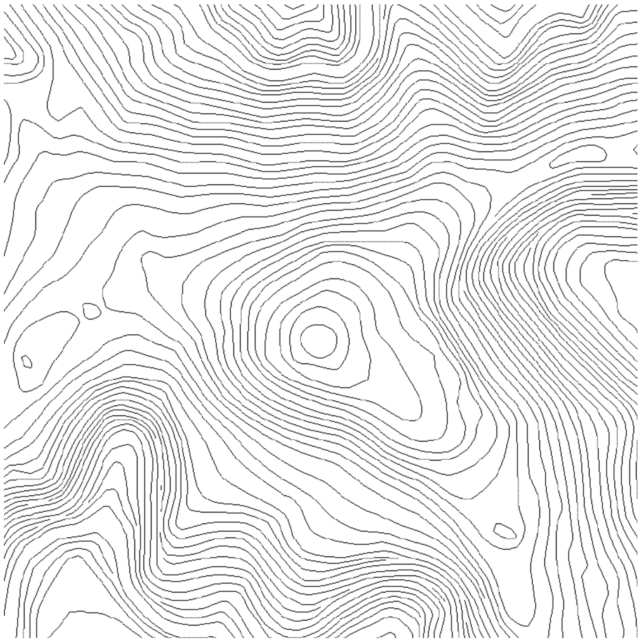

Figure 4: 800×800 **contours from a DLG of Mt. Washington, NH.**

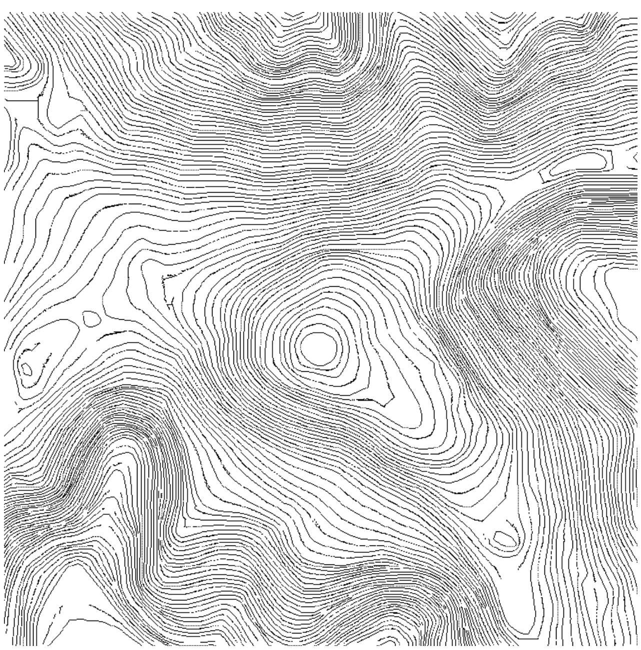

Figure 5: One iteration of intermediate contours.

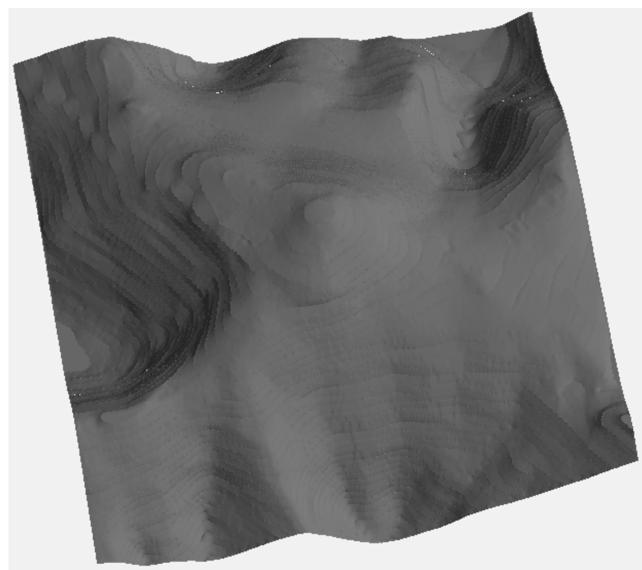

Figure 6: MIC interpolation with five iterations of smoothing.

Figure 10 shows a plot of the profile near the peak produced by the MIC interpolation and approximation methods as well as TOPOGRID; the vertical lines represent the contour elevations. The TOPOGRID produces a smooth surface through the contours, but because of the thin plate processing, the peak is rather flat and the surface bulges out somewhat on either side. Both the MIC interpolation and approximation create a rounded peak that seems to follow the slop trend better, with the approximation being smoother because the surface is allowed to deviate slightly from the original contours. This illustrates the common trade-off between absolute accuracy and smoothness.

Finally, Figure 11 shows the height classes for the methods. TOPOGRID exhibits smoothness, but has a high number of elevations very close to the contours. This accounts for the rather severe ghosting in the TOPOGRID DEM. Furthermore, the frequency of the height classes drops the farther a point is from the contours. This may indicate an undulating surface between contours, another problem inherent in thin plate procedures. Both MIC methods im-

Table 1: Results of applying methods to Mt. Washington data.

Method	C_{sq}	C_{ave}	*RMSE*	% of contour interval
MIC interpolation, 5 smoothing iterations	392616	0.26	0.0	0.0
MIC approximation, 1 smoothing iteration	106478	0.28	0.92	4.6
MIC approximation, 5 smoothing iterations	12170	0.08	1.40	7.0
TOPOGRID	134142	0.22	3.40	17.0

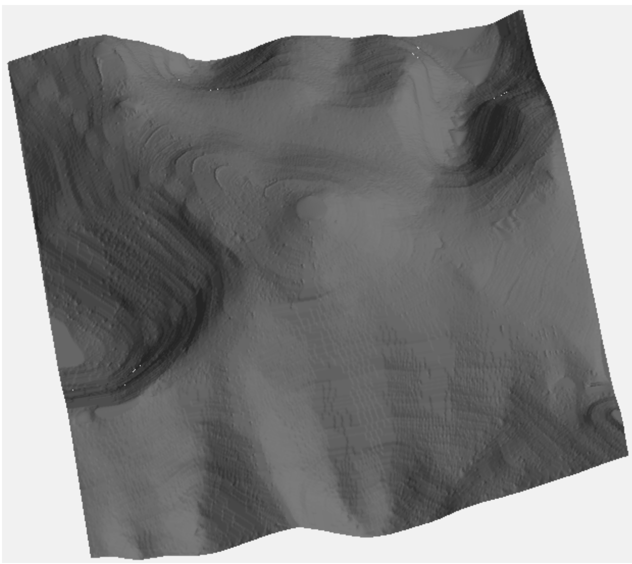

Figure 7: MIC approximation with one iteration of Gaussian smoothing.

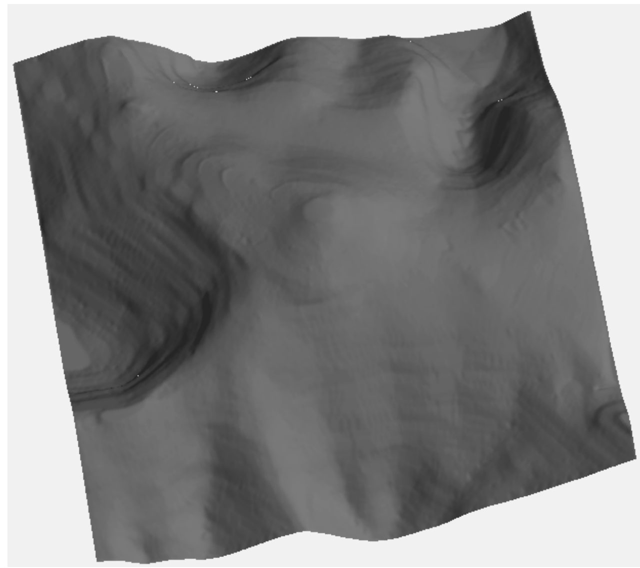

Figure 8: MIC approximation with five iterations of Gaussian smoothing.

prove on the frequency of elevations near the original contours, but the intermediate contours are apparent in the frequency pattern. More smoothing lessens this phenomenon, but at the expense of accuracy.

6. CONCLUSIONS

The maximum intermediate contours methods create visually good DEMs, exhibiting minimal artifacts. The surfaces also perform well in quantitative tests, as measured by both curvature and *RMSE.* To keep the *RMSE* within the tolerance of five per cent of the contour interval as recommended in [4], the number of smoothing iterations must be kept to a minimum in the MIC approximation, unless a smoother surface is desired. A true interpolation can be done at the expense of smoothness near the contours. The MIC DEMs were shown to be superior to the DEM produced by the popular TOPOGRID procedure, qualitatively by shaded relief maps and a profile plot of the peak, and quantitatively by superior curvature and *RMSE* measures. Future work includes fine tuning the intermediate contours construction to eliminate artifacts and minimizing the stratification of the surfaces, perhaps by incorporating thin plate algorithms or by computing splines in the direction orthogonal to the original contours.

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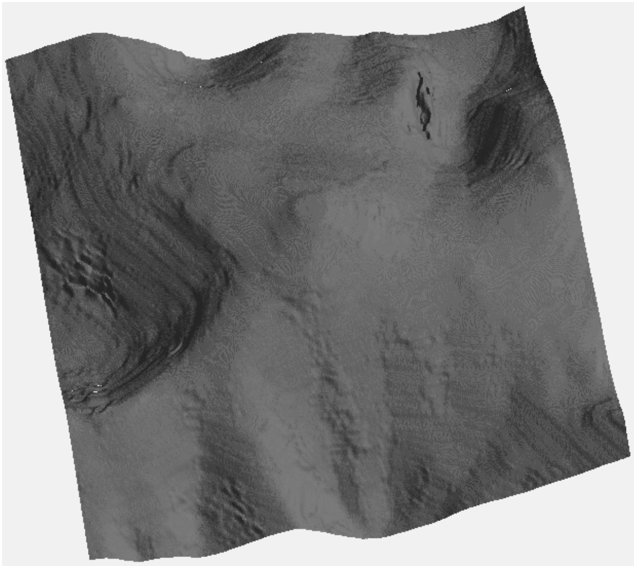

Figure 9: DEM computed with ArcInfo's TOPOGRID.

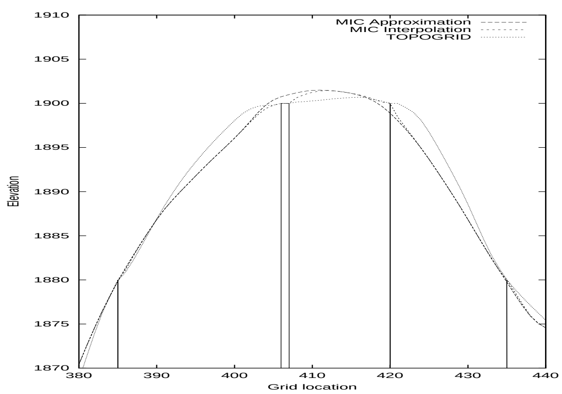

Figure 10: Plot of peak area profile.

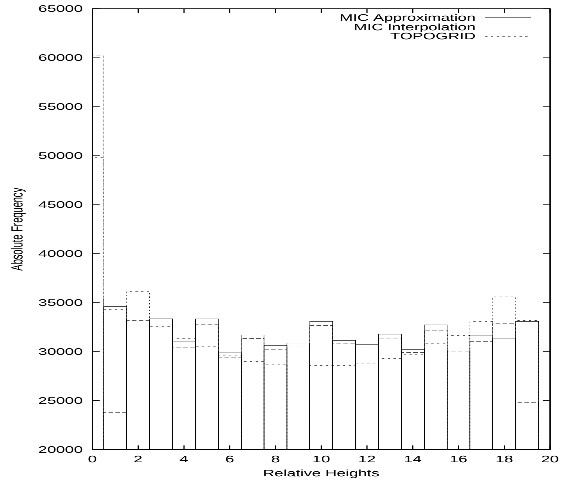

Figure 11: Plot of relative heights.

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