Contour to DEM

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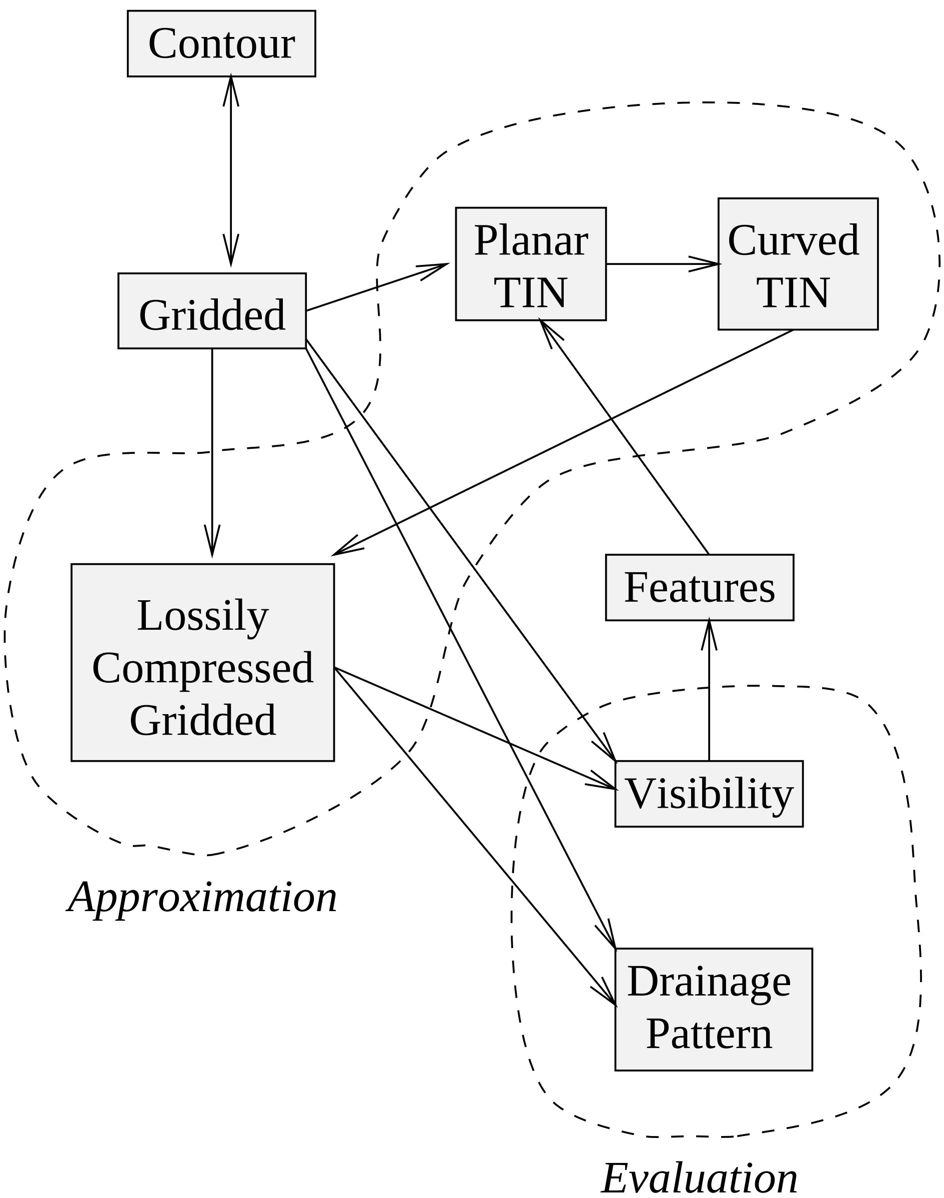
July 10, 1998

Why Viewgraphs, not Powerpoint?

- Brighter
- More robust
- Less visual clutter

Intro

This is a piece of my plan for converting, compressing, processing, elevation data.



So What's New?

People have converted data for several decades now.

- hardware,
- SW packages,
- specific algorithms.

Prior Art

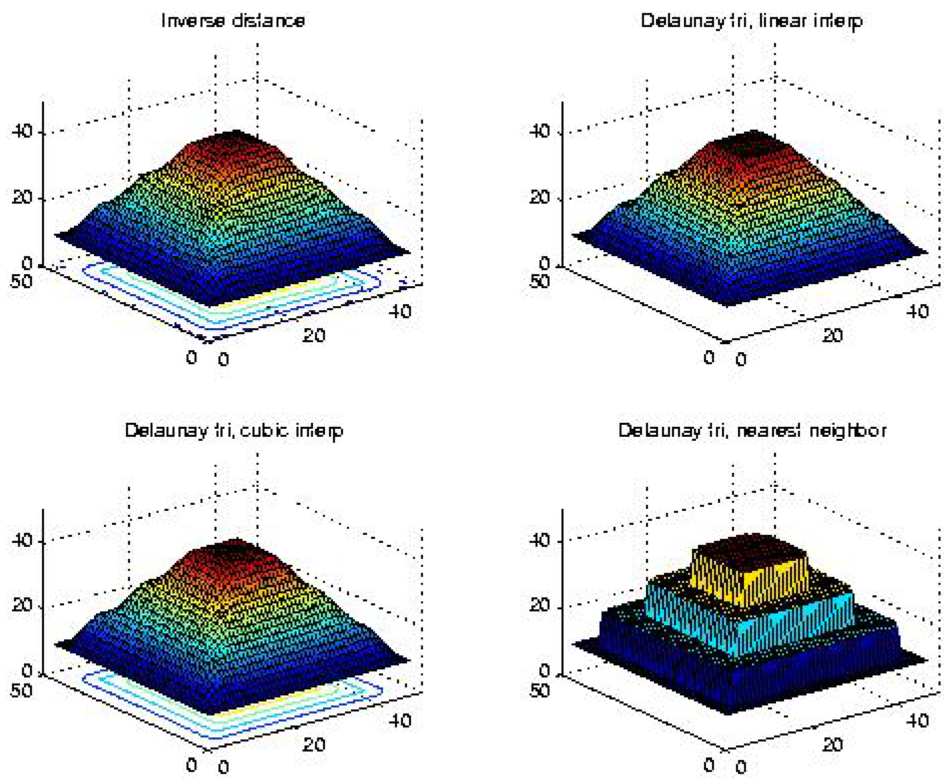
See 2 pages of refs in paper.

- Interpolate in vertical planes.
- PDE - Lagrange (heat flow) or thin-plate
- Extend straight lines out in 8 directions to the nearest contour
- Voronoi diagram / Delaunay triangulation, e.g. area stealing
- Medial axis
- Inverse distance weighting.
- Detect features, interpolate them, then fill in.

Limits of Existing Methods

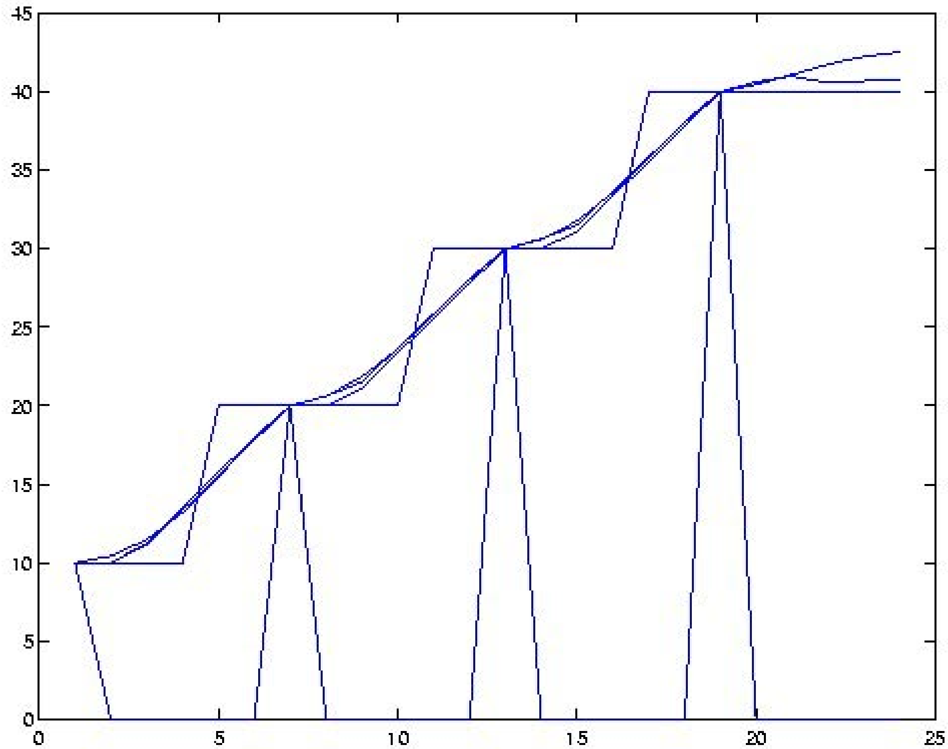
- Often tested only on synthetic, small, datasets.
- May require unbroken contours.
- May require contours be thinned.
- Generated triangles may be horizontal, or long & thin.
- Generated peaks may be flat.
- May generate terraces and ringing.

Example



Interpolating 3 Nested Squares with Inverse Distance Weighting, and 3 Ways with Delaunay Triangles

Diagonal Slice Interpolating Squares



Top to Bottom at Right: Cubic, Inverse Distance, Linear, Nearest, Orig Data

Interpolation & Approximation Introduction

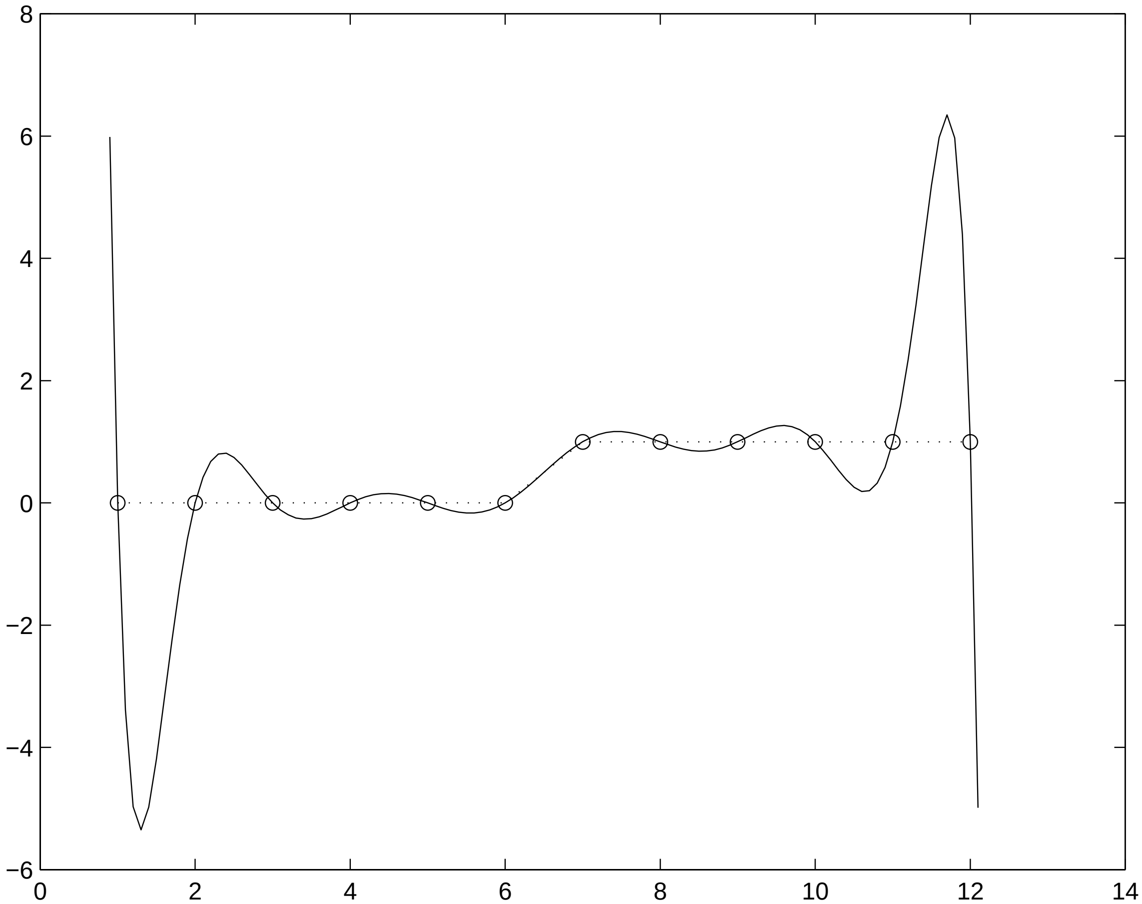
Desirable properties:

- local control
- variation minimization
- interpolation
- conformal
- Don't want to see the contours in the result.
- Peak inside top contour and valley outside outer contour.

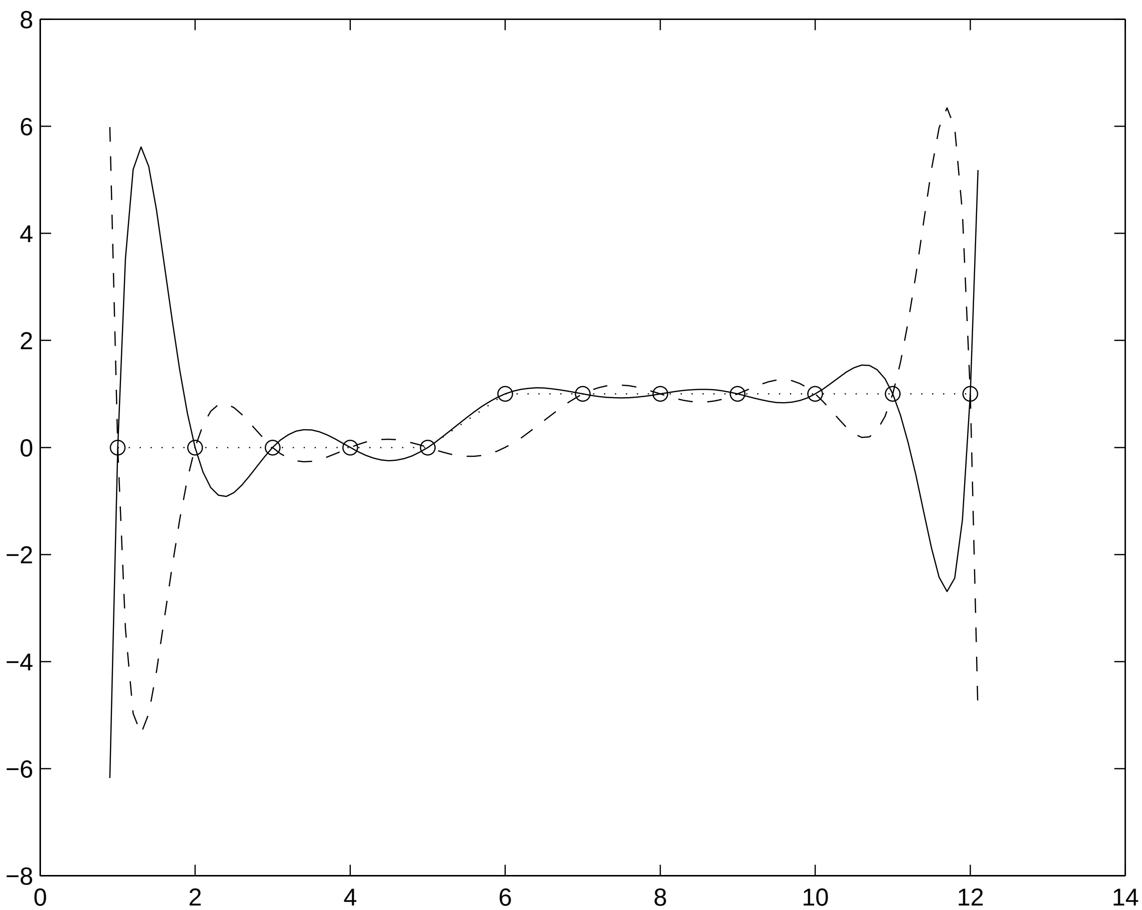
I&A Ctd

The math is counterintuitive, and the desired properties mutually contradictory.

E.g., Interpolating N-degree polynomial to N-1 points in 2D doesn't have local control or variation min.



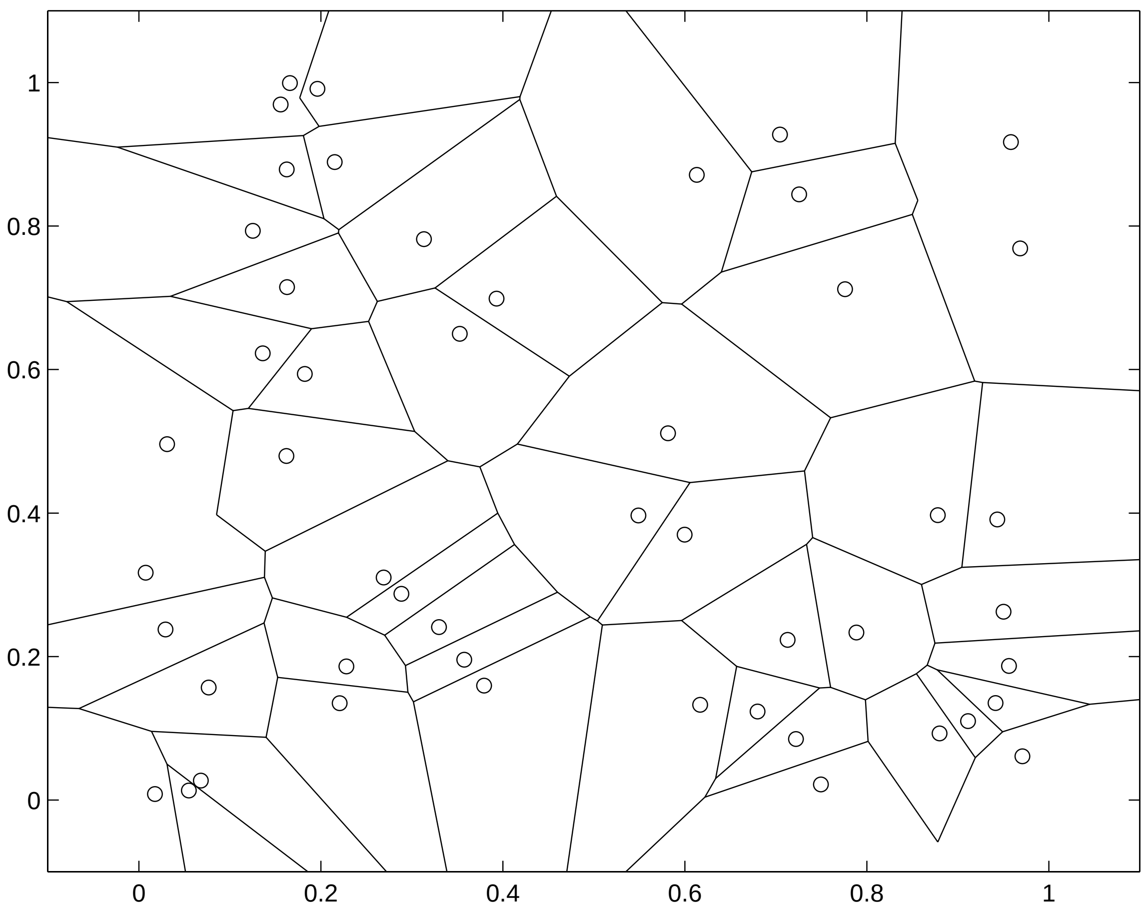
Interpolating 12 Points



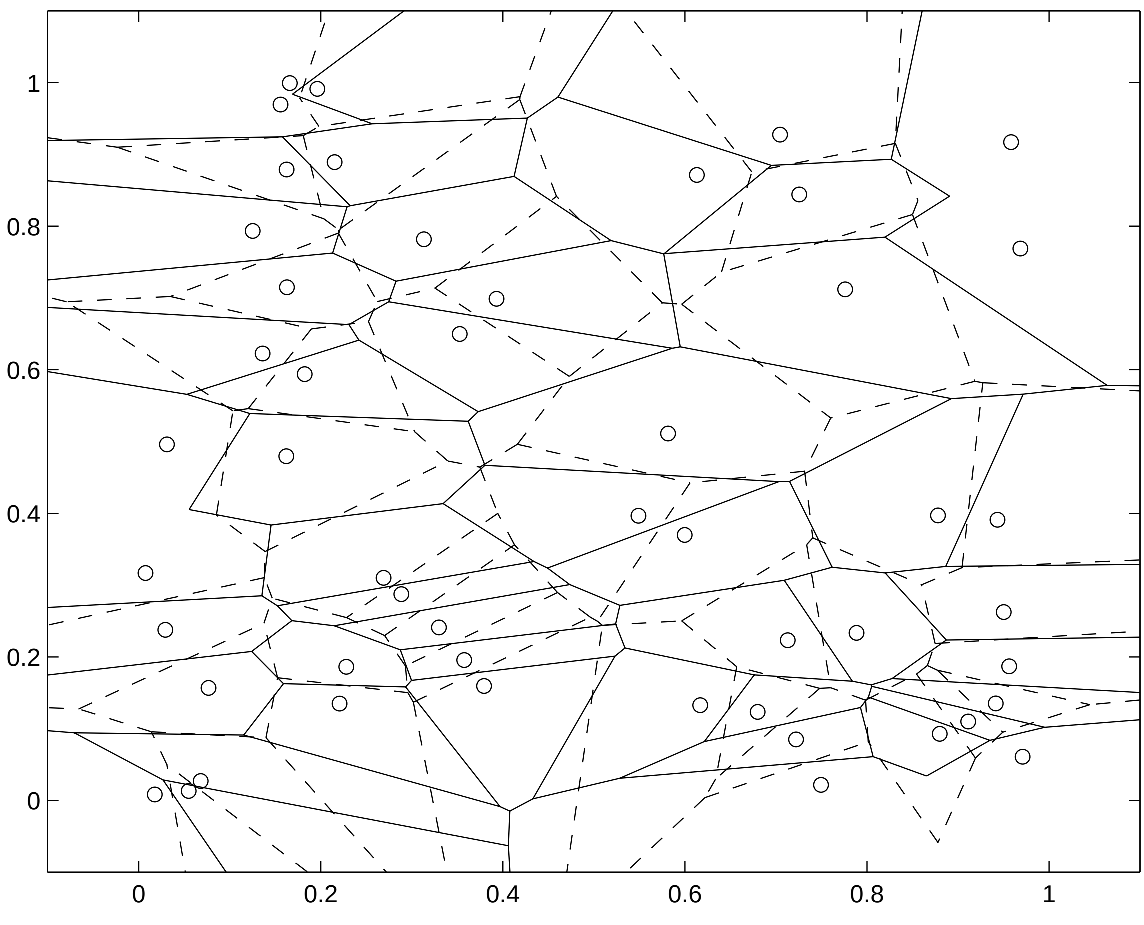
Moving One Point

I&A – 3

- running straight lines out from data points not conformal.
- Voronoi not conformal



Voronoi on 50 Uniform Random Points



Scale X by 2, Voronoi, Unscale

Goodness Criteria

Ideal:

- Maximum likelihood surface fitting the data.
- *Requires:* Formal probability model of terrain elevation.
- which is still incompletely solved. (e.g., long range correlations from drainage patterns).

Next best:

- What looks good?
- Shaded plots
- Profiles
- Statistically compare curvature at generated points on/near data points against farther points. *(future)*

New Math Techniques

- Sparse matrix techniques.
- Multigrid iteration.

Sparse matrix techniques. Solve linear equations, i.e.,

$$\left(\quad \mathcal{A} \quad \right) \begin{bmatrix} b \end{bmatrix} = \begin{bmatrix} c \end{bmatrix}$$

where $\mathcal{A}$ is mostly 0.

Example Solve a Laplacian PDE on a 257×257 grid. $\mathcal{A}$ is 66049×66049, but only about 330K elements (0.08%) are nonzero. CPU Time on a large IBM RS6000: 8 min.

On $N \times N$ grid, time perhaps N^4.

Multigrid Iteration

- Solves PDEs on larger grids ($30,000 \times 30,000$) than possible before.
- Method:
  - Solves first on a coarse grid, for speed.
  - Transfers that to a fine grid, for accuracy.
  - Loops back and forth between different grid sizes.
- On $N \times N$ grid, time perhaps N^3.
- Very widely used.

Our Test Cases

- Small synthetic case, 257×257.
- Crater Lake, 900×900.
- Bountiful, UT, 2100×2100.
- Tuckerman's Ravine, NH, 800×800

Our New Techniques

- Lofting (gradient interpolation)
- Solving overdetermined Laplacian system

Mods to Existing Systems

- Laplacian thin-plate PDEs
- Intermediate contours

Laplacian PDE

(PDE = Partial Differential Equation)

Definition:

- Models the flow of heat in a uniform material.
- The contours are at known temperatures.
- What temperatures are all the other points?

Math:

- PDE: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$, aka $z_{xx} + z_{yy} = 0$.
- On a grid: Each unknown point is the average of its 4 neighbors.

  $4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$

Laplace Properties

- The solution is *conformal*: curved transformations that don't change angles transform the solution together with the initial contour lines.
- Doesn't require continuous contours; can use isolated points.
- No info flows across closed contours: bad.
- *Historically:* impossible to solve realistic systems. A 1000×1000 grid is a linear system with 1 million equations and unknowns.
- *Currently:* Feasible, because of good sparse matrix and iterative (multigrid) methods.
- *Bad idea:* Make border points the average of their 2 or 3 neighbors. That flattens the surface there.

Overdetermined Laplace

- (This idea appears new.)
- *Problem:* No information flows across the contours. This causes terracing.
- *Solution:* Make the known data points also satisfy the equation.
- N^2 unknowns (even the known points are unknown).
- Two types of equations:
  1. For all points:
     $4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$
  2. For known points: $z_{ij} = h_{ij}$
- *Now:* more equations than unknowns.
- Do a least-squares solution in Matlab.

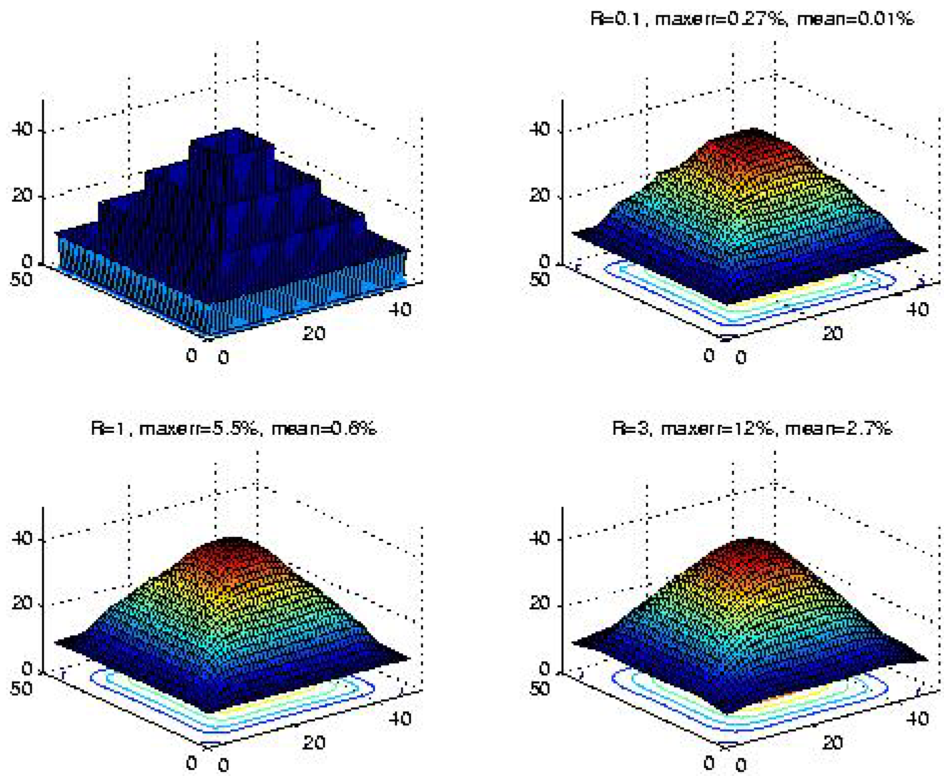
Overdetermined Tweaking —1

- This is an approximation, not interpolation.
- *Note:* Scaling an equation up makes it more important in the solution.
- Weighting the known-point equations higher $\Longrightarrow$ more accurate surface.
- ... lower $\Longrightarrow$ smoother surface.

Observation: A little inaccuracy goes a long way.

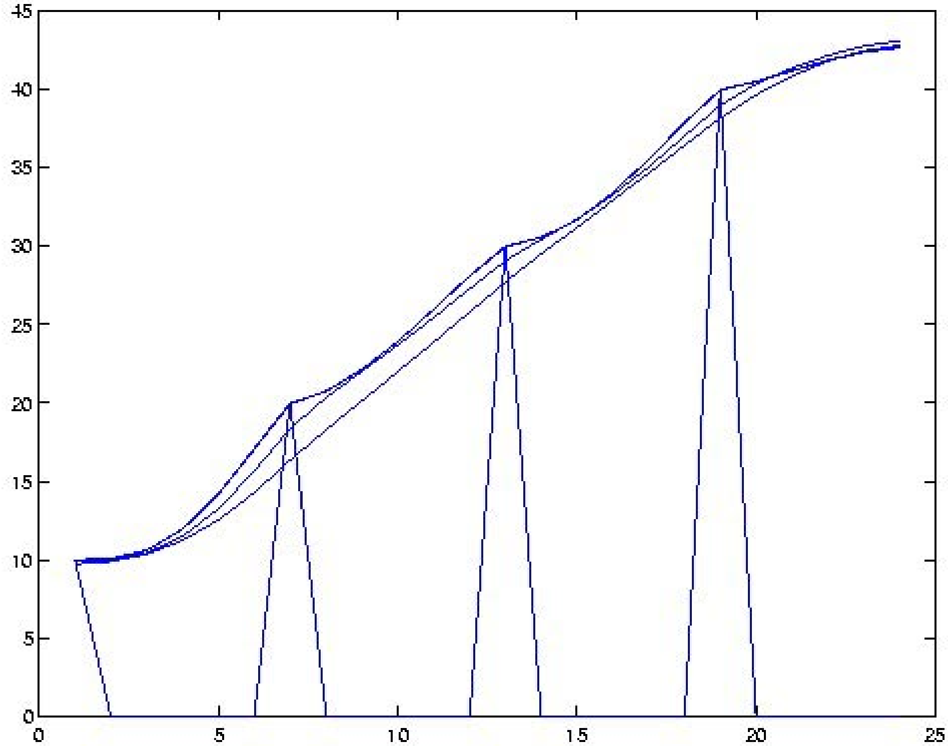
Note: This is different from springs on the data points.

Overdetermined Applied to Squares



Overdetermined Laplacian PDE, Weighting the Averaging Equation Differently

Diagonal Slices



Top to Bottom in Middle: R= 0.1, 1. 3, Original Data

R	max % err	Mean % err
0.1	0.27	0.01
1.	5.5	0.6
3	12	2.7

Overdetermined Tweaking —2

- Calculate the *error* on each data point.
- For the less-accurate points, increase their weights and re-solve.
- This reduces the max absolute error, but increases the mean.
- The surface is slightly less smooth.

Alternatively

- Do the points with large errors represent breaks in the surface slope?
- *Reduce* their weights and re-solve.

Overdetermined Laplace Tests

- Try an extreme case of nested squares.
- SW environment: Matlab, which has excellent sparse matrix routines.
- 257×257 takes a few minutes.
- On $N \times N$ grid, time perhaps N^4. Non sparse methods are N^6.
- Now working to extend it to larger cases.

Key Parts of Matlab Program

```
for i = 2:(N-1)        % Each interior point is
for j = 2:(N-1)        % the average of its 4
   NE=NE+1;        % neighbors.
   ns = ns+1;
   s(ns,:) = [NE, N*j+i-N, 4*R];
   ns = ns+1;
   s(ns,:) = [NE, N*j+i-1-N, -R];
   (3 more similar cases)
   C(NE) = 0;
end
end

for k = 1:np          % Each known point is equal
   NE=NE+1;       % to its value.
   i = px(k);
   j = py(k);
   ns = ns+1;
   s(ns,:) = [NE, N*j+i-N, 1];
   C(NE) = Z(i,j);
end
```

```
% Element list -> sparse matrix
B = spconvert(s);

% Solve the overdetermined linear system.
W1 = B \ C;
```

Thin Plate PDE

Let a thin plate of metal bend while fixing certain points.

Minimize the curvature energy.

PDE: $z_{xx}^2 + 2z_{xy}^2 + z_{yy}^2 = 0$

Iterative equation on a grid:

$$
\begin{aligned}
20z_{ij} &= \\
&\quad 8(z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}) \\
&\quad -2(z_{i-1,j-1} + z_{i-1,j+1} + z_{i+1,j-1} + z_{i+1,j+1}) \\
&\quad -(z_{i-2,j} + z_{i+2,j} + z_{i,j-2} + z_{i,j+2})
\end{aligned}
$$

Smoother but more complicated.

Mods to PDE Methods

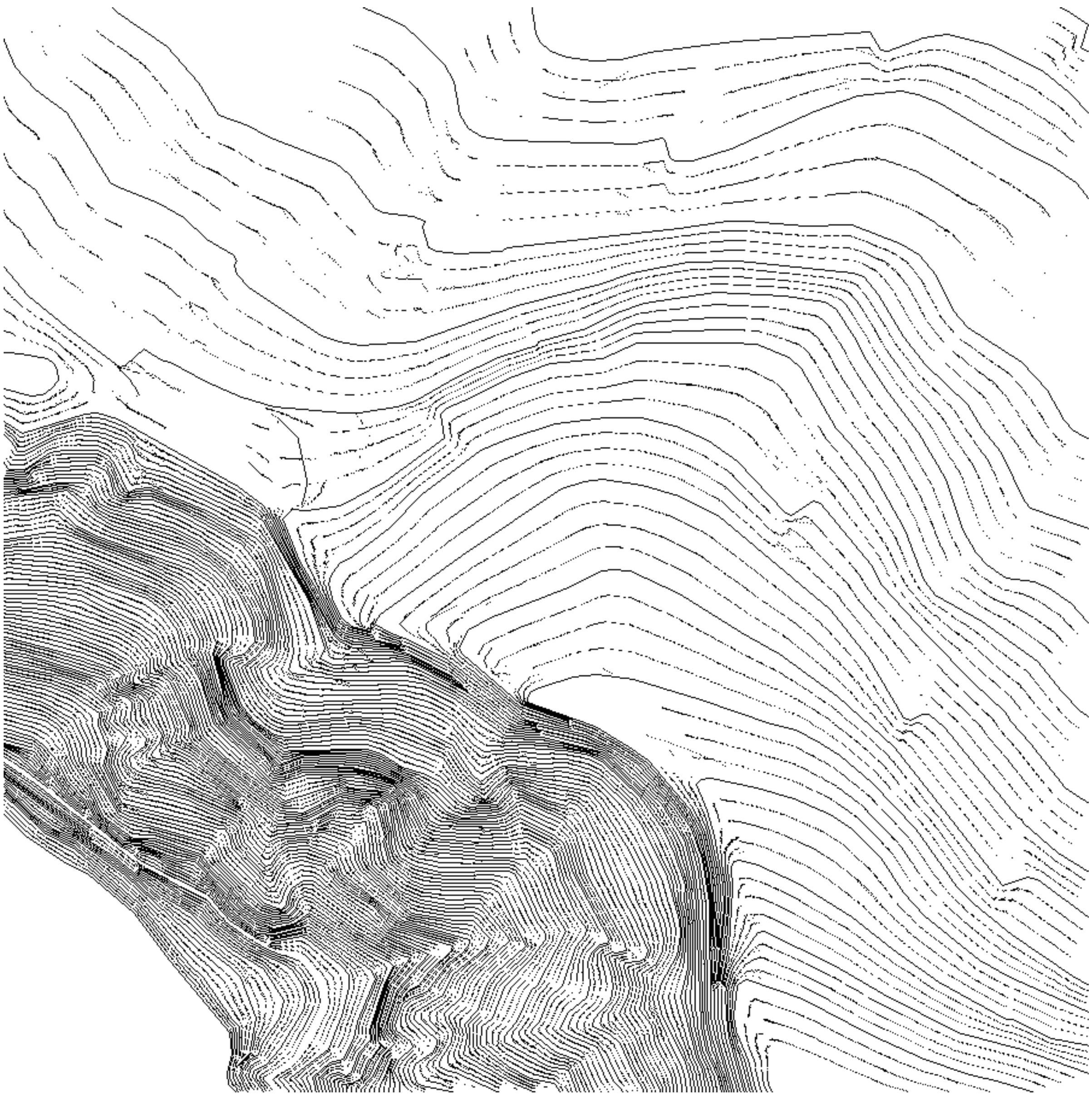
- *Springs* on the points.
- *Tension* to flatten the surface.
- *Break* the surface at inferred discontinuities.
- Note that Laplacian and thin-plate are conformal.

Intermediate Contours

- Similar to medial axis.
- Two adjacent contour lines are: $\mathcal{A}$, at elevation a, and $\mathcal{B}$, at b.
- Interpolate at elevation $(a + b)$ thus.
- Pick a point on $\mathcal{A}$.
- Find the closest point on $\mathcal{B}$ (approximating the gradient).
- The midpoint is on the intermediate contour.

Contrarian philosophy: Don't find elevation at certain points. Rather, find points with a certain elevation.

Crater Lake Original Contours



Crater Lake Interpolated Contours

Intermediate Contour Variants

- Maximum Intermediate Contours (MIC): Iterate the process, filling ever-finer contours, or:
- Do thin-plate, &
- Hermite-spline the peaks, &
- Inverse-distance weight other small gaps, &
- Gaussian-smooth the surface.

Gradient Lines

Problem: Generated surfaces terrace between contour lines.

Solution: Import *lofting* from CAGD.

- Generate a first version of a surface by any method.
- Find its *gradient lines.*
- On each gradient line, we know only the elevations where it crosses the contours.
- *Interpolate* its elevations in-between.

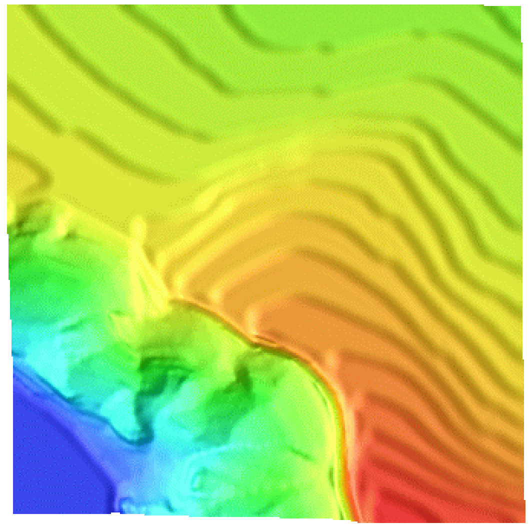
Smooths the surface, while keeping the interpolation.

Crater Lake Tests

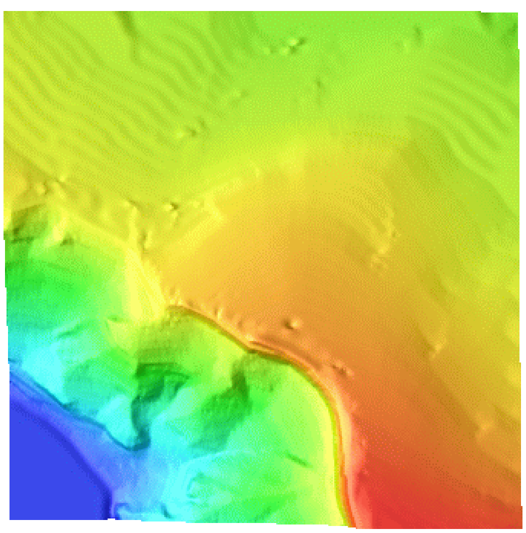
Criterion	*Thin-plate*	*Interm. Cont.*	*Grad. Lines*
Total squared curvature, $\sum\sum(z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} - 4z_{ij})^2$	72987	93170	72709
Average absolute curvature, $\frac{1}{(n-2)^2}\sum\sum(\lvert z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} - 4z_{ij}\rvert)$	0.138	0.118	0.107
Root-mean-square error relative to DEM, $\sqrt{\frac{1}{n^2}\sum_{i=1}^{n^2}(z_i - w_i)^2}$, where w_i is a DEM point.	8.69	5.28	5.48

Crater Lake

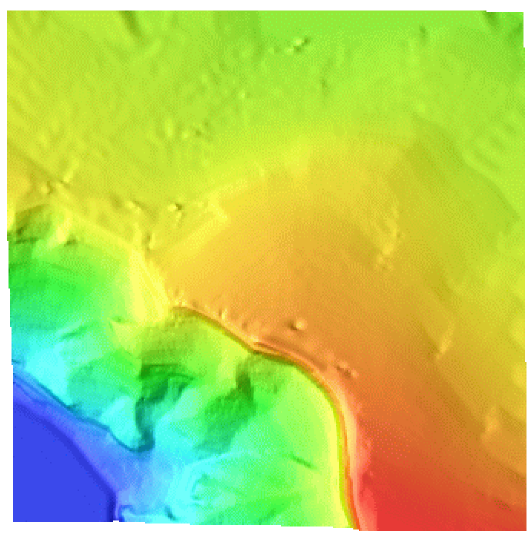
Original Contours



Thin-Plate Approx.

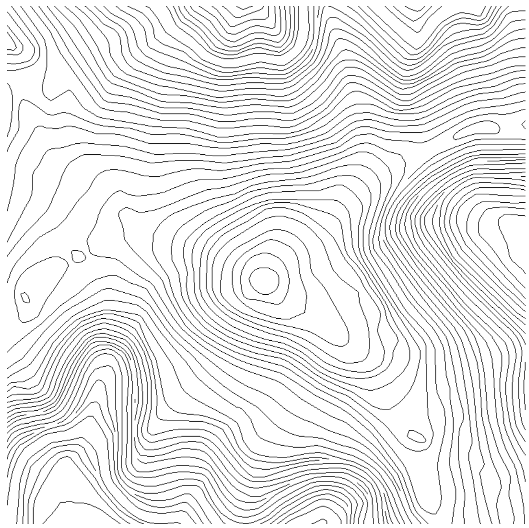


Intermediate Contour Approx.

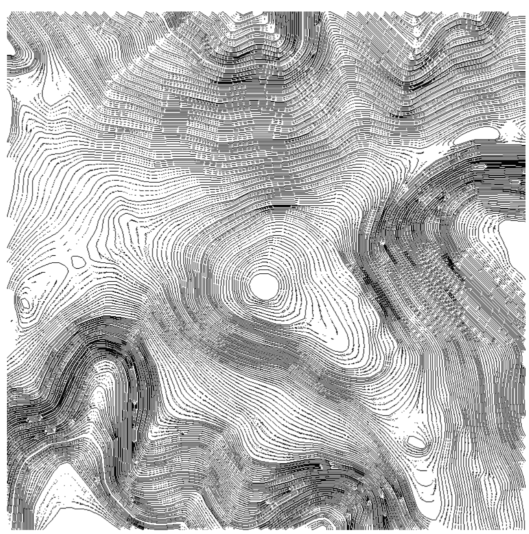


Gradient Line Approx.

Tuckermans Ravine Tests

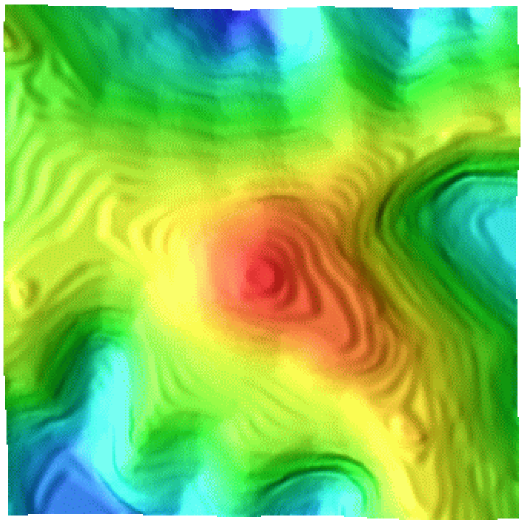


Original Contours

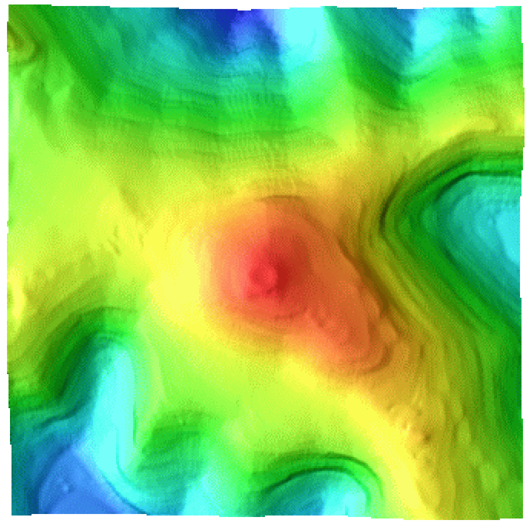


Intermediate Contours

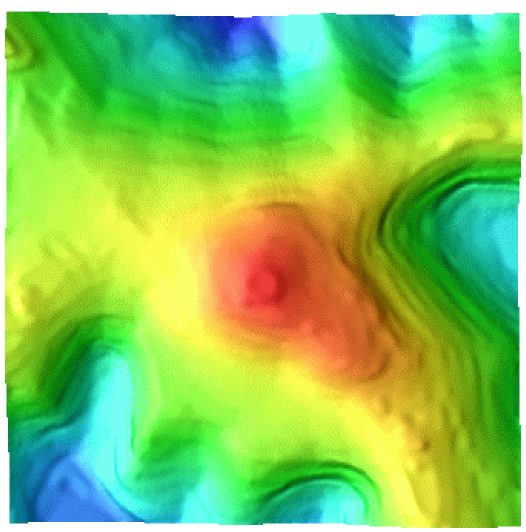
Tuckermans



Thin-Plate Approx.

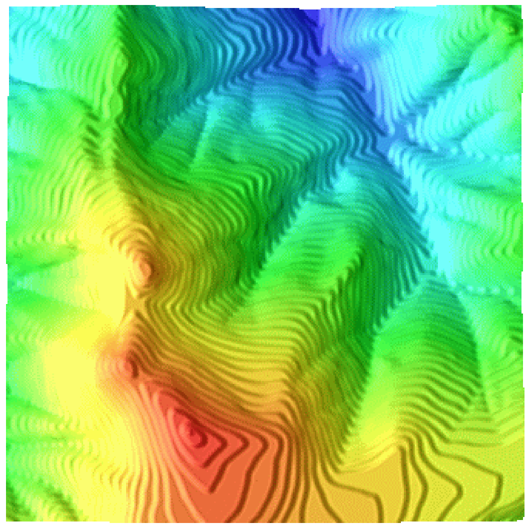


Intermediate Contour Approx.

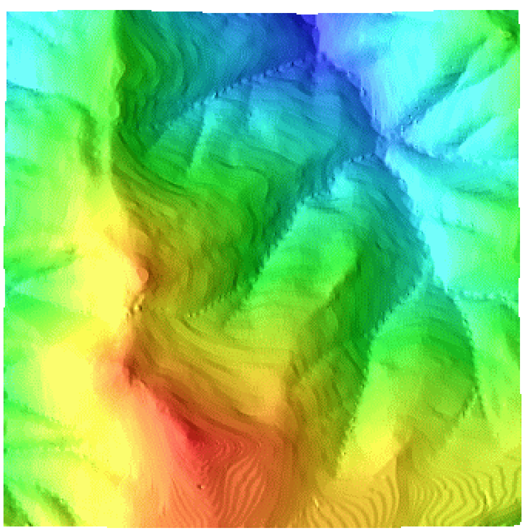


Gradient Line Approx.

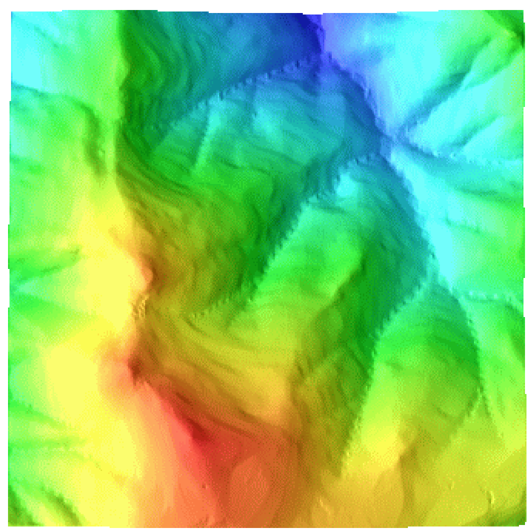
Bountiful, Utah, Tests



Thin-Plate Approx.



Intermediate Contour Approx.



Bountiful Max Contour Approxim.

Blending Patches Together

Process a large surface by

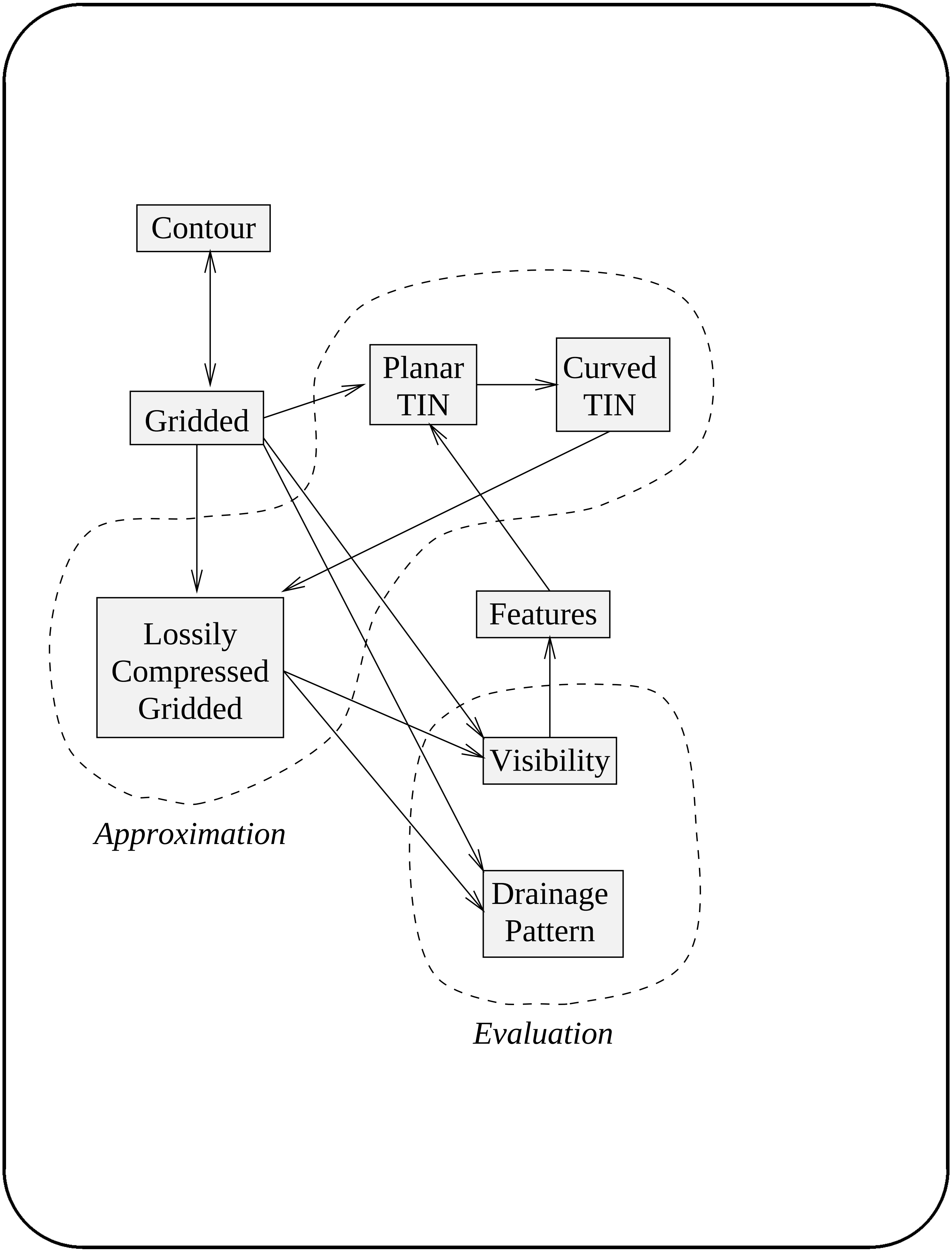
- Cutting into patches,
- Solving on each patch, &
- Blending the partial solutions.

How? (future)

- Solutions outside the outermost contour line are fictitious.
- Overlap the patches by twice the distance between contours.
- Discard the outermost several rows & columns of each patch, keeping some overlap.
- Smoothly blend the overlap.

Future Work

- Extend Overdetermined Laplacian to larger grids.
- Try Overdetermined Thin-Plate.
- Infer breaks in surface.
- ...
- Look at TINs again – Helio Pedrini


Contour
Gridded
Planar TIN
Curved TIN
Lossily Compressed Gridded
Features
Visibility
Drainage Pattern
Approximation
Evaluation