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Accelerating the exact evaluation of geometric predicates with GPUs

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Our Team

- Marcelo de Matos Menezes, UFV: master's student since 2018; interests are Computer Graphics, Computational Geometry and High-Performance Computing, long-term goal to apply the ideas described on this paper to other CG algorithms.
- Salles Viana Gomes Magalhães, RPI PhD grad, UFV prof
- W. Randolph Franklin, RPI prof.
- Matheus Aguilar de Oliveira, UFV: CS undergrad since 2018; interests are Computational Geometry and Competitive Programming.
- Rodrigo E. O. Bauer Chichorro, UFV: CS undergrad since 2017; interests are Competitive Programming, Computational Geometry and Artificial Intelligence





Our research strategy

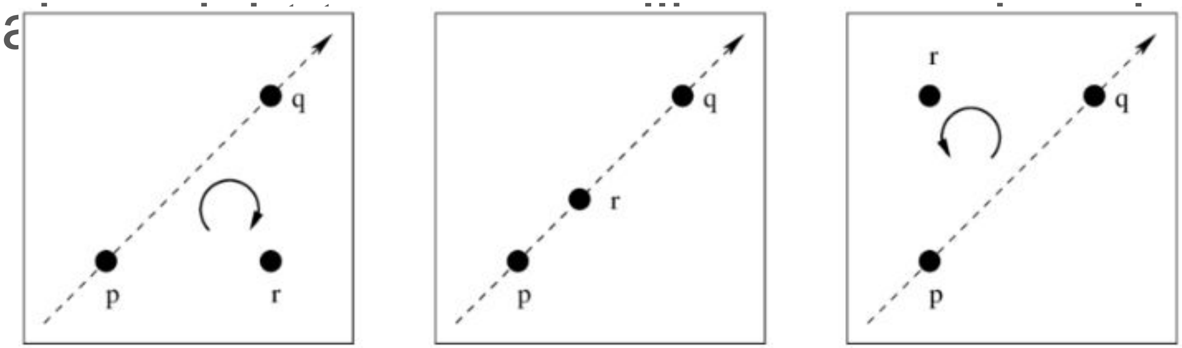
- Identify fundamental geometric operations
- used in higher-level systems
- that need to produce correct results.
- and should execute very fast.
- Devise new theory
- using simple data structures
- on current hardware.
- Implement
- Test.

This paper’s contribution

- A faster solution to erroneous computations caused by floating point finite precision computations.
- Errors can cause predicates (conditionals)to be evaluated wrong.
- That can cause topological errors.
- Existing solutions are either very slow or may fail.
- We synergize three software techniques and two hardware platforms.
  - filter input with uniform grid on CPU, then
  - filter survivors with interval arithmetic on GPU, finally
  - if necessary, compute exactly with multiprecision rationals back on CPU.
- Result: both fast and good.

The problem of roundoff errors

- Floating-point errors: computational geometry challenge.
- Generate topological inconsistencies: global impossibilities.
  - intersection point between two lines may not lie in either.
- Example: planar orientation predicate
  - Do three points p = (px, py), q = (qx, qy) and r = (rx, ry) ma ft turn?

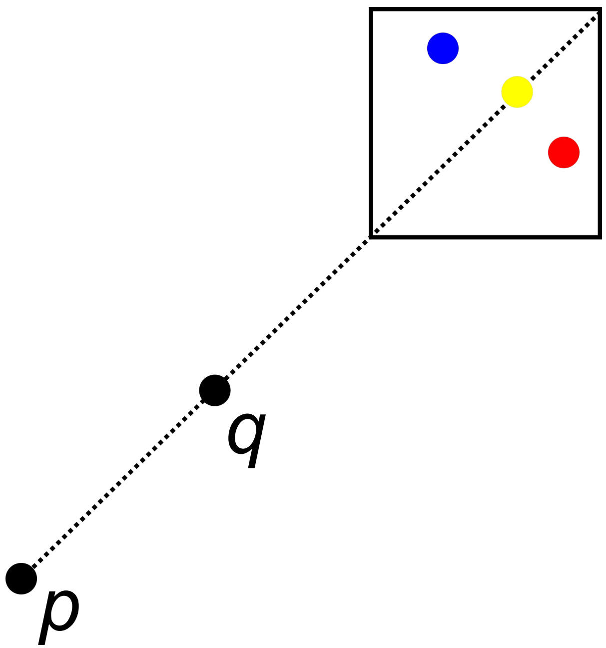


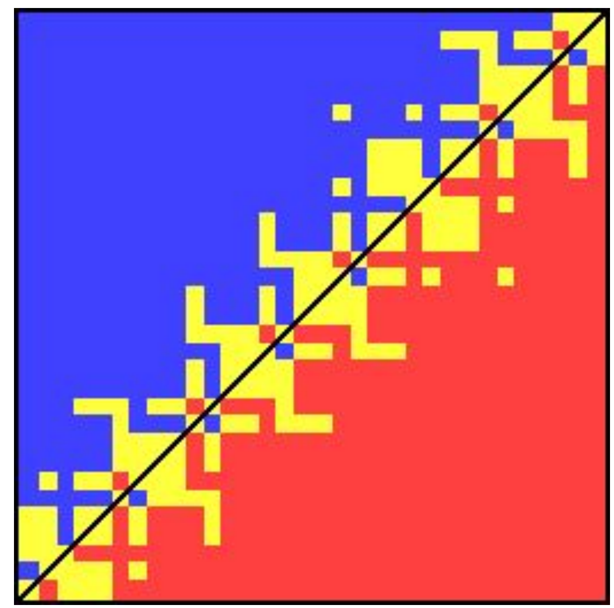
Source: https://www.researchgate.net/figure/The-orientation-predicate-of-3-po n-collinear-and_fig2_1959784

- Predicate = sign of the determinant: $\begin{vmatrix} p_x & p_y & 1 \\ q_x & q_y & 1 \\ r_x & r_y & 1 \end{vmatrix}$

Roundoff errors

- Evaluating the predicate using floating point arithmetic:





Source: Kettner et al., Classroom examples of robustness problems in geometric computations

- Common techniques (snap rounding, epsilon tweaking, etc): no guarantee

Rationals: one roundoff error solution

- Solution for roundoff errors: exact arithmetic (e.g. GMP rationals), but challenges:
  - Slower than floats
  - Size is exponential in depth of computation tree, although that's not a problem if the tree is shallow
  - Growing the size of a variable allocates memory on the global heap.
    - Total time may be superlinear in the number of objects, and
    - is serial,
- Apparently little prior art of working (not just proposed) rational number systems on GPUs

Arithmetic filters and Interval arithmetic (IA), 1

- Technique used in several CG implementations, e.g.: CGAL
- Basic idea: use exact arithmetic only when really necessary
- Predicate evaluation: typically “=” sign of an arithmetic expression
- Each value has:
  - an exact value (can be lazily computed), and
  - an approximation given by an interval [xl, xh].
- Predicates evaluated using the approximation
- If the sign of the exact result can be <u>safely</u> inferred based on results computed with the intervals → use that sign
- Otherwise (a.k.a. <u>filter failure</u>) → re-evaluate with exact arithmetic

Arithmetic filters and IA, 2

- IA used to compute the sign of an expression.
- If it reports a *non-zero* result, it's guaranteed to be correct.
- Sometimes it reports a failure. (Then we escalate.)
- Real *x* represented as $[\underline{x}, \overline{x}]$, where $\underline{x} <= x <= \overline{x}$

$$
\begin{aligned}
[x] + [y] &= [\underline{x} + \underline{y}, \overline{x} + \overline{y}] \\
[x] - [y] &= [\underline{x} - \overline{y}, \overline{x} - \underline{y}] \\
[x] \cdot [y] &= [\min\{\underline{xy}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{xy}\}, \max\{\underline{xy}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{xy}\}] \\
[x]/[y] &= \begin{cases} [x] \cdot [1/\overline{y}, 1/\underline{y}] & \text{if } 0 \notin [y], \\ \mathbb{R} & \text{otherwise} \end{cases} \\
[x]^{1/2} &= \begin{cases} [\underline{x}^{1/2}, \overline{x}^{1/2}] & , \quad \text{if } 0 \leq \underline{x} \\ \mathbb{R} & , \quad \text{otherwise} \end{cases}
\end{aligned}
$$

Source of the fig: Brönnimann, H., Burnikel, C., & Pion, S. (2001). Interval arithmetic yields efficient dynamic filters for computational geometry. *Discrete Applied Mathematics*, *109*(1-2), 25-47.

Arithmetic filters and IA, 3

- An interval is a pair of floating-point numbers.
- To satisfy the containment property, the operations must change the way floating point values are rounded.
- IEEE-754 standard:
  - Result = exact result rounded to the next (or previous) representable FP number
  - Can be rounded:
    - towards $-\infty$
    - to the closest FP (default)
    - towards $+\infty$
- Changing the rounding mode on a GPU is very fast (slow on CPU)

Arithmetic filters and Interval arithmetic, 4

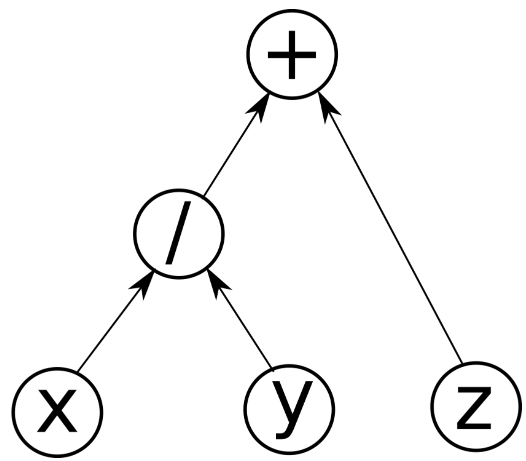
- CGAL uses arithmetic filters/IA- transparent to programmer but not thread safe.
- Illustration of a predicate one could implement:

```
// Predicate: returns true if the sum of x_exact with y_exact is positive
//            and false otherwise. x_interval and y_interval must contain,
//            respectively, x_exact and y_exact.

bool predicate(mpq_class x_exact, CGAL::Interval_nt<> x_interval,
               mpq_class y_exact, CGAL::Interval_nt<> y_interval) {
    try {
        if (x_interval + y_interval > 0)
            return true;
        else
            return false;
    }
    catch (CGAL::Interval_nt<>::unsafe_comparison& ex) {
        if (x_exact + y_exact > 0)
            return true;
        else
            return false;
    }
}
```

Arithmetic filters and Interval arithmetic, 5

- CGAL: arithmetic filtering can be performed “dynamically/automatically”
- Example:
  - A DAG may be created to keep track of results
  - If exact evaluation necessary → lazily re-evaluate the values

```
a = x/y
b = a + z
if(b>0) {
  ….
}
```



Exact fast parallel intersection of large 3-D triangular meshes

- Earlier work, presented last year
- Salles Magalhaes thesis
- Intersected 3D meshes using shared-memory multi-core CPUs. Combined:
  - Simulation of Simplicity
  - Arithmetic filtering/IA. “Manually” managed.
  - Parallel on multicore Intel Xeon with OpenMP
  - Big rationals.
- Today: start to incorporate GPUs.

Idea for using exact computation and GPUs

- GPUs:
  - excellent for floating-point arithmetic
  - however, warps of 32 threads should run same instruction stream on adjacent data
  - trees, hierarchical data structures, pointers are very inefficient.
- Implement the IA computation on the GPU
- CPU batch offloads evaluation of predicates to GPU.
- Indeterminate results are filtered and re-evaluated on the CPU.

What is CUDA?

- To program Nvidia GPUs.
- C++ with small syntax extensions and library.
- nvcc compiler separates program into code for CPU host and code for GPU device.
- GPU architecture is complicated.
    - thousands of cores, each 1/20 as powerful as Xeon core
    - SIMT, 32 thread warp
    - several memory classes:
        - varying speed,
        - size (to 48GB),
        - latency,
        - unified VM with host.
- A range of higher level abstract layers like Thrust and Kokkos trade off programmer time and execution time.

Implementation details, 1

- Created a class, based on Collange et al., to perform the necessary calculations → easier usage
- The rounding modes on CUDA C are selected via compiler intrinsics:
  - e.g.: For addition:
    - __dadd_rd() switches the rounding mode towards $-\infty$
    - __dadd_ru() switches the rounding mode towards $+\infty$
- These are hidden from the user through operator overloading

Implementation details, 2

Some methods in our *CudaInterval* class

```
class CudaInterval {
public:
    __device__ __host__ CudaInterval(const double l, const double u)
        : lb(l), ub(u) {}
    ...
    __device__ CudaInterval operator+(const CudaInterval& v) const {
        return CudaInterval(__dadd_rd(this->lb, v.lb),
            __dadd_ru(this->ub, v.ub));
    }
    ...
    __device__ int sign() const {
        if (this->lb > 0) // lb > 0 implies ub > 0
            return 1;
        if (this->ub < 0) // ub < 0 implies lb < 0
            return -1;
        if (this->lb == 0 && this->ub == 0)
            return 0;
        // If none of the above conditions is satisfied, the sign of the
        // exact result cannot be inferred from the interval, Thus, a flag
        // is returned to indicate an interval failure.
        return 2;
    }
    ...
private:
    double lb, ub; // Stores the interval's lower and upper bounds
};
```

Implementation details, 3

- Predicates: easily implemented using class instances
- Example: 2D orientation predicate

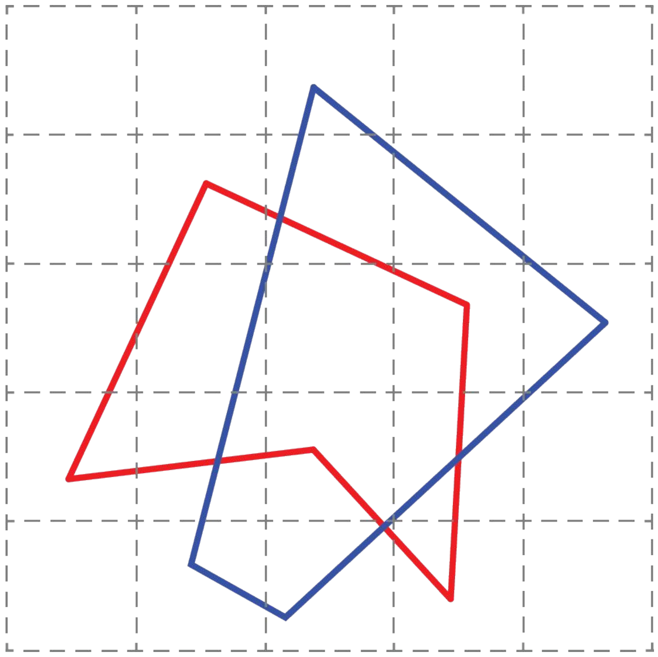
```
struct CudaIntervalVertex {
    CudaInterval x, y;
};

__device__ int orientation(
    const CudaIntervalVertex* p,
    const CudaIntervalVertex* q,
    const CudaIntervalVertex* r) {
    return ((q->x - p->x) * (r->y - p->y) -
            (q->y - p->y) * (r->x - p->x)).sign();
}
```

Fast red-blue intersection tests

- Case study: fast and exact algorithm for detecting red-blue intersection of line segments.
- Given two sets of segments *S1* (red segments) and *S2* (blue segments) $\rightarrow$ find pairs of red-blue intersections.
- Possible quadratic number of red-red and blue-blue intersections, even though few red-blue intersections.
- So, harder than finding all segment intersections.
  - Sweep line is too inefficient here.
- Algorithm steps:
  - Uniform grid preprocessing filter on CPU identifies pairs of segments that may intersect
  - Interval analysis tests further filters those pairs on GPU,
  - Exact rational arithmetic back on CPU exactly tests a few pairs.

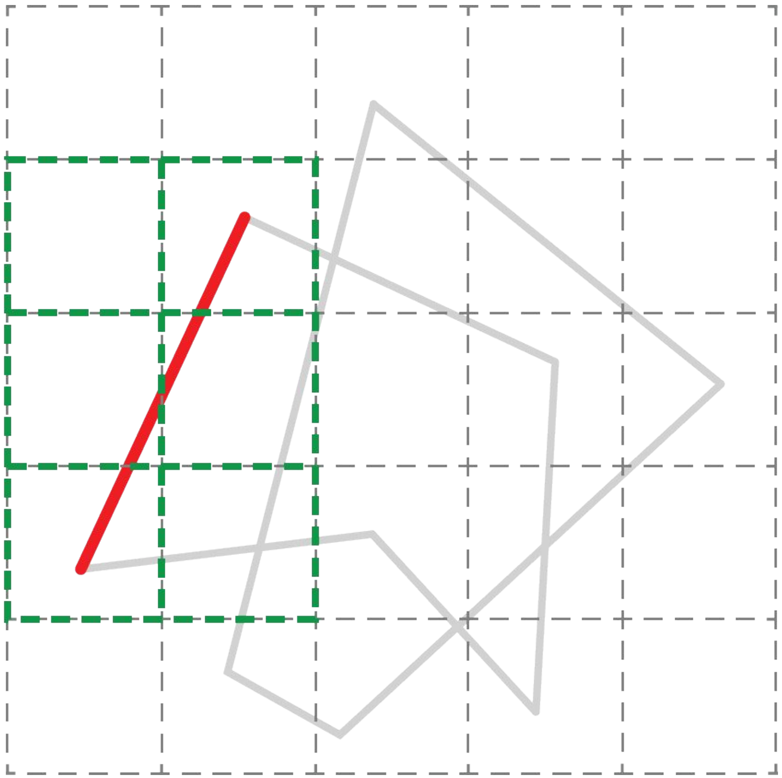
Fast red-blue intersections: Pre-processing, 1

- Consider the following segment sets *S1* (red) and S2 (blue):
- A uniform grid divides the domain into equally sized regions:



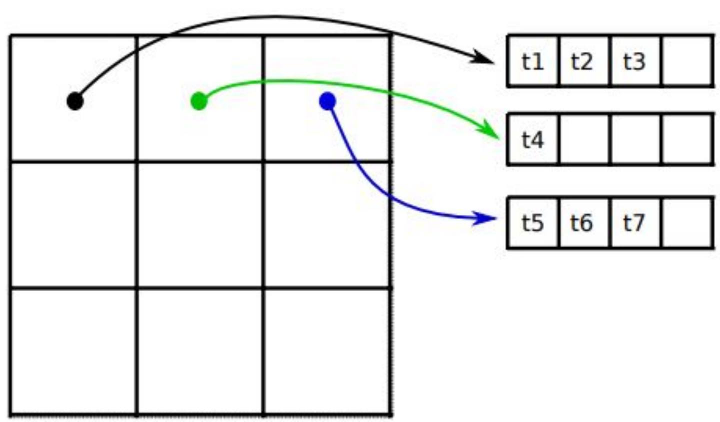
Fast red-blue intersections: Pre-processing, 2

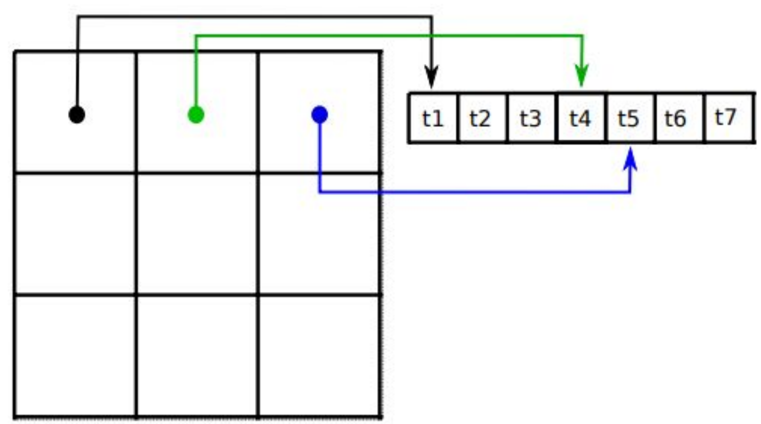
- Each segment (from both sets) is associated with the grid cells its bounding box intercepts.
- (Possible future mod would compute exactly which cells intersect the segment.)



Fast red-blue intersections: Pre-processing, 3

- For time and space efficiency, use a ragged array
  - One array containing all the elements, plus
  - Dope vector pointing to start of each cell's contents.
  - Constant time to read cell #i element #j.
- Creation requires two passes:
  - Count the number of elements in each cell, then
  - Insert the edges into the ragged array
- Both passes parallelize - faster than dynamic sized arrays



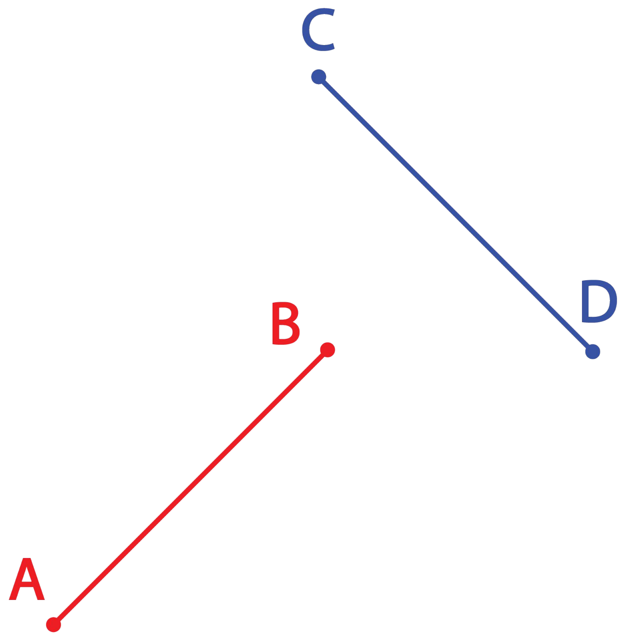


Fast red-blue intersections: Pre-processing, 4

- Once the uniform grid is constructed, a list of the pairs of red and blue segments from all the grid cells is created
- This list is generated in parallel using a strategy similar to the creation of the ragged-array
  - first pass to perform the count of pairs of segments
  - second pass to insert the pairs into the list
- The list can than be sent to the GPU, which will evaluate which of those pairs do intersect.

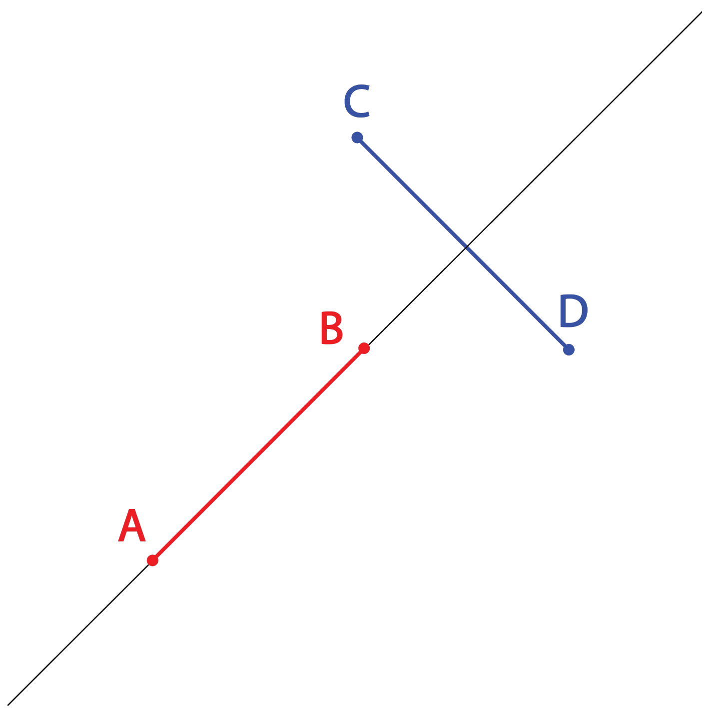
Intersection testing, 1

- Consider the segments AB and CD pictured below
- Four orientation predicates are sufficient to determine if they intersect or not
- **intersect((A,B) , (C,D)) = orientation(A, B, C) $\neq$ orientation(A, B, D) $\wedge$ orientation(C, D, A) $\neq$ orientation(C, D, B)**

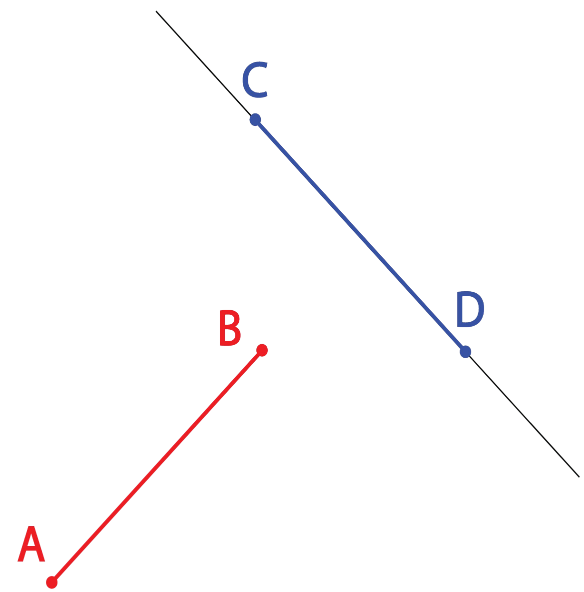


Intersection testing, 2

- C and D have different orientations w.r.t. (A,B)
- → CD **intersects** the supporting line of AB



- A and B have the same orientation w.r.t. (C,D)
- → AB **does not intersect** the supporting line of CD



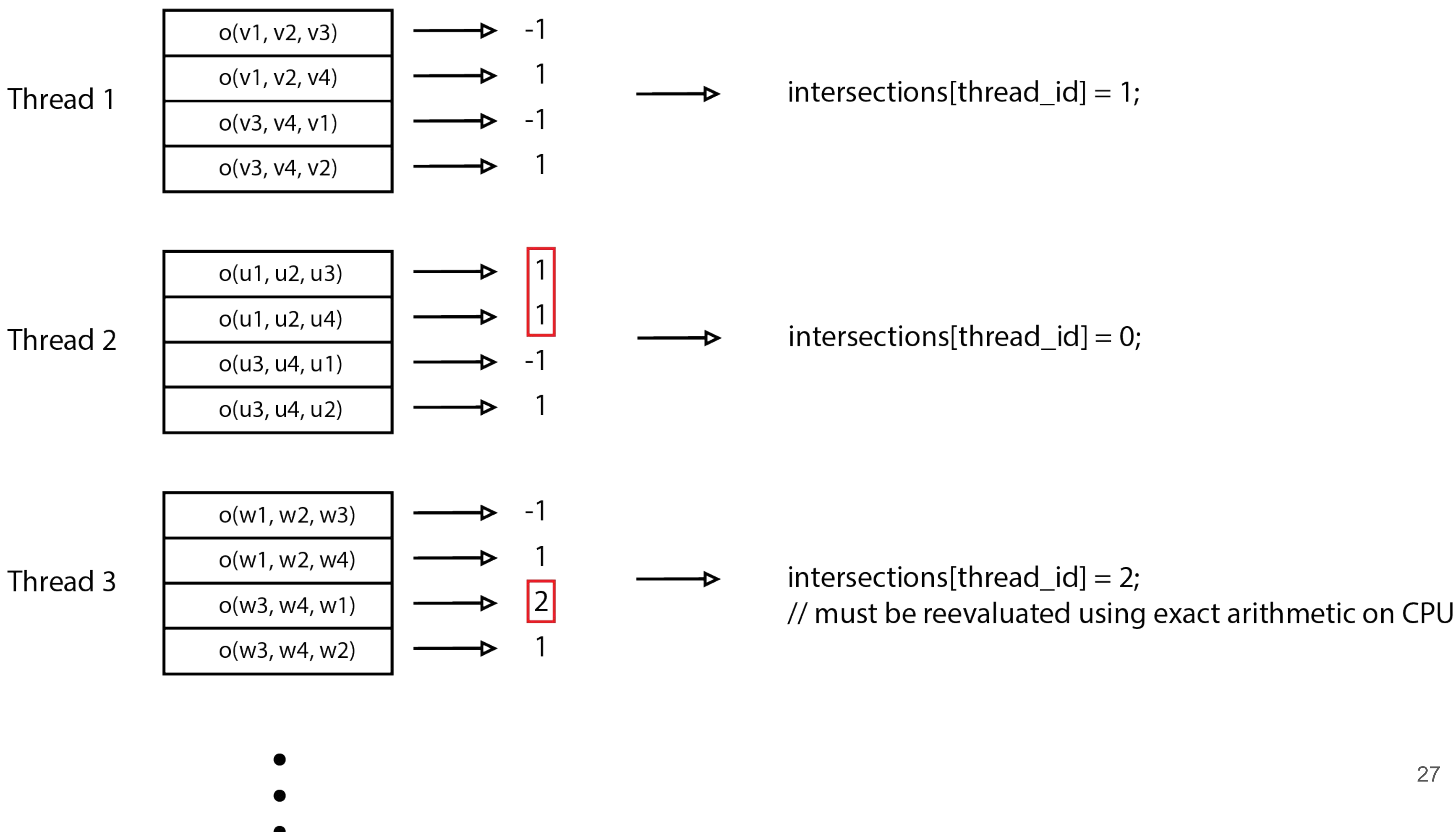
Intersection testing, 3

- A CPU implementation checks one pair of segments at a time, evaluating the four predicates in a for loop:

```
...
for (int i = 0; i < n; ++i) {
    Vertex v1 = set_1_edges[i].v1;
    Vertex v2 = set_1_edges[i].v2;
    Vertex v3 = set_2_edges[i].v1;
    Vertex v4 = set_2_edges[i].v2;

    int o1 = orientation(v1, v2, v3);
    int o2 = orientation(v1, v2, v4);
    int o3 = orientation(v3, v4, v1);
    int o4 = orientation(v3, v4, v2);

    intersections[i] = (o1 != o2) && (o3 != o4);
}
...
```

Intersection testing, 4

- List of pairs sent in one batch to GPU.
- One thread does one intersection test.



Experiments, 1

- Environment:
  - AMD Ryzen 5 processor with 6 3.2GHz cores (12 hyperthreads)
  - 16 GB of RAM
  - NVIDIA GeForce GTX 1070 Ti GPU
- Arbitrary precision arithmetic provided by the GMP library
- OpenMP for parallelizing the CPU code
- Cuda for the GPU side
- Compared against CGAL:
  - Sequential method for detecting intersections of dD Iso-oriented Boxes (pre-processing)
  - Arithmetic filtering and lazy evaluation

Experiments, 2

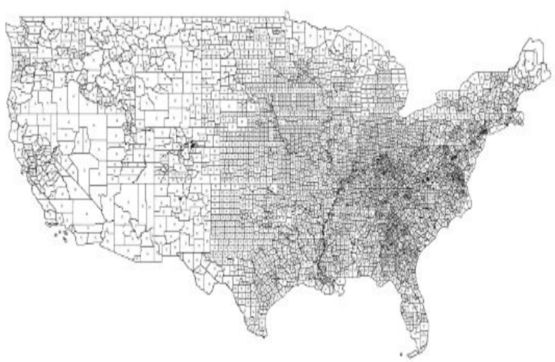
- Experiments have been performed using segments from four polygonal maps from two countries
- The intersection tests were made in pairs, using a 2500x2500 resolution uniform grid:
  - BrSoil x BrCounty
  - UsCounty x UsAquifers
  - UsCounty x UsCountyRotated
- Properties of each map:

	Pairs of maps evaluated					
	BrSoil	BrCounty	UsCounty	UsAquifers	UsCounty	UsCountyRot.
Number of segments	211,011	326,193	3,740,989	352,924	3,740,989	3,740,989
Average segment length (% of bb.)	5×10^{-4}	4×10^{-4}	8×10^{-7}	1×10^{-4}	8×10^{-7}	8×10^{-7}
Percentage of empty grid cells		86%		98%		98%
Average # pairs of segments/cell		0.3		2.0		34.7
Number of pairs of segments		300,039		12,756,283		216,542,974
Number of intersections		20,860		11,948		11,751

Experiments, 3

BrSoil and BrCounty

UsCounty and Us Aquifers



Notes on the test data

- The edge segments are very unevenly distributed.
- Most grid cells are empty, a few have many edges.
- Yet the uniform grid works well.
- Quadtrees etc are not necessary (and are slower and don’t parallelize well).
- Most intersection tests fail.
- That’s ok because they’re very fast and parallelize.

no filtering

filtering, lazy evaluation...

vs. Interval*

BrCounty and BrSoil							
	Rational*	Interval*	CGAL*	Rational	Interval	GPU	Speedup
Pre.	1.242	0.225	0.478	0.549	0.324	0.099	2
Inter.	1.444	0.152	0.015	0.385	0.040	0.018	9
Total	2.686	0.377	0.493	0.934	0.364	0.117	3
# tests	300K	300K	70K	300K	300K	300K	-

UsCounty and UsAquifers							
	Rational*	Interval*	CGAL*	Rational	Interval	GPU	Speedup
Pre.	7.884	0.812	2.628	1.610	0.392	0.164	5
Inter.	42.816	4.059	0.023	11.198	0.612	0.096	42
Total	50.700	4.871	2.651	12.808	1.004	0.260	19
# tests	13M	13M	159K	13M	13M	13M	-

UsCounty and UsCountyRotated							
	Rational*	Interval*	CGAL*	Rational	Interval	GPU	Speedup
Pre.	14.532	1.422	7.482	2.798	0.454	0.251	6
Inter.	675.616	63.677	1.027	194.918	9.422	1.367	47
Total	690.148	65.099	8.509	197.716	9.876	1.618	40
# tests	217M	217M	11M	217M	217M	217M	-

Parallel

BrCounty and BrSoil	Rational*	Interval*	CGAL*	Rational	Interval	GPU	Speedup
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# tests	217M	217M	11M	217M	217M	217M	-

CGAL: better pre-processing culling (but slower)

Interval*: faster culling and can be parallelized

Time not exactly proportional to number of tests (faster if pair does not intersect)

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Effect of arithmetic filtering

Filters failed in only 0.000002% to 0.0005% of the predicates

→ Rationals rarely necessary

→ In the GPU implementation, CPU rarely had to re-evaluate with rationals.

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# tests	217M	217M	11M	217M	217M	217M	-

Pre-processing: not entirely on the CPU -- GPU computes in which cell each vertex is.
(this is not a predicate, but can be computed with IA and filtering)

GPU pre-processing: includes copying intervals (coordinates) to the GPU.

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# tests	217M	217M	11M	217M	217M	217M	-

Inter.: 0.685s prepare + 62.992s eval. (prep. = generate (parallel) list of edges to test)

GPU : 1.149s prepare + 0.218s eval. (prep. = same as CPU + copy ids to/from GPU)

289x speedup in evaluation→Possibly better speedups in algs. w/less communication

Conclusions and future work, 1

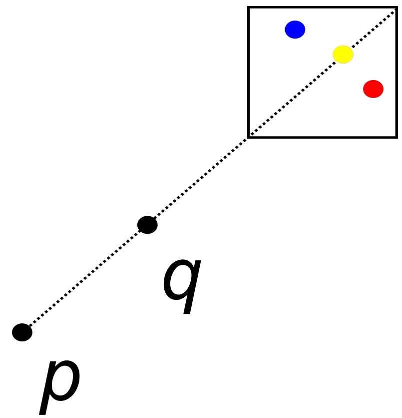
- Good for interactive applications (CAD, GIS, CG, ...)
- Intervals do not fail often (fail → re-evaluation)
- More efficient to keep data on the GPU and re-use
    - If coordinates will be re-used, copy to the GPU at the beginning of the program.
    - Use communication only for what is really necessary. (e.g.: for intersections, copy the ids of the pairs of the edges)
    - E.g.: boolean operations: detecting intersection is only one step → data can be kept on the GPU and re-used in all steps.

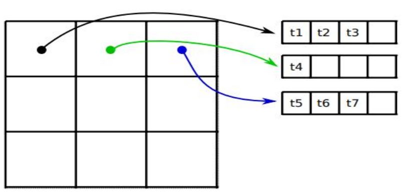
Conclusions and future work, 2

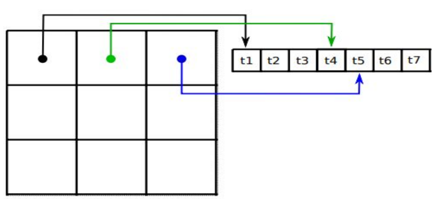
- Future work:
  - Apply to 2D/3D point location, mesh intersection and other CG algorithms.
  - Improve the performance of the predicates.
    - E.g.: reduce CPU-GPU communication overhead (move combinatorial part of the algorithm to GPU, overlap communication/processing, etc).
- Challenges:
  - Predicates must be evaluated in batch
  - Have to “manually” keep track of how each interval was generated (ok mainly when depth of the computation tree is small)
  - Intervals may fail more often in applications with deep computation trees.

Mesh intersection

Thank you!



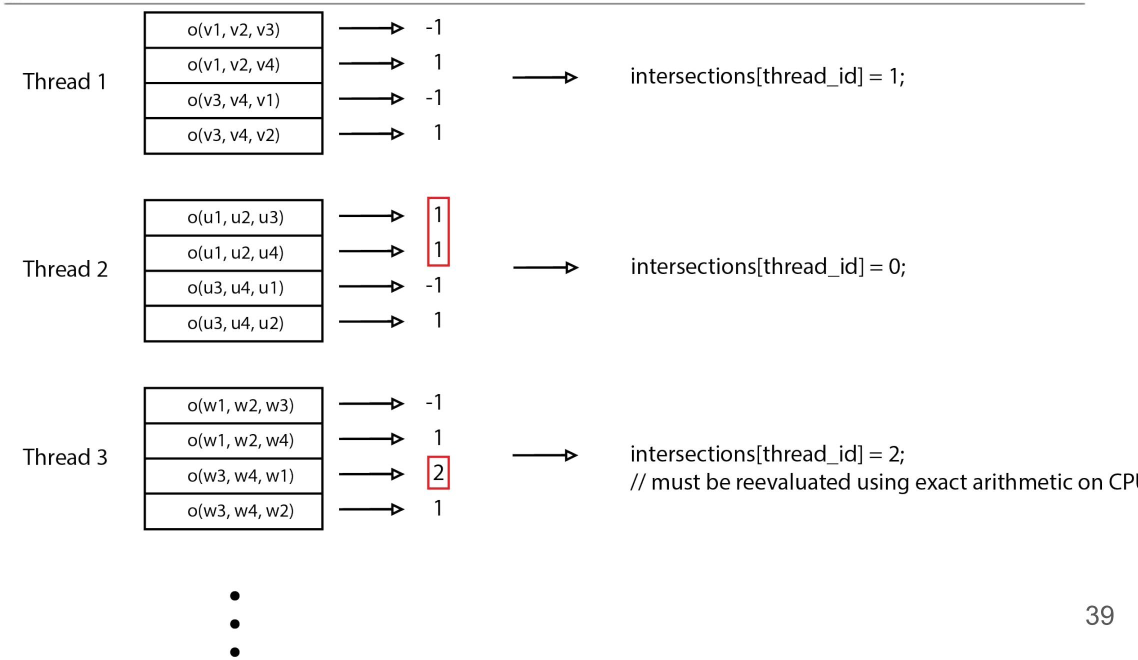




BrCounty and BrSoil							
	Rational*	Interval*	CGAL*	Rational	Interval	GPU	Speedup
Pre.	1.242	0.225	0.478	0.549	0.324	0.099	2
Inter.	1.444	0.152	0.015	0.385	0.040	0.018	9
Total	2.686	0.377	0.493	0.934	0.364	0.117	3
# tests	300K	300K	70K	300K	300K	300K	-

UsCounty and UsAquifers							
	Rational*	Interval*	CGAL*	Rational	Interval	GPU	Speedup
Pre.	7.884	0.812	2.628	1.610	0.392	0.164	5
Inter.	42.816	4.059	0.023	11.198	0.612	0.096	42
Total	50.700	4.871	2.651	12.808	1.004	0.260	19
# tests	13M	13M	159K	13M	13M	13M	-

UsCounty and UsCountyRotated							
	Rational*	Interval*	CGAL*	Rational	Interval	GPU	Speedup
Pre.	14.532	1.422	7.482	2.798	0.454	0.251	6
Inter.	675.616	63.677	1.027	194.918	9.422	1.367	47
Total	690.148	65.099	8.509	197.716	9.876	1.618	40
# tests	217M	217M	11M	217M	217M	217M	-



Acknowledgement: CAPES CNPq