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RPI UF

Fast analysis of upstream features on spatial networks

(winner, 1st place, 2018 GIS Cup)

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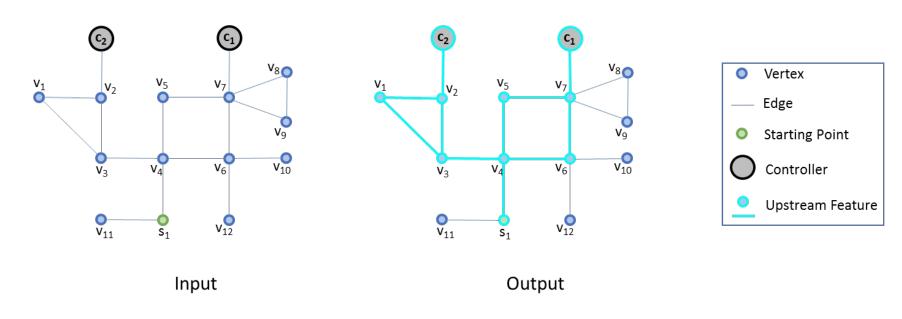
Ricardo dos Santos Ferreira, UFV





Introduction

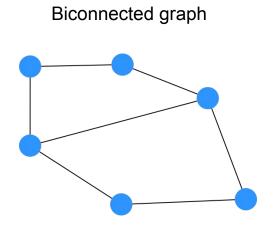
- Input: graph (JSON file), set of starting points (vertices/edges) and controllers (vertices)
- We call starting points/controllers important vertices
- Output: edges and vertices in a simple path between a controller and a starting point.
- Challenge: the number of paths between a pair of vértices may be huge (e.g.: exponential in a complete graph) → paths cannot enumerate

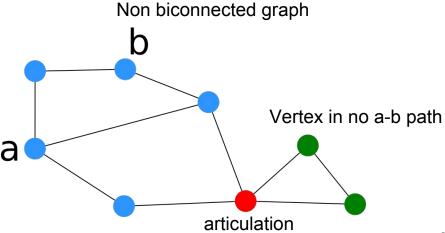


Source: GISCUP 2018

Key ideas

- Key idea: avoid enumerating all paths between controllers and starting points.
- Main concepts: articulations, biconnected components, Block-Cut trees
- Articulation: vertex which, when removed, increases the number of connected components (CCs) in a graph
- Biconnected graph: graph without articulations
- Biconnected components (or blocks): maximal biconnected subgraphs of a graph.
- **Important observation**: given two vertices a,b of a block B, any vertex of B is in an a-b path.

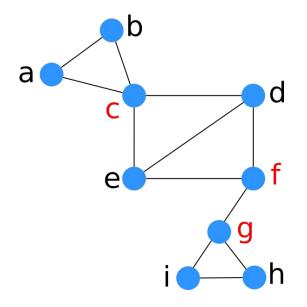




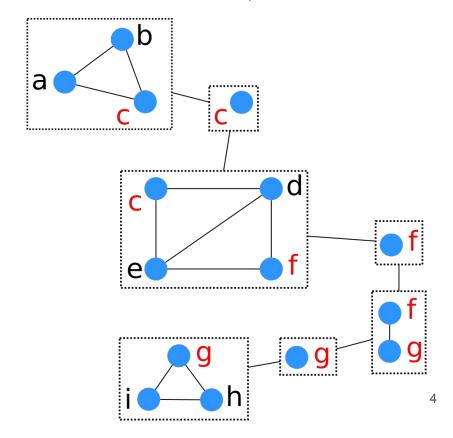
Key ideas

- Block-cut tree of a graph G (BC(G)): tree where:
 - Vertices are the blocks and articulations of G
 - There is an edge between each block B and the articulations from B.

Graph G (articulations in red)



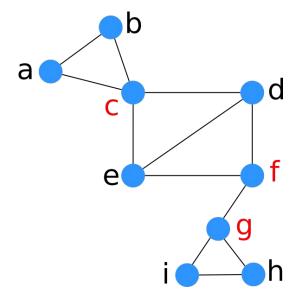
Block-cut tree of G (rectangles represent the vertices of the BC-Tree)



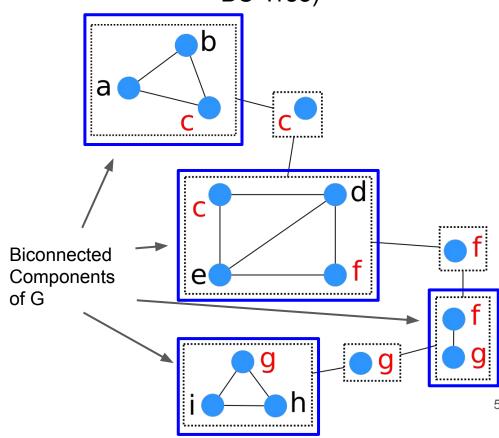
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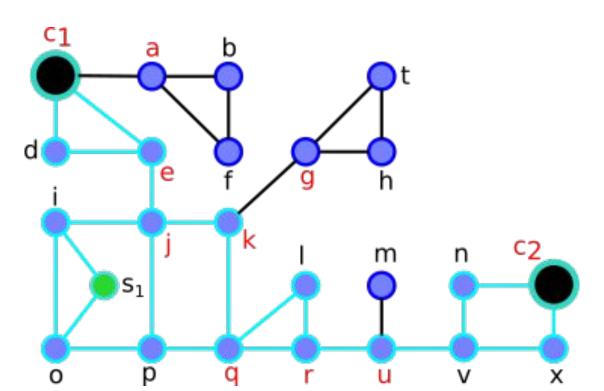
Graph G (articulations in red)



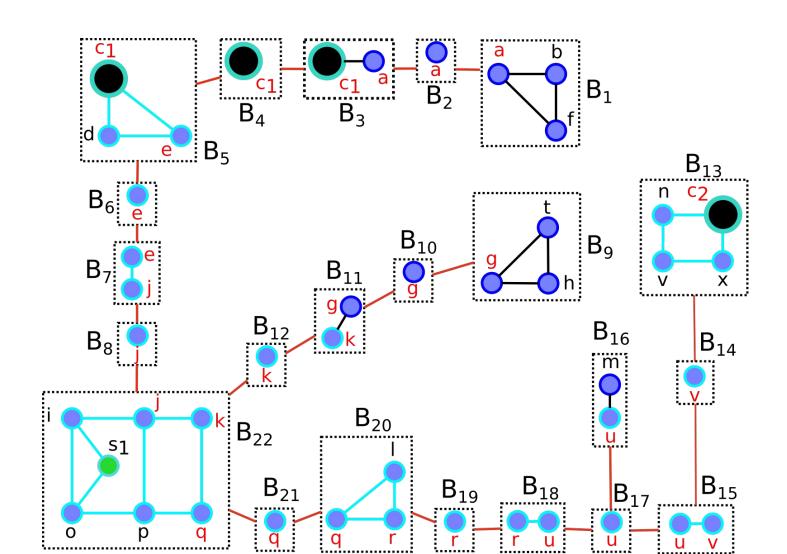
Block-cut tree of G (rectangles represent the vertices of the BC-Tree)



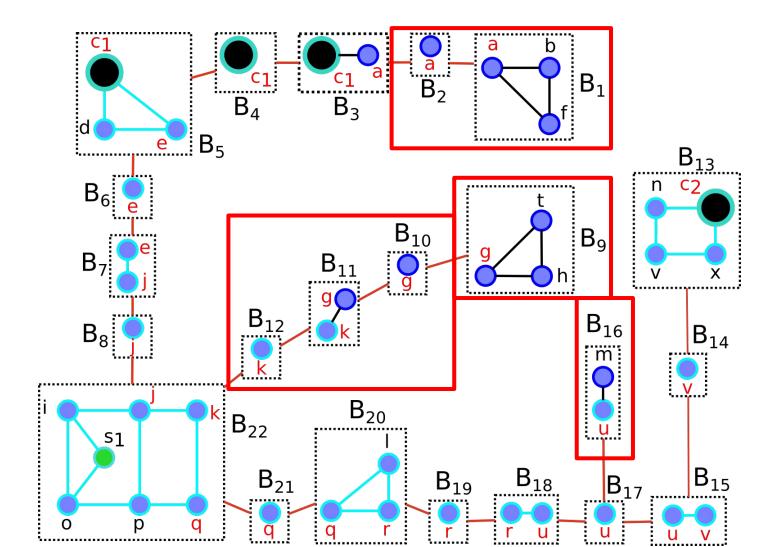
- Consider this Graph with some controllers/starting points.
 - If graph does not contain both → solution is empty
- Graph is connected (otherwise, solve for each connected component)
- Initially, suppose important features can only be vertices.
- Black vertices: controllers
- Green vertex: starting point
- Red labels: articulations
- Features detached in light blue: output features



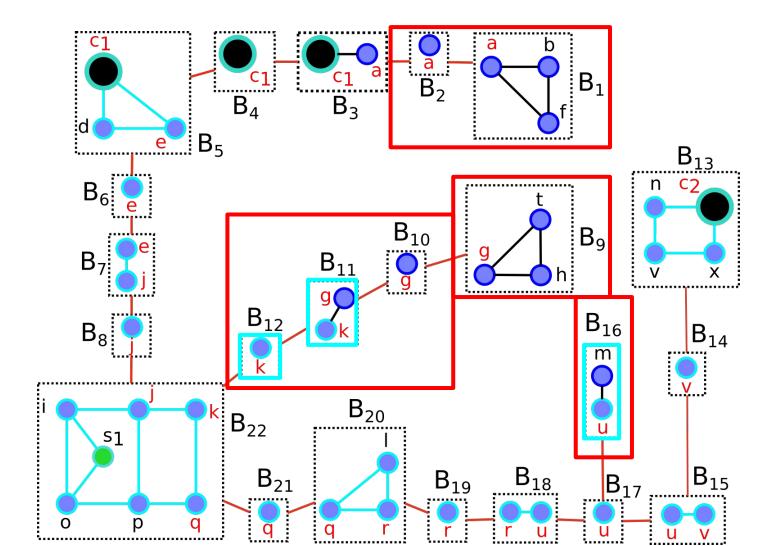
First step: create block-cut tree



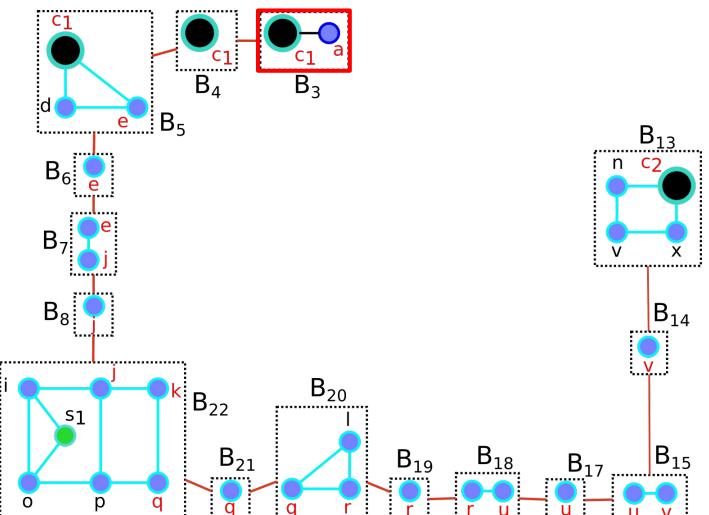
Leaves without important vértices → are in no path between important vértices
 → can be iteratively removed



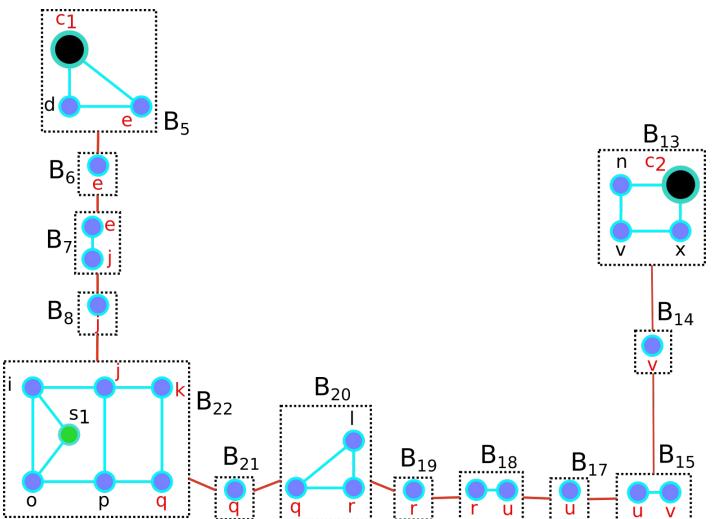
- Some of the removed blocks contain features that should be in the output.
 - This is ok → these vertices are also in neighbor (non-removed) blocks



- Now, any vertex of the BC-Tree is in a path between a vertex with a controller and one with a starting point.
- Special case: a in B_3 won't be in the output \rightarrow remove leaves where the only important vertex is an articulation still in the tree.



- Now, any vertex of the BC-Tree is in a path between a vertex with a controller and one with a starting point.
- If a block is in a path between a controller and a starting point → all vertices/edges inside this block will be (biconnected comp. property)



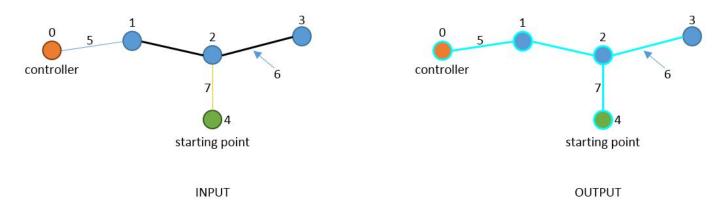
Summary of the algorithm

- 1. Find blocks and articulations (Tarjan's algorithm: O(V+E))
- 2. Create a Block-Cut tree (O(|blocks + articulations|) = O(V+E) (worst case))
- 3. Iteratively remove leaves w/o important vértices (O(|blocks + articulations|))
- 4. Output features in the remaining leaves (O(V+E))

Total complexity: O(V+E)

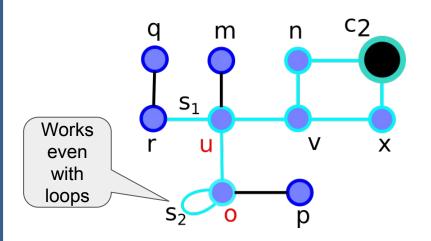
Edge starting points

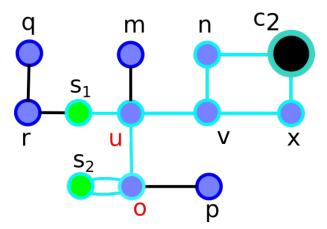
- Edges may be starting points
- Example (from GISCUP mailing list): vertex 4 and edge 6 are starting points



Source: GISCUP 2018

- Instead of treating this as a special case in the algorithm → modify the input:
 - Starting point edge e=(u,v) → edges e=(u,s), e=(s,v) and s is a starting point vertex.





Implementation details

- According to the GISCUP website, solutions are ranked by efficiency (total elapsed wall clock time) and correctness.
- Several design choices carefully taken based on experiments with varying sized datasets.

- Parser: since the total time was evaluated, we created a fast custom parser.
 - Makes some assumptions about the input file.
 - According to the organizers, feature (vertex/edges) ids are always the format "{7FC28536-6F4A-4A9A-B439-1D87AE2D8871}" → we encoded them as two 64-bit integers → faster graph creation (which requires map lookups) at a small cost of encoding/decoding the ids.
 - Algorithm is loosely coupled → other parsers can be employed if a better (reusable) code is desired.

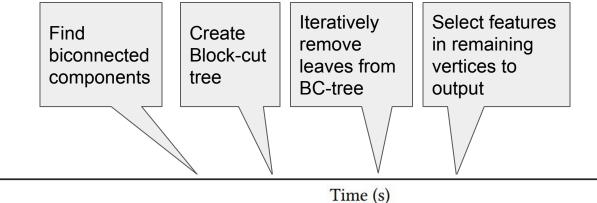
Implementation details

- Tarjan's algorithm for computing biconnected components:
 - We implemented a custom version
 - Since entire graph has to be traversed:
 - Also finds connected components (CCs)
 - Ignores CCs without both controllers and starting points
 - Labels the biconnected component with information about the types of vertices in them (controllers, starting points, regular vertices)
 - Tarjan's algorithm is typically recursive. We implemented it iteratively to avoid stack overflows in big graphs.

- Parallel programming: we parallelized the main bottlenecks of the algorithm using OpenMP.
 - Graph (adjacency list) is represented using a ragged array and constructed in parallel.
 - Maps with the ids of the vértices (0,1,....V-1) and edges are created in parallel.
 - etc.

- 8-core Xeon Processor, 64 GB of RAM, Linux OS
- First dataset: GISCUP Esri's Naperville Electric Network Dataset
- Other ones: randomly created

Number of



Dataset	Vertices	Edges	Biconnected Components	Output features	Graph creation	Tarjan	BC-tree	Remove leaves	Select output	Total	Total w/ IO
EsriNaperville	8465	8302	7859	34	0.001	0.000	0.001	0.000	0.000	0.002	0.016
Graph1	1M	16M	1	17000000	0.780	0.565	0.013	0.000	0.292	1.650	6.368
Graph2	3M	3.3M	1866755	2566493	0.221	0.599	0.209	0.062	0.057	1.148	2.233
Graph3	3M	5M	435857	7128288	0.303	0.769	0.105	0.011	0.091	1.279	2.842
Graph4	5M	8M	843737	11312528	1.227	1.603	0.276	0.023	0.193	3.322	6.160
Graph5	13M	16M	5414204	18171594	0.939	3.047	0.842	0.163	0.449	5.440	10.703
Graph6	16M	16M	15999979	39	1.020	2.937	1.216	0.536	0.117	5.826	10.882
Graph6_100	16M	16M	15999979	1142	1.028	2.939	1.445	0.533	0.112	6.057	11.065
Graph6_1M	16M	16M	16500369	7336103	1.064	3.055	2.658	0.444	0.753	7.974	15.049
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- 8-core Xeon Processor, 64 GB of RAM, Linux OS
- First dataset: GISCUP Esri's Naperville Electric Network Dataset
- Other ones: randomly created
- Bottleneck: I/O and Tarjan's algorithm (DFS, data dependency, non parallelizable)
- Other steps are more parallelizable:
 - 5x speedup on Graph Creation (largest test case)
 - 2x speedup on the creation of the BC-Tree (largest test case)

2											
		N	lumber of	Time (s)							
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- 8-core Xeon Processor, 64 GB of RAM, Linux OS
- First dataset: GISCUP Esri's Naperville Electric Network Dataset
- Other ones: randomly created
- Little variation on graphs with different amounts of starting points/controllers.
 - Detached: 1 starting point, 1 controller

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- 8-core Xeon Processor, 64 GB of RAM, Linux OS
- First dataset: GISCUP Esri's Naperville Electric Network Dataset
- Other ones: randomly created
- Little variation on graphs with different amounts of starting points/controllers.
 - Detached: 100 starting points 100 controllers

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	8465 1M 3M 3M 5M 13M 16M 16M	Vertices Edges 8465 8302 1M 16M 3M 3.3M 3M 5M 5M 8M 13M 16M 16M 16M 16M 16M 16M 16M	Components 8465 8302 7859 1M 16M 1 3M 3.3M 1866755 3M 5M 435857 5M 8M 843737 13M 16M 5414204 16M 16M 15999979 16M 16M 15999979	Vertices Edges Biconnected Components Output features 8465 8302 7859 34 1M 16M 1 17000000 3M 3.3M 1866755 2566493 3M 5M 435857 7128288 5M 8M 843737 11312528 13M 16M 5414204 18171594 16M 16M 15999979 39 16M 16M 15999979 1142	VerticesEdgesBiconnected ComponentsOutput featuresGraph creation846583027859340.0011M16M1 170000000.7803M3.3M186675525664930.2213M5M43585771282880.3035M8M843737113125281.22713M16M5414204181715940.93916M16M15999979391.02016M16M1599997911421.028	Vertices Edges Biconnected Components Output features Graph creation Tarjan creation 8465 8302 7859 34 0.001 0.000 1M 16M 1 17000000 0.780 0.565 3M 3.3M 1866755 2566493 0.221 0.599 3M 5M 435857 7128288 0.303 0.769 5M 8M 843737 11312528 1.227 1.603 13M 16M 5414204 18171594 0.939 3.047 16M 16M 15999979 39 1.020 2.937 16M 16M 15999979 1142 1.028 2.939	Vertices Edges Biconnected Components Output features Graph creation Tarjan BC-tree 8465 8302 7859 34 0.001 0.000 0.001 1M 16M 1 17000000 0.780 0.565 0.013 3M 3.3M 1866755 2566493 0.221 0.599 0.209 3M 5M 435857 7128288 0.303 0.769 0.105 5M 8M 843737 11312528 1.227 1.603 0.276 13M 16M 5414204 18171594 0.939 3.047 0.842 16M 16M 15999979 39 1.020 2.937 1.216 16M 16M 15999979 1142 1.028 2.939 1.445	Vertices Edges Biconnected Components Output features Graph creation Tarjan Tarjan BC-tree leaves Remove leaves 8465 8302 7859 34 0.001 0.000 0.001 0.000 1M 16M 1 17000000 0.780 0.565 0.013 0.000 3M 3.3M 1866755 2566493 0.221 0.599 0.209 0.062 3M 5M 435857 7128288 0.303 0.769 0.105 0.011 5M 8M 843737 11312528 1.227 1.603 0.276 0.023 13M 16M 5414204 18171594 0.939 3.047 0.842 0.163 16M 16M 15999979 39 1.020 2.937 1.216 0.536 16M 16M 15999979 1142 1.028 2.939 1.445 0.533	Vertices Edges Biconnected Components Output features Graph creation Tarjan BC-tree leaves Remove leaves Select output 8465 8302 7859 34 0.001 0.000 0.001 0.000 0.000 1M 16M 1 17000000 0.780 0.565 0.013 0.000 0.292 3M 3.3M 1866755 2566493 0.221 0.599 0.209 0.062 0.057 3M 5M 435857 7128288 0.303 0.769 0.105 0.011 0.091 5M 8M 843737 11312528 1.227 1.603 0.276 0.023 0.193 13M 16M 5414204 18171594 0.939 3.047 0.842 0.163 0.449 16M 16M 15999979 39 1.020 2.937 1.216 0.536 0.117 16M 16M 15999979 1142 1.028 2.939 1.445 0.533 0.112	Vertices Edges Biconnected Components Output features Graph creation Tarjan BC-tree leaves Remove leaves Select output Total output 8465 8302 7859 34 0.001 0.000 0.001 0.000 0.000 0.002 1M 16M 1 17000000 0.780 0.565 0.013 0.000 0.292 1.650 3M 3.3M 1866755 2566493 0.221 0.599 0.209 0.062 0.057 1.148 3M 5M 435857 7128288 0.303 0.769 0.105 0.011 0.091 1.279 5M 8M 843737 11312528 1.227 1.603 0.276 0.023 0.193 3.322 13M 16M 5414204 18171594 0.939 3.047 0.842 0.163 0.449 5.440 16M 16M 15999979 39 1.020 2.937 1.216 0.536 0.117 5.826

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- First dataset: GISCUP Esri's Naperville Electric Network Dataset
- Other ones: randomly created
- Little variation on graphs with different amounts of starting points/controllers.
 - Detached: 1M starting points, 1M controllers
 - 37% slower than the dataset with 1 controller (same network)

Graph1	and Ed	VI N			Time (s)										
Graph1	les Eu	O	Biconnected Components	Output features	Graph creation	Tarjan	BC-tree	Remove leaves	Select output	Total	Total w/ IO				
	465 83	3302	7859	34	0.001	0.000	0.001	0.000	0.000	0.002	0.016				
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Graph6_1M 16	6M 10	16M	16500369	7336103	1.064	3.055	2.658	0.444	0.753	7.974	1 <u>5.0</u> 49				

Conclusions

- Linear-time algorithm to find upstream features in utility networks
- Implemented in parallel
- (asymptotic) Time independent of the number of starting points/controllers

More broadly:

We could process a billion element (a full continent) dataset in under 10 minutes on a \$5000 workstation. We would not need

- Cloud computing Hadoop Spark
- Supercomputing
- → MPI

Thank you

