Exact intersection of 3D geometric models

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Abstract. *This paper presents* 3D-EPUG-OVERLAY*, an exact and parallel algorithm for computing the intersection of 3D triangulated meshes, a problem with applications in several fields such as GIS and CAD.* 3D-EPUG-OVERLAY *presents several innovations: it employs exact arithmetic and, thus, the computations are completely exact, which avoids topological impossibilities that are often created by floating point arithmetic. Also, it uses a uniform grid to index the geometric data and was designed to be easily parallelizable. Furthermore, we propose the use of Simulation of Simplicity to effectively ensure that all the special cases are properly handled by the algorithm.*

1. Introduction

Computing intersections or overlays is a very important operation for CAD systems, GIS, computer games and computational geometry. In 2D, given a pair of maps A and B, that are composed of faces or polygons representing partitions of the E^2 plane, the overlay of A with B is a map C where each polygon is the intersection of a polygon of A with a polygon of B. For example, the intersection of a map representing the states of the United States with a map representing the American drainage basins is another map where the polygons represent the portion of each basin that is in each state. These operations also extend to 3D datasets. An example of application in 3D is to compute the intersection between solids representing layers of soil with a solid representing a section of the soil that will be excavated.

According to Feito et al. [Feito et al. 2013], although 3D models have been widely used in computer science, processing them is still a challenge. Due to the algorithm implementation complexity (that usually needs to handle several special cases), the necessity of processing big volumes of data and precision problems caused by floating point arithmetic, software packages occasionally "fail to give a correct result, or they refuse to give a result at all" [Feito et al. 2013]. The likelihood of failure increases as datasets get bigger.

Even though in some situations an algorithm that occasionally fails is acceptable, it is often important to have an algorithm that is both efficient and robust. This is mainly important when the algorithm is used as subroutines for other algorithms.

Since in many applications it is important to have exact algorithms, [Hachenberger et al. 2007] presented an algorithm for computing the exact intersection of Nef polyhedra. The basic idea of a Nef polyhedron is to represent the polyhedron as a finite sequence of complement and intersection operations on half-spaces [Hachenberger et al. 2007]. Because of their importance, the algorithms proposed by Hachenberger were implemented in the CGAL library[CGAL 2016]. According to Leconte et al. [Leconte et al. 2010], even though these algorithms are exact they have some limitations such as their poor performance. Besides the performance problems, another limitation of the Hachenberger's algorithms is the fact that they were designed to handle geometric data represented as Nef Polyhedra, what is not as widely used as other representations such as triangular meshes.

[Bernstein and Fussell 2009] also presented an intersection algorithm that tries to achieve robustness. Their basic idea is to represent the polyhedra using binary space partitioning (*BSP*) trees with fixed-precision coordinates. As the authors mention [Bernstein and Fussell 2009], the main limitation is that the process to convert BSPs to widely used representations (such as meshes) is slow and inexact.

In previous works we have developed exact and efficient algorithms for processing 2D (polygonal maps) and 3D models (triangulated meshes). More specifically, we have successfully developed algorithms for intersecting polygonal maps [Magalhães et al. 2015] and performing point location queries [Magalhães et al. 2016] in both polygonal maps and 3D meshes. These algorithms employ a combination of 5 separate techniques to achieve both robustness and efficiency. Exact arithmetic is employed to completely avoid errors caused by floating point numbers. Special cases are treated using *Simulation of Simplicity* (SoS) [Edelsbrunner and Mücke 1990]. The computation is performed using simple local information to make the algorithm easily parallelizable and to easily ensure robustness. Since the use of exact arithmetic is expected to add an overhead, efficient indexing techniques and High Performance Computing (HPC) are used to mitigate this.

In all these algorithms our spatial data is represented using simple topological formats. The 2D maps are represented using sets of oriented edges where each edge contains the information (labels) of the polygons on its positive and negative sides. In 3D, the meshes are represented using a set of oriented triangles and each triangle has labels to the polyhedra on its positive and negative sides.

In this paper we will present a brief description of these previous works and present our current research: 3D-EPUG-OVERLAY (3D-Exact Parallel Uniform Grid-Overlay), a parallel algorithm for exactly intersecting 3D triangulated meshes.

2. Roundoff errors

Usually, non-integer numbers are approximately represented in computers with floating-point values. The difference between the value of a non-integer number and its approximation is often referred as roundoff error. Even though these differences are usually small, arithmetic operations frequently create more errors, which accumulate becoming larger.

In geometry, roundoff errors can generate topological inconsistencies causing globally impossible results for predicates like point inside polygon. For example, [Kettner et al. 2008] presented an interesting study of the failures caused by roundoff errors in geometric problems such as the planar orientation computation.

Several techniques have been proposed in order to overcome this problem. The simplest one consists of using an ϵ tolerance to consider two values x and y are equal if $|x -$

$y| \leq \epsilon$. However this is a formal mess because equality is no longer transitive, nor invariant under scaling. In practice, epsilon-tweaking fails in several situations [Kettner et al. 2008].

Snap rounding is another method to approximate arbitrary precision segments into fixed-precision numbers [Hobby 1999]. However, snap rounding can generate inconsistencies and deforms the original topology if applied consecutively on a data set. Some variations of this technique attempt to get around these issues [Hershberger 2013, Belussi et al. 2016].

[Shewchuk 1996] presents the Adaptive Precision Floating-Point technique, that focus on exactly evaluating predicates. The idea is to perform this evaluation using the minimum amount of precision necessary to achieve correctness. As mentioned by the author, this technique focus on geometric predicates and it is not suitable to solve all geometric problems. For example, “a program that computes line intersections requires rational arithmetic” [Shewchuk 1996].

The formally proper way to eliminate roundoff errors and guarantee robustness is to use exact computation based on rational number with arbitrary precision [Li et al. 2005, Hoffman 1989, Kettner et al. 2008]. In this work, our algorithms perform computation using arbitrary precision rationals provided by GMP library. Computing in the algebraic field of the rational numbers over the integers, with the integers allowed to grow as long as necessary, allows the traditional arithmetic operations to be computed exactly, with no roundoff error. The cost is that the number of digits in the result of an operation is about equal to the sum of the numbers of digits in the two inputs. This behavior is acceptable if the depth of the computation tree is small, which is true for the algorithms we will present.

Besides ensuring exact results in the predicates and arithmetic computations, the use of arbitrary precision rational numbers has other advantages. First, Simulation of Simplicity [Edelsbrunner and Mücke 1990], the technique we use for treating the special cases, requires exact arithmetic. Second, our algorithms will be able to support input geometrical data where the coordinates are represented using rationals and, thus, we will be able to process meshes that cannot be exactly represented using floating point arithmetic.

3. Previous works

In this section, EPUG-OVERLAY [Magalhães et al. 2015] and PINMESH [Magalhães et al. 2016], two previous algorithms developed for, respectively, intersecting maps and performing point location queries in 3D meshes will be presented.

Before presenting these two algorithms, two important techniques applied in these works will be briefly described: the use of a uniform grid for indexing the data and the application of the Simulation of Simplicity technique for handling special cases. Both techniques will be also applied in the intersection algorithm described in this paper.

The understanding of EPUG-OVERLAY, PINMESH and of the sections 3.1 and 3.2 is important because techniques similar to the ones described in these sections are applied on 3D-EPUG-OVERLAY.

3.1. Indexing data with a uniform grid

[Franklin et al. 1989] proposed a uniform grid to accelerate his algorithm for computing the area of overlaid polygons. When a polygonal map (or triangular mesh) is indexed with a uniform grid, a 2D grid (or 3D grid for meshes) is created, superimposed over the input datasets and, then, the edges (or triangles) intersecting each cell c are inserted into c. After the grid is created, it can be employed to accelerate the geometric algorithms. For example, given two maps indexed by the grid, the intersection of pairs of edges from the two maps can be found by processing each cell and comparing the edges in that cell pair-by-pair (one edge from each map) to compute the intersection points.

The uniform grid works well even for unevenly distributed data for various reasons [Akman et al. 1989, Franklin et al. 1988]. First, the total time is the sum of one component (constructing the grid) that runs slower with a finer grid, plus other components (e.g., intersecting edges) that run faster. The total running-time varies slowly with changing grid resolutions. Second, an empty grid cell is very inexpensive, so that sizing the grid so that the geometric objects in the dense part of the data are well distributed works.

Nevertheless, to process very uneven data, in EPUG-OVERLAY and PINMESH we have incorporated a second level grid into those few cells that are densely populated. The exact criteria for determine what cell to refine depends on the algorithm that will use the grid. For example, since in the intersection computation pairs of edges in the cells are tested for intersection, one could refine the grid cells where the number of intersection tests (i.e, the number of pairs of edges from the two maps) is greater than a threshold.

This nesting could be recursively repeated until all grid cells have fewer elements than a given threshold, creating a structure similar to quadtree (or octree), although with more branching. However, the general solution uses more space for pointers (or is expensive to modify) and is irregular enough that parallelization is difficult. Also, experiments have shown that the best performance is achieved using just a second level [Magalhães et al. 2015]. This can be explained because the first level grid, in general, has many cells with more elements than the threshold justifying the second level refinement. But, in the second level, only a few number of cells exceed the threshold and the overhead (processing time and memory use) to refine those cells is never recaptured.

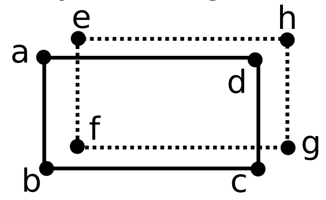
3.2. Simulation of Simplicity

To correctly handle the special cases (for example, coincident edges or triangles) we apply the Simulation of Simplicity (SoS) [Edelsbrunner and Mücke 1990] technique. This is a general purpose symbolic perturbation technique designed to treat special (degenerate) cases. The inspiration for SoS is that if the coordinates of the points are perturbed the degeneracies disappear. However, too big a perturbation may create new problems, while a too small one may be ineffective because of the limited precision of floating point numbers.

SoS is a solution that uses a symbolic perturbation by an indeterminate infinitesimal value ϵ^i, for some natural number i. Its mathematical formalization extends some exactly computable field, such as rationals, by adding orders of infinitesimals, ϵ^i. Floating point numbers with roundoff error cannot be the base. The infinitesimal ϵ is an *indeterminate*. It has no meaning apart from the rules for how it combines. All positive first-order infinitesimals are smaller than the smallest positive number. All positive second-order infinitesimals are smaller than the smallest positive first-order infinitesimal, and so on. All this is logically consistent and satisfies the axioms of an abstract algrebra field.

The result of SoS is that degeneracies are resolved in a way that is globally consistent. For example, consider Figure 1 : two identical rectangles ($abcd$ represented using solid edges and *efgh* represented using dashed edges) are overlaid, but all the vertices of *efgh* are slightly translated using the vector (ϵ, ϵ^2). This translation is globally consistent, i.e., even if the rectangle is stored as separate edges an intersection test with edge *ef* will return true only when this test is performed against the edge *ad* while an intersection test performed with *gf* will return true only when the test is performed against *cd*.

Figure 1. Effect caused by SoS during the intersection computation.



The infinitesimals do not need to be explicitly used in the program since they will be used only to determine signs of expressions. The only time that the infinitesimals change the result is when there is a tie in a predicate. Then, the infinitesimals break the tie. The effect is to make the code harder to write and longer. However, unless a degeneracy occurs, the execution speed is the same. When a degeneracy does occur, the code is slightly slower.

3.3. Point location

PINMESH [Magalhães et al. 2016] is an exact and efficient algorithm for performing point location queries in 3D meshes. It is based on the idea of ray-casting: given a query point q, a semi-infinite vertical ray r starting on q is traced and, then, the mesh triangle t that intersects r in the lowest point is used to determine q's location. Since t is the lowest triangle to intersect r, because of the Jordan Curve Theorem, q will necessarily be on the polyhedron below t (this polyhedron can be quickly determined since all triangles contain the labels of the two polyhedra it bounds).

For performance, a uniform grid is applied to reduce the amount of ray-triangle intersection computation tests. Furthermore, empty grid cells (that will, necessarily, be completely inside a polyhedron) are labeled with the polyhedra where they are located (these labeles are used to accelerate the queries). As a result of a careful implementation and use of parallelization, PINMESH is very efficient, being able to index a dataset and perform 1 million queries on a 16-core processor up to 27 times faster than RCT [Liu et al. 2010], a sequential and inexact algorithm (that was the current fastest one).

In PINMESH all coordinates are represented using rational numbers and, as a consequence, there is no roundoff error. Furthermore, special cases are handled using simulation of simplicity: the idea is that all the triangles are translated using an infinitesimal vector (ϵ,ϵ^2,ϵ^3) and, as showed in [Magalhães et al. 2016], after this translation no special case (for example, the one that happens when r hits a triangle vertex and, thus, there is a tie in the process of selecting the lowest triangle hit by r) will happen.

3.4. Exact 2D map overlay

EPUG-OVERLAY [Magalhães et al. 2015] is an exact and efficient algorithm for overlaying two polygonal maps. Given two maps $\mathcal{A}$ and $\mathcal{B}$ composed of faces represented implicitly as a set of edges, the goal is to create a map where each face represents the intersection of a face of $\mathcal{A}$ with a face of $\mathcal{B}$. Parallel programming associated with an efficient indexing made EPUG-OVERLAY very efficient. Indeed, according to the experiments shown in [Magalhães et al. 2015], it was able to process maps with more than 50 million edges faster than GRASS GIS (that is sequential, but does not use exact arithmetic).

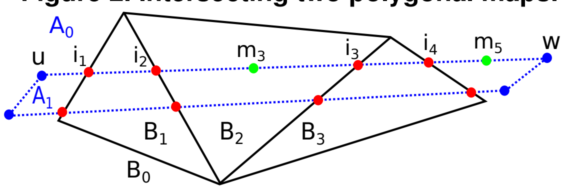
As described in [Magalhães et al. 2015], EPUG-OVERLAY is composed of the following basic steps:

- **Create the 2-level uniform grid**: First, a two-level uniform grid is created to index the edges from the two input maps A and B.
- **Compute the intersection points between all edges of maps A and B**: the uniform grid is applied to accelerate the process of computing the intersection of the edges in the two maps. For each grid cell c, pairs of edges (from the two input maps) in c are tested for intersection and the intersections are computed. The intersecting edges are split at the intersection point and, after that, edges intersections will happen only in vertices.
- **Classify the resulting edges**: after the input edges are split at the intersection points, the labels indicating the polygons bounded by each edge are updated.

Figure 2 illustrates this process: the map A (in dotted blue) contains 4 edges and two polygons (polygon A_1 and polygon A_0, representing the exterior of the map) while map B (solid black lines) contains 7 edges and 4 polygons. After the intersections are detected and the edges are split at the intersection points (in red) the resulting edges are classified. For example, the edge (u, w) bounds polygons A_0 (positive side) and A_1 (negative side). Edge (i_2, i_3) (generated after (u, w) was split) is inside polygon B_2 of the other map and, thus, in the output map (i_2, i_3) will bound polygon $A_0 \cap B_2$ (this polygon is equivalent to the exterior of the resulting map) on its positive side and $A_1 \cap B_2$ on the negative side.

Since the edges are split at the intersection points, after this process all the edges will be completely inside a polygon of the other map. Thus, one strategy to determine in what polygon an edge e is consists in using a fast 2D point location algorithm to locate a point from e in the other map (for example, the location of m_3 from Figure 2 can determine in what polygon (i_2, i_3) is).

Figure 2. Intersecting two polygonal maps.



This strategy uses only local information to compute the intersection, i.e., instead of intersecting pairs of faces the individual edges are intersected and classified and the resulting faces will be represented implicitly by the edges. This has several advantages. First, it is easier to test a pair of edges for possible intersection than to test a pair of faces (which would devolve to testing pairs of edges anyway). Second, knowing an intersection of a pair of edges contributes information about four output faces. Third, as an edge is fixed size but a face is not, parallel operations on edges are more efficient.

Degenerate cases are handled with *Simulation of Simplicity (SoS)*. The idea is to pretend that map $\mathcal{A}$ is slightly below and to the left of map $\mathcal{B}$. Thus no edge from $\mathcal{A}$ will coincide with an edge from $\mathcal{B}$ during the intersection computation. Oversimplified slightly, the process proceeds by translating map $\mathcal{B}$ by (ϵ, ϵ^2), where ϵ is an infinitesimal.

As mentioned before, we do not actually compute with infinitesimals, but instead determine the effect that they would have on the predicates in the code, and modify the predicates to have the same effect when evaluated as if the variables could have infinitesimal values. For instance, the test for $(a_0 \leq b_0)\&(b_0 \leq a_1)$ becomes $(a_0 \leq b_0)\&(b_0 < a_1)$. With SoS, no point in $\mathcal{A}$ is identical to any point in $\mathcal{B}$ neither do two any edges coincide.

4. Exact 3D mesh intersection

Similarly to our 2D intersection algorithm, in 3D the computation is performed using only local information stored in the individual triangles. That is, the triangles from one mesh are intersected with the triangles from the other one. Then, a new mesh containing the triangles from the two original meshes is created and the original triangles are split in the intersection points. That is, if a pair of triangles in this new mesh intersect, then this intersection will happen necessarily in a common edge or vertex. Finally, the adjacency information stored in each triangle is updated to ensure that the new mesh will consistently represent the intersection of the original ones.

4.1. Intersecting triangles and remeshing

For performance, a strategy similar to the one used in EPUG-OVERLAY was adopted: for each uniform grid cell, the intersections between pairs of triangles from the two triangulations are computed. The pairs of triangles are intersected using the algorithm presented by Möller [Möller 1997], that uses several techniques to avoid unnecessary computation by detecting as soon as possible if the pair of triangles does not intersect.

More specifically, a two-level 3D uniform grid is employed to accelerate the computation using an strategy similar to the one we used in the 2D map intersection algorithm. That is, the grid will be created by inserting in its cells triangles from both meshes M_1 and M_2. Then, for each grid cell c, the pairs of triangles from both meshes in c are intersected. If the resolution of the uniform grid is chosen such that the expected number of triangles per grid cell is a constant K, then it is expected that each triangle will be tested for intersection with the other K triangles in its grid cell. Thus, the expected total number of intersections tests performed will be linear on the size of the input maps.

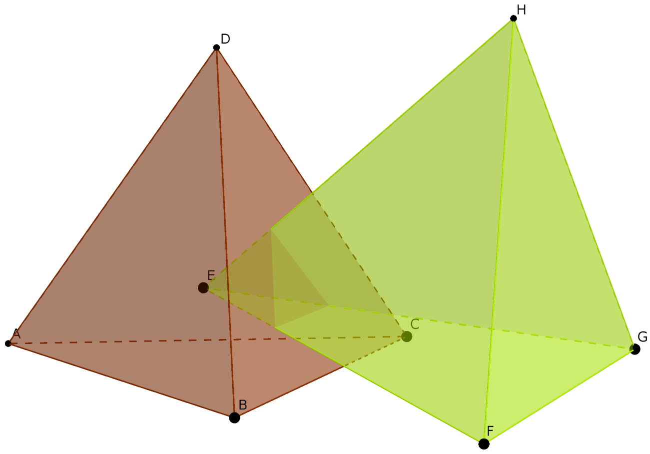
Since there is no influence of one cell in the other ones, the process of intersecting the triangles can be trivially parallelized: the grid cells can be processed in parallel by different threads using a parallel programming API such as OpenMP.

After computing the intersections between each pair of triangles, the next step is to split the triangles where they intersect (creating new ones) such that, after this processing, all the intersections will happen only in common vertices or edges. When a triangle is split, the attributes (that is, the ids of the two objects it bounds) will be copied to the new triangles. This process is similar to the 2D map overlay step where the edges are split at the intersection points to ensure that all intersections happen in vertices.

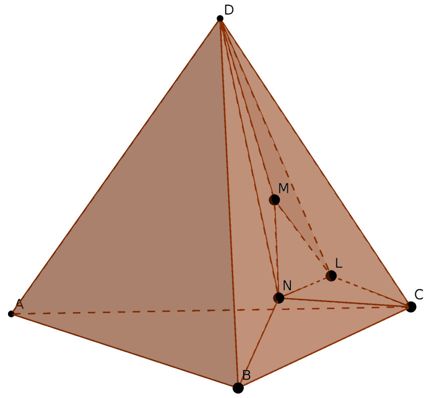
Figure 3 presents an example of intersection computation. In Figure 3 (a), we have two meshes representing two tetrahedral with one region in each one: the brown mesh (mesh M_1) bounds the exterior region and region 1 while the yellow mesh (mesh M_2) bounds the exterior region and region 2.

After the intersections between the triangles are computed, the triangles from one mesh that intersect triangles from the other one are split in several triangles, creating meshes

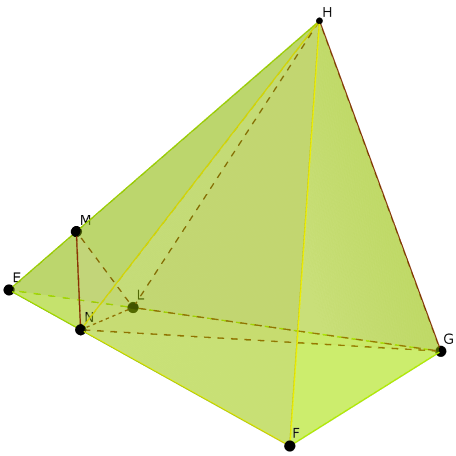
Figure 3. Computing the intersection of two tetrahedra.



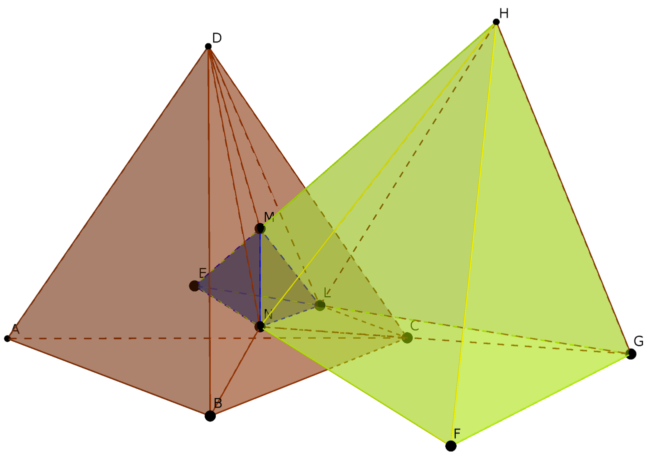
(a)



(b)



(c)



(d)

M_1' and M_2' (for clarity, these two meshes are displayed separately in Figures 3 (b) and (c), respectively). The only triangle from mesh M_1 that intersects mesh M_2 is the triangle BCD. Since BCD intersects three triangles from M_2, it was split in 7 triangles when M_1' was created (triangles LMN, CLN, CBN, BDN, DMN, DLM and CDL). Similarly, each of the three triangles from M_2 intersecting M_1 was split in 3 smaller triangles.

4.2. Classifying triangles

After the intersections are detected and all the triangles that intersect other triangles are split at the intersection points, two new meshes M_1' and M_2' are created such that each new mesh M_i' will have the following two kinds of triangles:

- Triangles from the original mesh: if a triangle t from M_i did not intersect any triangle from the other mesh (or if this intersection was located on a vertex or edge), then t will be in M_i'.
- New triangles: if a triangle t from M_i intersects one or more triangles from the other mesh (and this intersection is not located on a common vertex or edge), then t will be split into several smaller triangles and these smaller triangles will be inserted into M_i'.

It is clear that each mesh M_1' will exactly represent the same regions that M_1 represents. In fact, if no triangle from M_1 intersects the mesh M_2, then M_1' will be equal to M_1. Otherwise, each triangle t from M_i that intersects M_2 will be split in n triangles $t_1, t_2, ..., t_n$ and these new triangles will be inserted into M_i' instead of t. Since the union of the triangles $t_1, t_2, ..., t_n$ is t and these split triangles contain the same attributes as t, then M_1' represents the same regions M_1 represents. This observation is also valid for M_2'.

Thus, computing the intersection between M_1' and M_2' is equivalent to computing the intersection of M_1 with M_2. However, M_1' and M_2' are easier to process: since the triangles from one mesh intersect with the triangles of the other one only in common vertices or edges, then each triangle t from M_1' will be completely inside a region from M_2'. Suppose a triangle t from M_1' bounds regions R_a and R_b and is completely inside region R_c from mesh M_2'. When M_1' is intersected with M_2', t will be in the resulting mesh and it will bound regions $R_a \cap R_c$ and $R_b \cap R_c$. The same process can be performed with the triangles from M_2.

Therefore, the process of classifying the triangles to create the output mesh consists in processing each triangle t from the mesh M_1', determining in what region of M_2' t is and, then, updating the information about the regions t bounds such that we will have a consistent mesh. The same process needs to be performed with triangles from M_2'.

To determine in what region from the other mesh a triangle is, the point location algorithm presented in section 3.3 is applied. That is, since point location queries can be quickly performed, an efficient way to locate a triangle that is completely inside a region consists in locating one of its interior points (for example, its centroid).

Similarly to the other steps, the classification can also be performed in parallel: since updating the regions that a triangle bounds does not influence other triangles, all the triangles from the two meshes can be processed in parallel.

If a triangle t is in the exterior of the other mesh, in the resulting mesh the two regions t bounds will be the exterior region. To maintain the mesh consistency, the triangles bounding only the exterior region can be ignored and not stored in the output mesh.

Figure 3 (d) illustrates the classification step. All the intersections happen in common edges and the only triangle from M_1' that is completely inside region 2 (of M_2') is triangle LMN. Since LMN bounds region 1 and the exterior region in M_1', in the resulting intersection LMN will bound region $1 \cap 2$ and the exterior region. All the other triangles from M_1' are in the exterior region of M_2' and, thus, they will only bound the exterior region in the resulting intersection (therefore, they will be ignored when the output mesh is computed). Similarly, in M_1' the only triangles that are inside region 1 of M_1' are triangles EMN, ELM and ELN. These three triangles will also bound the exterior region and region $1 \cap 2$ in the resulting mesh.

4.3. Handling the special cases

The current version of 3D-EPUG-OVERLAY does not handle special cases (degeneracies) yet. However, the ideas we intend to apply in order to handle these cases have already been successfully implemented for EPUG-OVERLAY and PINMESH and, therefore, we believe they will suitable to 3D-EPUG-OVERLAY.

Since it is usually difficult to guarantee that all degeneracies are considered (this is particularly true in 3D, where more special cases may happen than in 2D), we intend to develop a symbolic perturbation scheme similar to the one used to ensure the special cases in the point location algorithm (PINMESH) are treated. The idea is that one of the meshes will be infinitesimally translated and, as a result, singularities such as the one that happens when a triangle from one mesh intersects a co-planar triangle from the other mesh (and, thus, the intersection of them is not necessarily a line segment) will never happen.

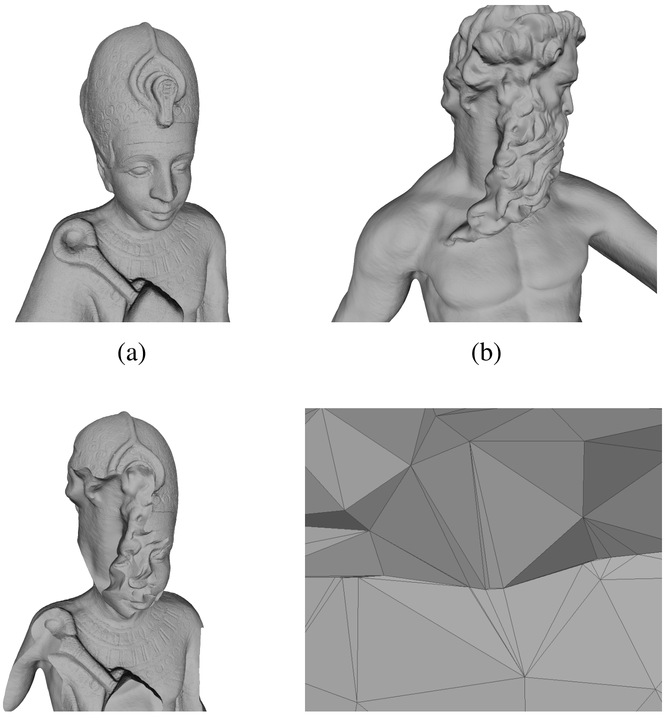
An adequate perturbation scheme associated with the use of exact arithmetic and a careful implementation will ensure our intersection algorithm is robust.

5. Preliminary results

3D-EPUG-OVERLAY was implemented in C++ and several experiments have been performed. Figure 4 presents an example of intersection computed using 3D-EPUG-OVERLAY: the model Ramesses (a) and Neptune (b) were intersected. These two models were downloaded from the AIM@SHAPE repository [AIM@SHAPE 2016] and were produced by, respectively, the users Marco_Attene and Laurent_Saboret. The Ramesses model contains more than 1 million triangles while the Neptune model contains more than 4 million triangles. Figure 4 (c) presents the result of the intersection.

Figure 4 (d) presents a zoom that detaches the region of the resulting mesh where the triangles from the two models intersect. As it can be seen, the remeshing process generate several thin triangles (displayed in the vertical center of the figure), that are usually hard to process mainly by methods based on floating-point arithmetic.

Figure 4. Computing the intersection of two 3D models.



Since some features of 3D-EPUG-OVERLAY (such as the use of SoS to handle special cases) are still under implementation and the main feature of 3D-EPUG-OVERLAY is its exactness, we have decided to optimize its performance after these features are implemented. However, we intend to employ as the main optimization techniques the same strategies that were successfully employed to optimize previous works such as EPUG-OVERLAY and PINMESH: trading memory for computation, i.e., pre-computing results that will be necessary often such that these results can be reused; parallelization of the bottlenecks of the algorithm using OpenMP: similarly to our previous work, 3D-EPUG-OVERLAY was designed specifically for being easily parallelizable ; reduction of the memory allocations on the heap: heap allocations cannot be performed in parallel efficiently and, thus, our previous experience has showed that it should be avoided mainly inside parallelized blocks of code. Since as rationals grows memory needs to be allocated, in previous works we pre-allocated the temporary rationals necessary in computations and, thus, avoided creating them inside the parallelized functions.

Since these techniques were sucessfully applied to our previous works (and, as result, they even outperformed inexact algorithms), we believe they will also make 3D-EPUG-OVERLAY very efficient.

6. Conclusions and future works

This paper presented 3D-EPUG-OVERLAY, an exact and parallel algorithm for computing the intersection of 3D models represented by triangulated meshes. 3D-EPUG-OVERLAY uses arbitrary precision rational numbers to store all the geometric coordinates and perform computation and, thus, is roundoff error free.

Even though the current implementation of 3D-EPUG-OVERLAY does not treat special cases, preliminary experiments have indicated that 3D-EPUG-OVERLAY can successfully intersect some big meshes available in public repositories.

As future work, we intend to implement a symbolic-perturbation scheme on 3D-EPUG-OVERLAY to ensure that all the special cases are properly handled. Furthermore, optimization techniques similar to the ones applied in some of previous works will be also applied to 3D-EPUG-OVERLAY.

7. Acknowledgement

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