Local topology and parallel overlaying large planar graphs

W. Randolph Franklin and Salles V. G. de Magalhães, RPI

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Goal of this talk

- minimal geometry representations for polygons etc.
- applied to overlaying two plane graphs (GIS maps), combining
  - minimal reps, for simplicity,
  - uniform grid, for fast intersection detection,
  - rational numbers, to prevent roundoff errors,
  - Simulation of Simplicity, for degeneracies,
  - OpenMP, for parallel speedup.
- big example: overlay two maps (US Water Bodies, US Block Boundaries)
- 54,000,000 vertices, 737,000 faces
- 149 elapsed seconds (plus 116s for I/O).
- next step: overlay 3D meshes.

My background

- Philosophically a Computer Scientist.
- PhD officially in Applied Math.
- Working in Electrical, Computer, and Systems Engineering Dept.
- Students in Computer Science
- Teaching Engineering Parallel Computing.
- Collaborating with Geographers for a long time.
- Enjoy applying computer science & engineering to geometry & GIS.

Aim

- new ways to look at relations between objects in space
- to facilitate spatial operations
  - area
  - overlay
- what is minimal explicit type of info need?
  - fewer special cases
  - less code
  - less debugging
- goal: to do something
  - better,
  - faster,
  - in parallel,
  - on bigger datasets
- All this is intended to be used.

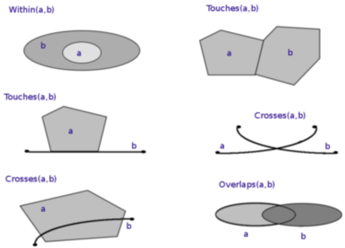
to·pol·o·gy

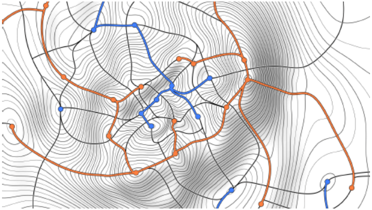
tpälj/
noun

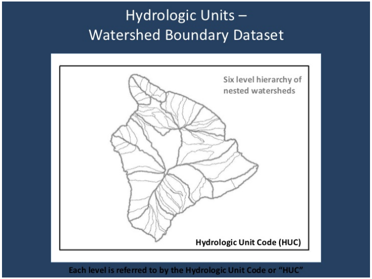
1. . . .
2. the way in which constituent parts are interrelated or arranged. "the topology of a computer network"
3. I'll include local geometry
   - location
   - directions
4. Contrast to more global topology
   - complete edges, faces (however, will use these sometimes)
   - edge loops, face shells
   - hierarchies of inclusions

Prior art

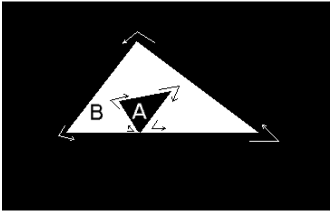
- 9 relations in topology
- Morse complexes
- hydrography hierarchy
- winged edges, half edges
- manifold objects
- regularized set ops







More prior art



Winged edge

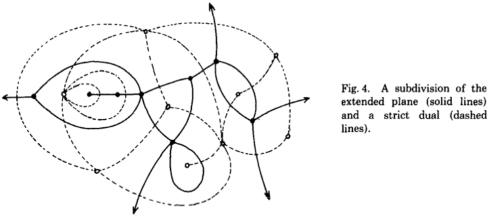
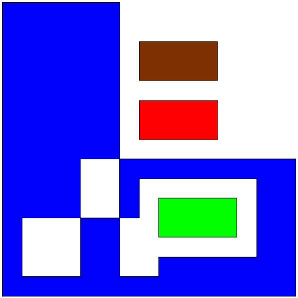


Figure 4 in Guibas and Stolfi. Primitives for the manipulation of general subdivisions and the computation of Voronoi Diagrams

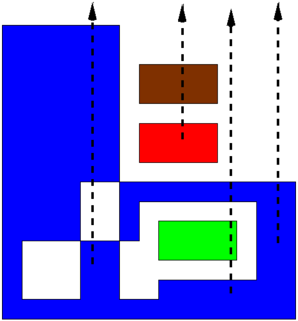
How little info does a polygon need?

- Set of vertices is ambiguous.
- Set of edges is good.
  - point in polygon
  - area, center of gravity
- The computation is a map-reduce.



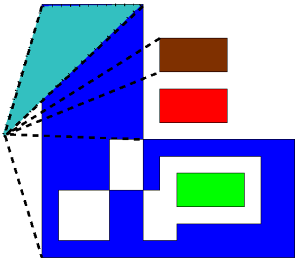
Point Inclusion Testing on a Set of Edges

- "Jordan curve" method
- Extend a semi-infinite ray.
- Count intersections.
- Odd $\equiv$ inside.
- *Obvious but bad alternative:* sum subtended angles. Implementing w/o arctan, and handling special cases wrapping around 2π is tricky and reduces to Jordan curve.



Area Computation on a Set of Edges

- Each edge, with the origin, defines a triangle.
- Sum.
- Extends to any mass property, including (using a characteristic function) point inclusion.

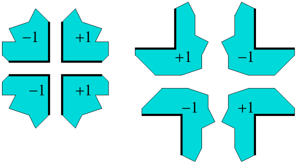


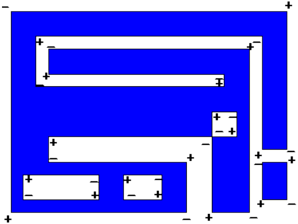
Advantages of Set of Edges Data Structure

- Simple enough to debug.
- "SW can be simple enough that there are obviously no errors, *or* complex enough that there are no obvious errors."
- Less space to store.
- Easy parallelization.
  - Partition edges among processors.
  - Each processor sums areas independently, to produce one subtotal.
  - Total the subtotals.

Augmented vertices: another minimal polygon representation

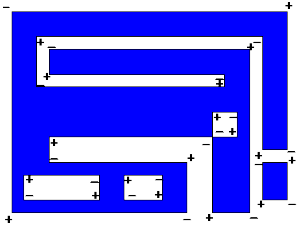
- Augmented vertices: add a little to each vertex.
- My examples will use rectilinear polygons, but all this works on general polygons
- 8 types of vertices.
- Assign a sign, $s = \pm 1$ to each type.
- Now, each vertex defined as $v_i = (x_i, y_i, s_i)$





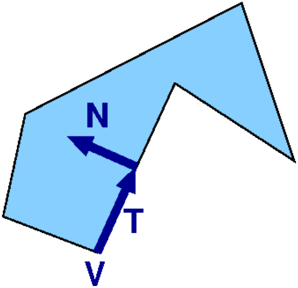
What augmented vertices can do

- Area: $A = \sum x_i y_i s_i$



Vertex incidences: YAMPR

- Another minimal data structure.
- like half edges.
- Only data type is incidence of an edge and a vertex, and its neighborhood. For each such:
  - V = coord of vertex
  - T = unit tan vector along edge
  - N = unit vector normal to T pointing into the polygon.
- Polygon: {(V, T, N)} (2 tuples per vertex)
- Perimeter $= -\sum(V \cdot T)$.
- Area $= 1/2 \sum(V \cdot T)(V \cdot N)$
- Multiple nested components ok.
- Parallelizable



But... don't we always know the edges?

(so what's the point of this?)

- Not always.
- Compute the area of the intersection of two polygons.
- Application: how much do they interfere?
- We know the input polygons' edges.
- However finding the output polygon's edges is harder than merely finding the augmented vertices.
- Two types of output vertices:
  - Some input vertices,
  - Some intersections of input edges.
- All output vertices must be inside an input polygon.
- Find candidate output vertices by intersecting pairs of input edges.
- Filter.
- Apply area equation to surviving vertices.

Map overlay

- Input: two maps containing sets of polygons (aka faces).
- Output: all the nonempty intersections of one polygon from each map.
- Example: Census tracts with watershed polygons, to estimate population in each watershed.
- Salles Viana Gomes de Magalháes presented this at BIGSPATIAL in Nov.
- However, first some foundations:

Parallel and memory notes

Massive shared memory

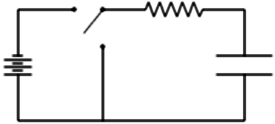
- is an underappreciated resource.
- External memory algorithms are not needed for many problems.
- Virtual memory is obsolete.
- $40K buys a workstation with 80 cores and 1TB of memory.

parallel computing

- Almost all processors, even my smart phone, are parallel.
- Algorithms that don't parallelize are obsolete.
- One Xeon core is 20x more powerful than one CUDA core.
- Nvidia GPUs are almost ubiquitous.

Why parallel HW?

- More processing $\rightarrow$ faster clock speed.
- Faster $\rightarrow$ more electrical power. Each bit flip (dis)charges a capacitor through a resistance.
- Faster $\rightarrow$ requires smaller features on chip
- Smaller $\rightarrow$ **greater** electrical resistance !
- $\Longrightarrow\Longleftarrow$.
- Serial processors have hit a wall.



Parallel HW features

- IBM Blue Gene / Intel / NVidia GPU / other
- Most laptops have NVidia GPUs.
- Thousands of cores / CPUs / GPUs
- Lower clock speed 750MHz vs 3.4GHz
- Hierarchy of memory: small/fast $\rightarrow$ big/slow
- Communication cost $\gg$ computation cost
- Efficient for blocks of threads to execute SIMD.
- OS, per 6/2013 `http://top500.org` :

runs on 187th fastest machine

& variants run on 1st through 186th.

Massive Shared Memory

- Massive shared memory is an underappreciated resource.
- External memory algorithms are not needed for most problems.
- Virtual memory is obsolete.
- $40K buys a workstation with 80 cores and 1TB of memory.

```
const long long int n(5'000'000'000);
static long long int a[n];
int main() {
double s(0);
for (auto &e in a) e = i;
for (auto e in a)  s += e;
std::cout << "n=" << n << ", s="
          << s << std::endl; }
```

Runtime: 60 secs w/o opt to loop and r/w 40GB. (6 nsec / iteration)

Parallel computing

- We use OpenMP (w. shared memory) and CUDA/Thrust (w. Nvidia GPU).
- Our machine:
  - dual 8-core Intel Xeon: 32 hyperthreads.
  - 128GB main memory.
  - Peak Linpack speed: 358Gflops.
  - (Compare: Apple 6s iPhone: 1Gflops.)
  - Nvidia K20Xm compute processor: 2496 CUDA cores @ 706MHz, 6GB memory.
  - cost in 2012 $<$ \$15K.

OpenMP

- Shared memory, multiple CPU core model.
- Good for moderate, not massive, parallelism.
- Easy to get started.
- Options for protecting parallel writes:
  - Sum reduction: no overhead.
  - Atomic add and capture: small overhead.
  - Critical block: perhaps 100K instruction overhead.
- Only valid cost metric is real time used.

- Programs with 2 threads can execute more slowly than with one.

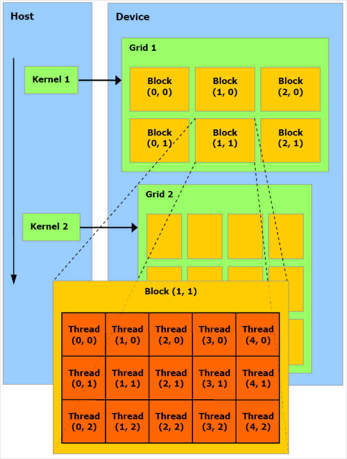
OpenMP Example

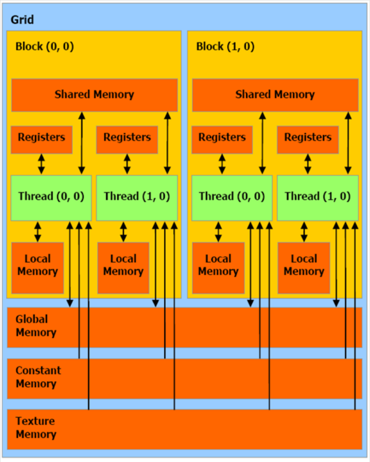
```
const int n(500000000);
int a[n], b[n];
int k(0);
int main () {
  #pragma omp parallel for
  for(int i = 0; i < n; i++) a[i]=i;
  #pragma omp parallel for
  for(int i = 0; i < n; i++) {
    #pragma omp atomic capture (or critical)
    j = k++;
    b[j] = j; }
  double s(0.);
  #pragma omp parallel for reduction(+:s)
  for (int i=0;i<n;i++) s+=a[i];
  cout << "sum: " << s << endl; }
```

CUDA

- NVIDIA's parallel computing platform and programming model.
- C++ small language extensions and functions
- CUDA compiler nvcc picks this apart.
- Direct access to complicated GPU architecture.
- Nontrivial learning curve: Efficient programming is an art.
- Assists like Unified Virtual Addressing trade execution vs programming speed.
- My advice: don't over optimize; next generation will be different.

GPU Architecture





Thrust

- C++ template library for CUDA based on STL.
- Functional paradigm: can make algorithms easier to express.
- Hides many CUDA details: good and bad.
- Powerful operators all parallelize: scatter/gather, reduction, reduction by key, permutation, transform iterator, zip iterator, sort, prefix sum.
- Surprisingly efficient algorithms like bucket sort.
- Possible back ends: CUDA, OpenMP, sequential on host.

Thrust Example

```
struct dofor {
  __device__  void operator()(int &i) { i *=2; } };
int main(void) {
  thrust::device_vector<int> X(10);
  thrust::sequence(X.begin(), X.end()); // init to 0,1,
  thrust::fill(Z.begin(), Z.end(), 2);   // fill with 2
  // compute Y = X mod 2
  thrust::transform(X.begin(), X.end(), Z.begin(),
      Y.begin(), thrust::modulus<int>());
  thrust::for_each(X.begin(), X.end(), dofor());
  thrust::copy(Y.begin(), Y.end(),      // print Y
  std::ostream_iterator<int>(std::cout, "\n")); }
```

Other techniques used in big example

- rational numbers
- simulation of simplicity
- uniform grid

Multiprecision big rationals

- Solves problem of roundoff error when intersecting lines.
- Slivers no longer matter.
- Code runs slower, but ok.
- Efficiency concerns:
  - Number size depends on computation tree depth. Ok.
  - Millions of heap allocations are inefficient, esp. in parallel. Not ok.
    - Not mentioned in documention; must infer from experiments.
    - Use Google's allocator.
    - Refactor code to minimize allocations.

Simulation of simplicity

- Solves problem of geometric degeneracies.
- E.g., vertex of one map coinciding with vertex of the other map.
- Pretends to add a different order of infinitesimal to each coordinate in one map.
- $(x_i, y_i, z_i) \rightarrow (x_i + \epsilon^{3i}, y_i + \epsilon^{3i+1}, z_i + \epsilon^{3i+2})$
- Now, coincidences cannot happen, even in intersections.
- Implementation: analyze what effect these infinitesimals would have on every predicate in the program, and
- Recode all the predicates.
- $if(a_1 \leq b \& b \leq a_2)$ becomes $if(a_1 \leq b \& b < a_2)$

Uniform grid

Summary

- Overlay a uniform 3D grid on the universe.
- For each input primitive — face, edge, vertex — find overlapping cells.
- In each cell, store set of overlapping primitives.

Properties

- Simple, sparse, uses little memory if well programmed.
- Parallelizable.
- Robust against moderate data nonuniformities.
- Bad worst-case performance on extremely nonuniform data.
- As do octree and all hierarchical methods.

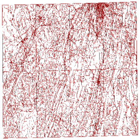
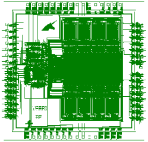
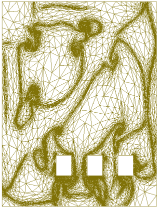
How it works

- Intersecting primitives must occupy the same cell.
- The grid filters the set of possible intersections.

Uniform Grid Qualities

- Major disadvantage: It's so simple that it apparently cannot work, especially for nonuniform data.
- Major advantage: For the operations I want to do (intersection, containment, etc), it works very well for any real data I've ever tried.
- Outside validation: used in our 2nd place finish in November's ACM SIGSPATIAL GIS Cup award.

USGS Digital Line Graph; VLSI Design; Mesh

Uniform Grid Time Analysis

For i.i.d. edges (line segments), show that time to find edge–edge intersections in E^2 is linear in size(input+output) regardless of varying number of edges per cell.

- N edges, length 1/L, $G \times G$ grid.
- Expected # intersections $= \Theta\left(N^2 L^{-2}\right)$.
- Each edge overlaps $\leq 2(G/L+1)$ cells.
- $\eta \triangleq$ # edges per cell, is Poisson; $\overline{\eta} = \Theta(N/G^2(G/L+1))$.
- Expected total # xsect tests: $G^2\overline{\eta^2} = N^2/G^2(G/L+1)^2$.
- Total time: insert edges into cells + test for intersections. $T = \Theta\left(N(G/L+1) + N^2/G^2(G/L+1)^2\right)$.
- Minimized when $G = \Theta(L)$, giving $T = \Theta\left(N + N^2L^{-2}\right)$.
- $= \Theta$ (size of input + size of output).

$\blacksquare$

Five components of big example

- simple flat topologically local data structures
- parallelizable
- uniform grid
- simulation of simplicity
- rational numbers

Next: Salles's ACM BIGSPATIAL talk

Future Modeling of Valid Terrain

My big long-term unsolved problem is to devise a mathematics of terrain.

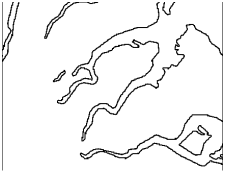
Goals: Math that

- allows the representation of only legal terrain (= height of land above geoid),
- minimizes what needs to be stated explicitly, and
- enforces global consistencies.

Why? To put compression and other ops on a logical foundation.

Terrain properties

- Messy, not theoretically nice.
- Often discontinuous (C^{-1}).
- Many sharp local maxima.
- But very few local minima.
- Lateral symmetry breaking — major river systems.
- Different formation processes in different regions.
- Features do not superimpose linearly; two canyons cannot cross and add their elevations.
- C^{∞} linear systems, e.g,. Fourier series, are wrong.
- Multiple related layers (elevation, slope, hydrology).



Peninsulas or fjords?

Current representations

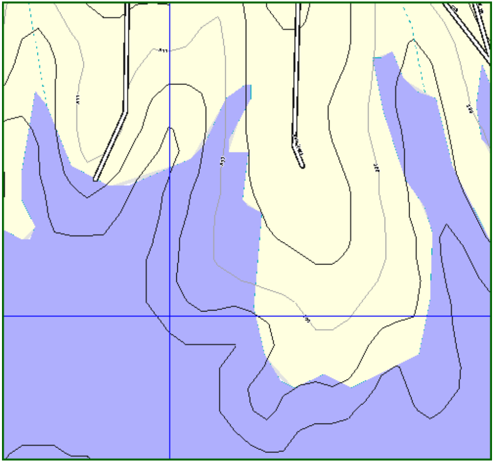
- Array of elevation posts.
- Triangular splines, linear or higher.
- Fourier series.
- Wavelets

Theory vs practice:

- Slope is derivative of elevation, but
- that amplifies errors, and
- lossy compression has errors, so
- maybe we want to store it explicitly.

Also, shoreline is a level set, but see next slide.

Inconsistencies between layers



Elevation contours crossing shoreline

Math should match physics

- Fourier series appropriate for small vibrations, not terrain.
- Truncating a series produces really bad terrain.
- Anything, like Morse complexes, assuming continuity is irrelevant.
- Fractal terrain is not terrain.
- Wavelets: how to enforce long-range consistency?
- Topology, by itself, is too weak.
- Terrain is not linear, not a sum of multiples of basis function.

Terrain formation by scooping

- Problem: Determine the appropriate operators, somewhere inside the range from conceptually shallow (ignoring all the geology) to deep (simulating every molecule).
- One solution: Scooping. Carve terrain from a block using a scoop that starts at some point, and following some trajectory, digs ever deeper until falling off the edge of the earth.
- Properties: Creates natural river systems w cliffs w/o local minima.
- Every sequence of scoops forms a legal terrain.
- Progressive transmission is easy.

(Chris Stuetzle, *Representation and generation of terrain using mathematical modeling*, PhD, 2012.)

Terrain formation by features

- Represent terrain as a sequence of features — hills, rivers, etc ..
- plus a combining rule.
- This matches how people describe terrain.
- Progressive transmission.
- The intelligence is in the combining rule.

How compact is this rep? How to evaluate it?

Implications of a better rep

- Put earlier empirical work on a proper foundation.
- Formal analysis and design of compression.
- Maximum likelihood interpolation, w/o artifacts.
- Treat more sophisticated metrics, like suitability for operations like path planning, or recognizability.
- Close the loop to pre-computer descriptive geometry.

Rensselaer Polytechnic Institute
Universidade Federal de Viçosa

RPI - Rensselaer Polytechnic Institute

PhD research: An efficient algorithm for computing the exact overlay of triangulations

Salles Viana Gomes de Magalhães, PhD. Student
Prof. Dr. W Randolph Franklin, RPI/Supervisor
Prof. Dr. Marcus V. A. Andrade, UFV
Wenli Li, PhD. Student

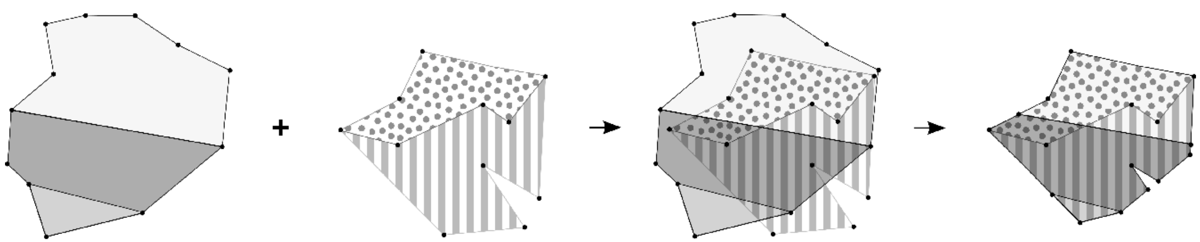
Myself

- Universidade Federal de Vicosa, Brazil – 2005-2010.
  - GIS since 2007
  - Areas: HPC, GIS, algorithms ...
  - Dr. Andrade

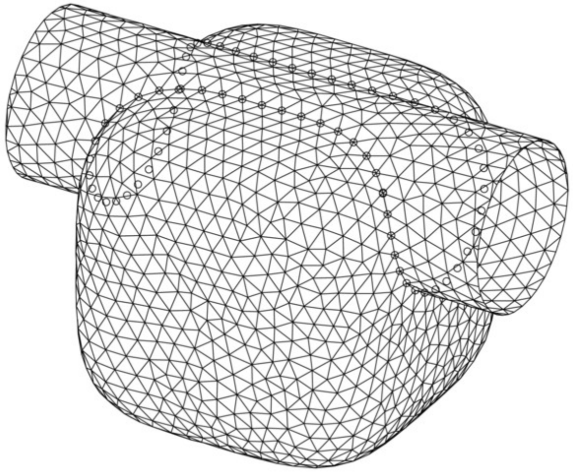
- 2014: Rensselaer Polytechnic Institute.
  - Dr. Franklin
  - Dr. Andrade
  - Wenli Li

Map overlay

- Two vectorial maps are superimposed.
- The intersection between polygons from the two maps is computed.
- Several applications. Ex: counties and watersheds.



- This problem extends to 3D objects (triangulations).
- Example: layers of soil *x* polyhedron representing excavation section.

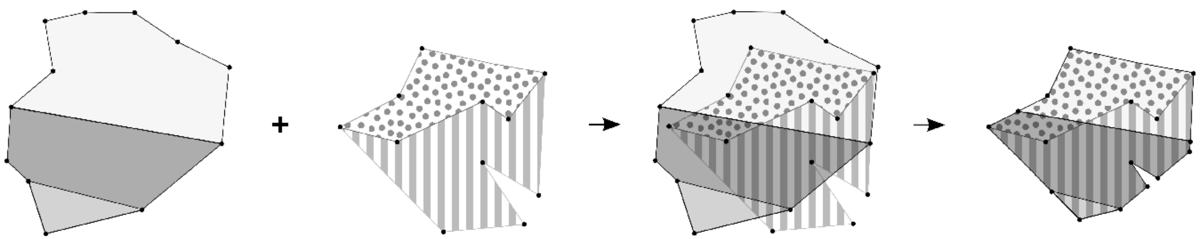


Challenge

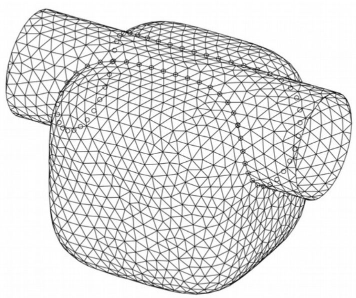
- Finite precision of floating point → roundoff errors.
  - Common techniques: no guarantee.
- Big amount of data & 3D → increase problem.
- Proposed solution: EPUG-OVERLAY and 3D-EPUG-OVERLAY

EPUG-OVERLAY and 3D-EPUG-OVERLAY

- EPUG-OVERLAY
  - **E**xact: uses rational numbers.
  - **P**arallel.
  - **U**niform **G**rid for indexing.



- Next steps: 3D-EPUG-OVERLAY
  - Will use the same techniques, but for 3D triangulations



source: wikipedia

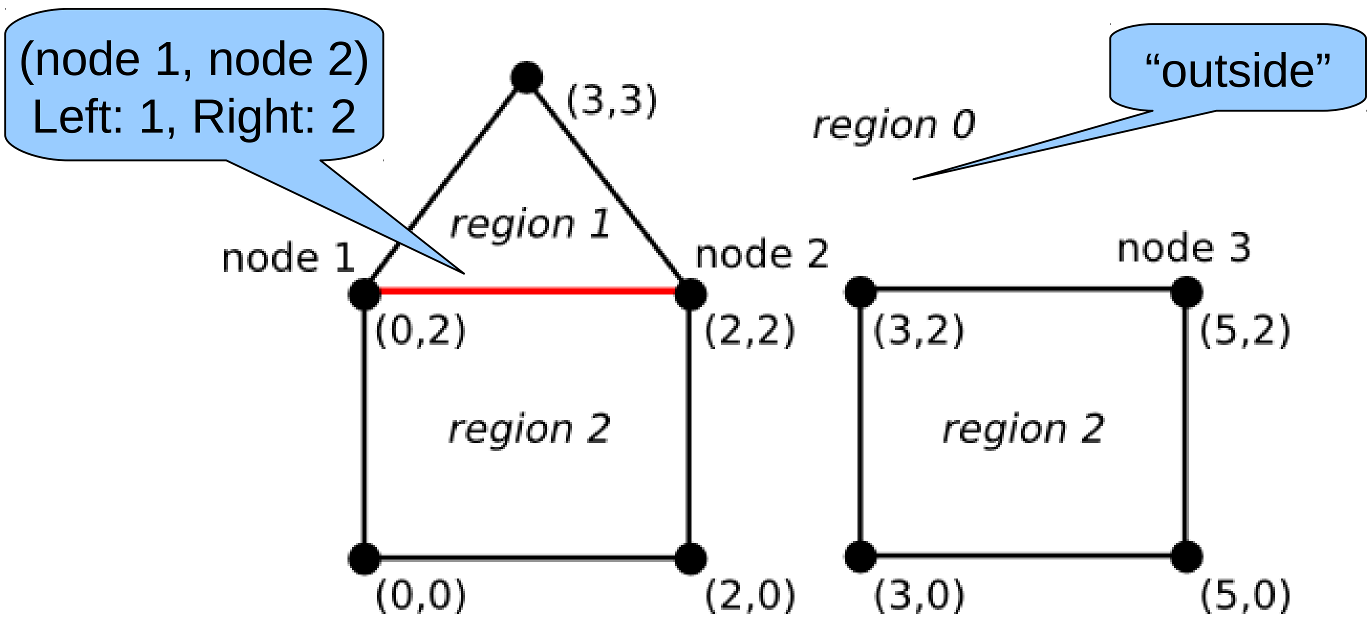
EPUG-OVERLAY

- Simple map representation.
  - No explicit global topology → easy to maintain and avoid topological errors.
  - Easy to process in parallel.

- Simple data structures.
  - Easy to parallelize
  - Efficient

Map representation

- Topological representation.
- Each region has one id.
- Edges represent boundaries.



Overlay algorithm

- Find all intersections.
- Locate vertices in the other map.
- Compute output polygons.

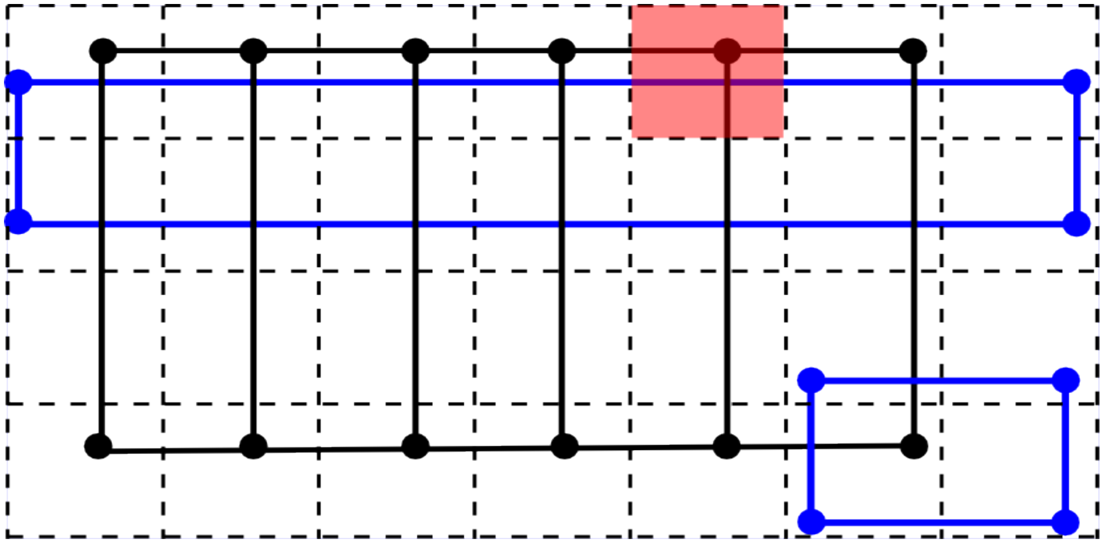
Computing intersections

- “Brute force”: O(|A| x |B|)
- Other possible technique:
  - Chazelle-Edelsbrunner O(n log n + k)
  - Complicate and doesn't parallelize
- In this work: uniform grid
  - Tests: very efficient

Computing intersections

In this work: uniform grid.

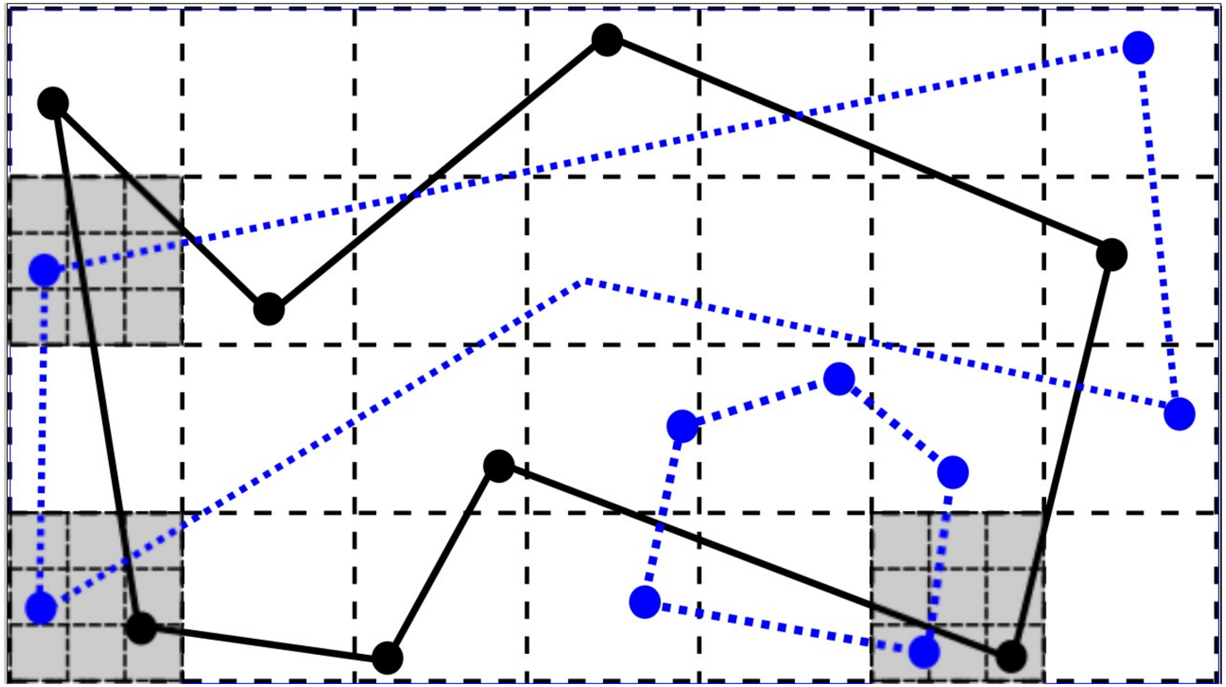
- Insert edges in grid cells (edge may be in several cells).
- For each grid cell *c*, compute intersections in *c*.



4x7 uniform grid.
Blue map: 8 edges
Black map: 16 edges

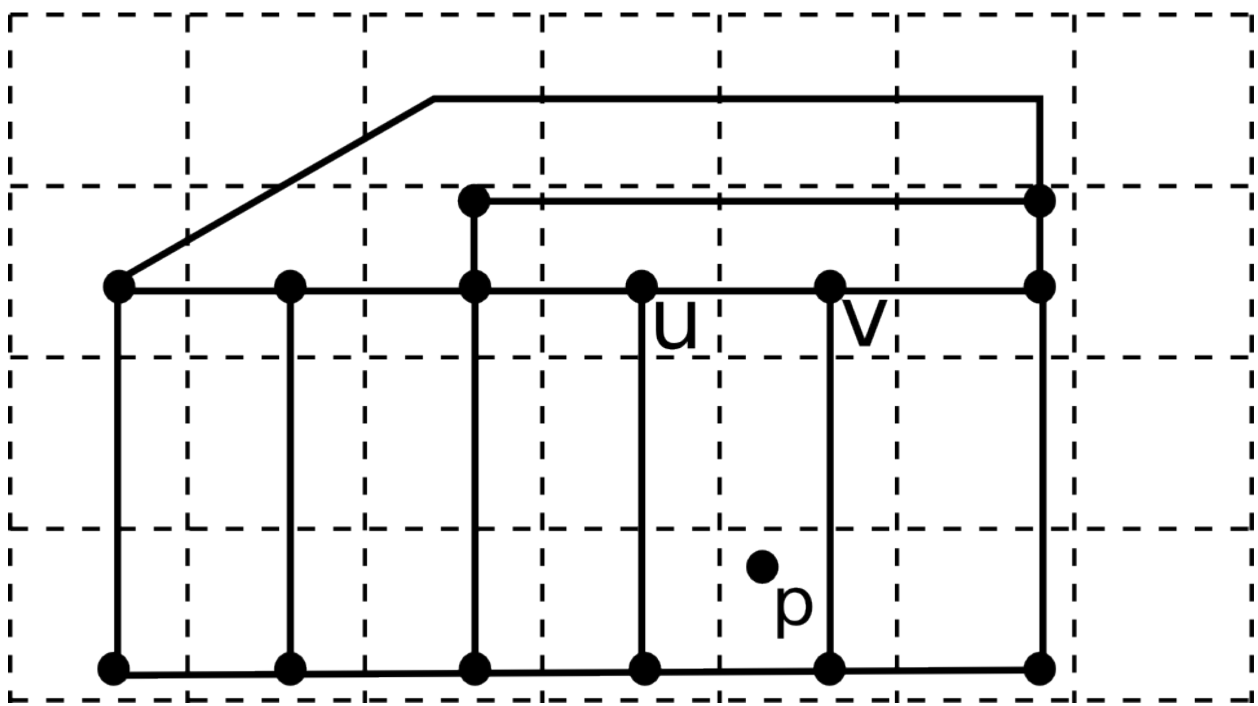
Computing intersections

- Uniform Grids work well for uneven data.
- For very uneven data: 2-level uniform grid.



Locating vertices in other map

- Also implemented using a uniform grid.
- Given *p*, find the lowest edge above *p*.



Locating vertices in other map

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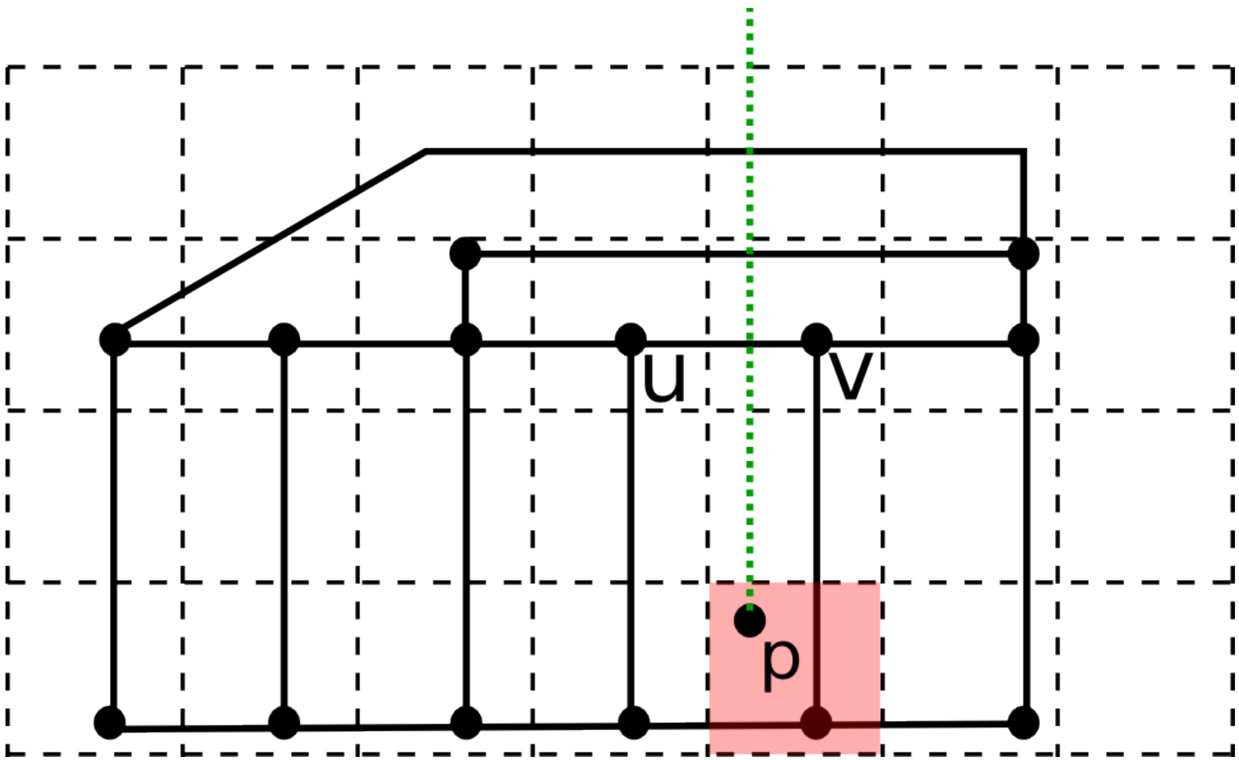
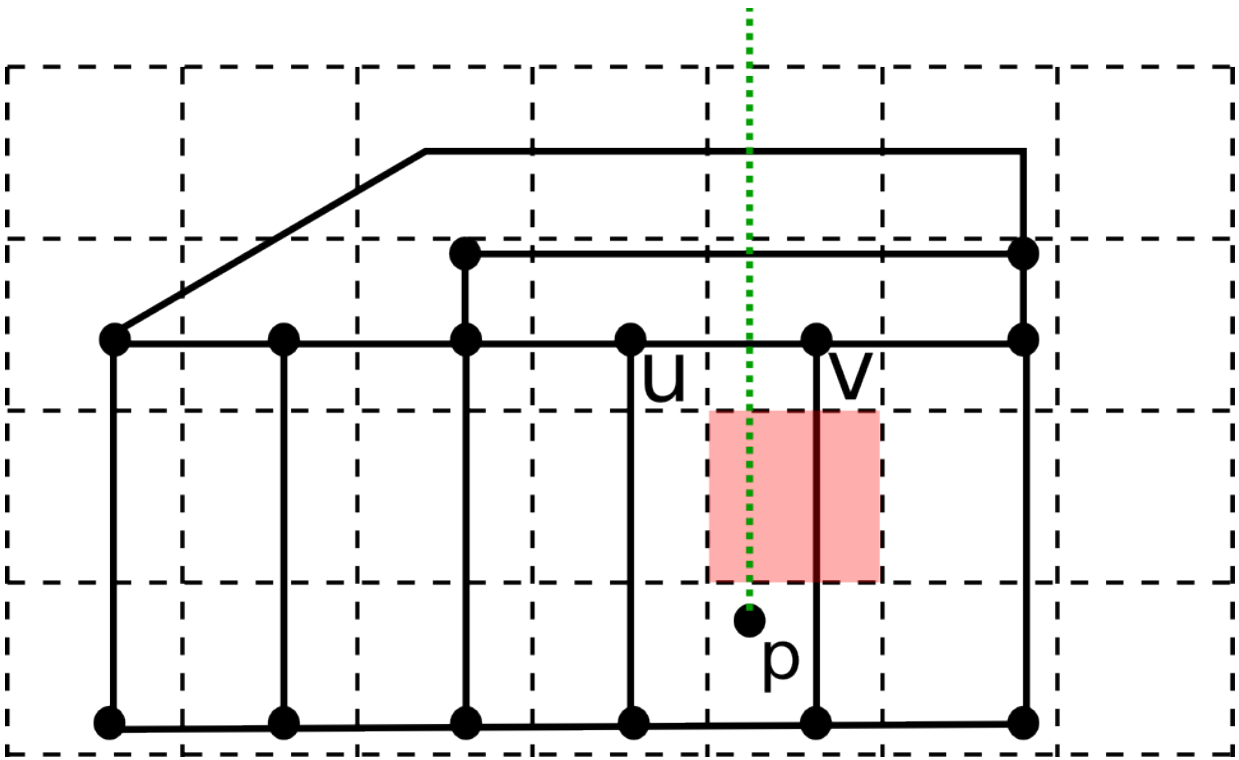


Locating vertices in other map

- Also implemented using a uniform grid.
- Given *p*, find the lowest edge above *p*.



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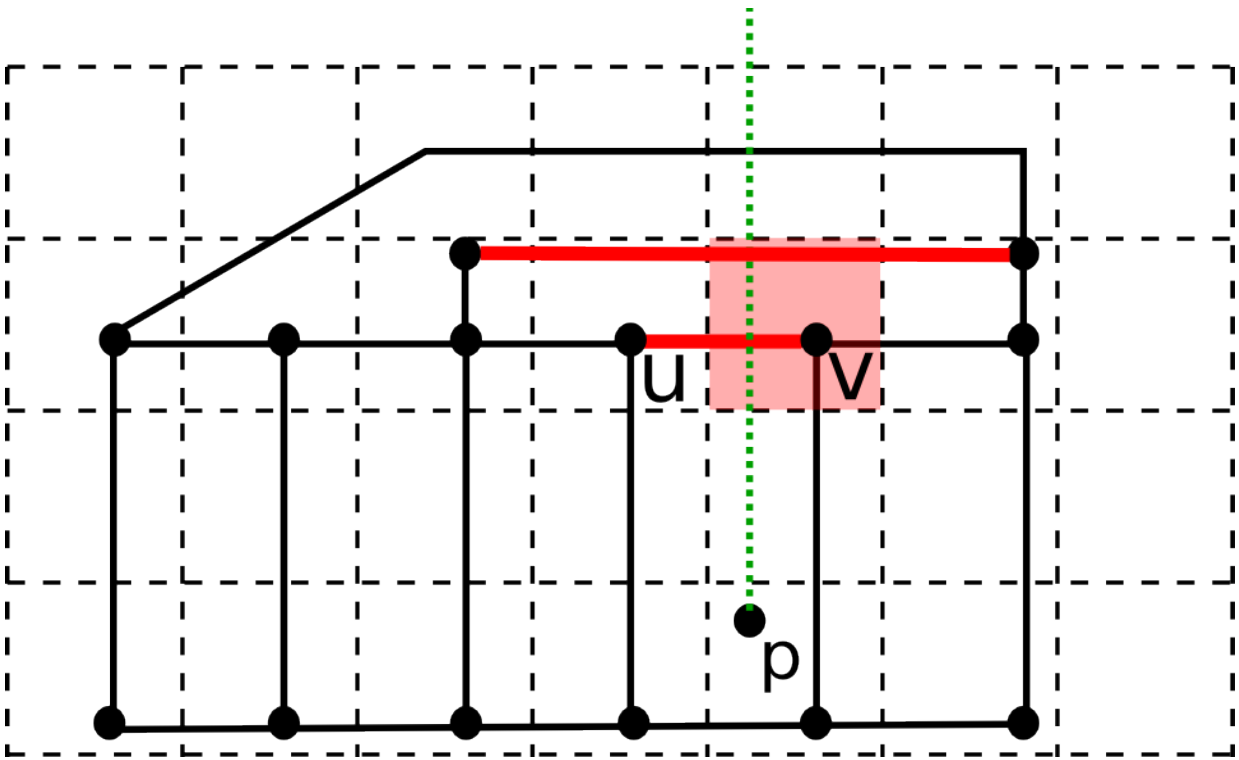

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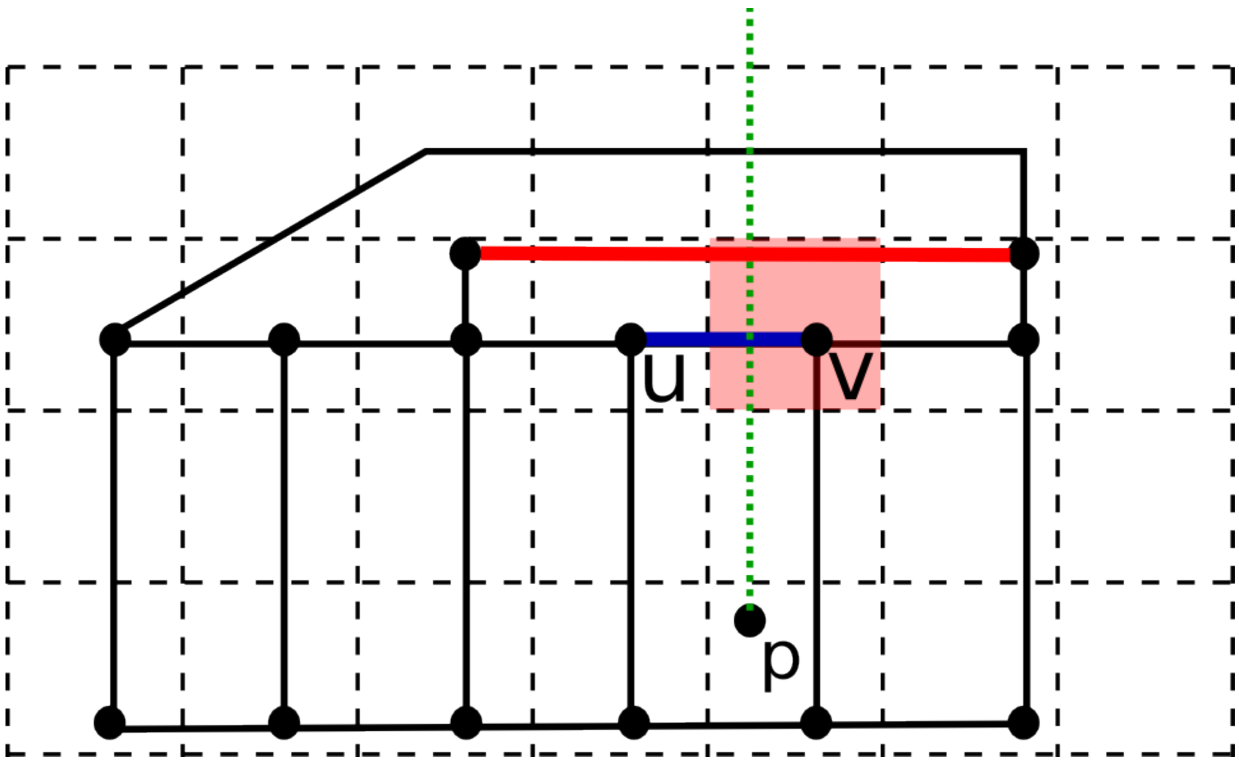


Computing output polygons

- Edges of the output polygons → computed based on input edges.
- For each input edge → three scenarios.

Computing output polygons

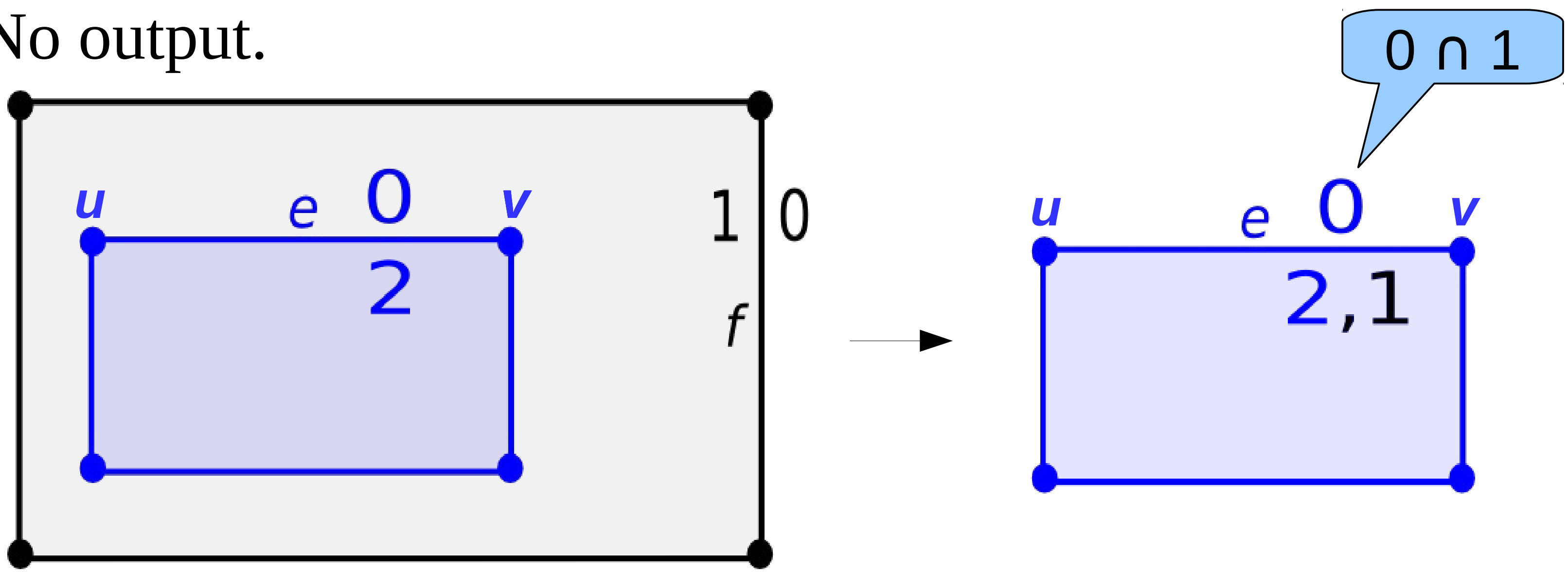
No intersection.

1 - edge completely inside a polygon (ex: *e*).

- Create output edge.

2 - edge completely outside a polygon (ex: *f*).

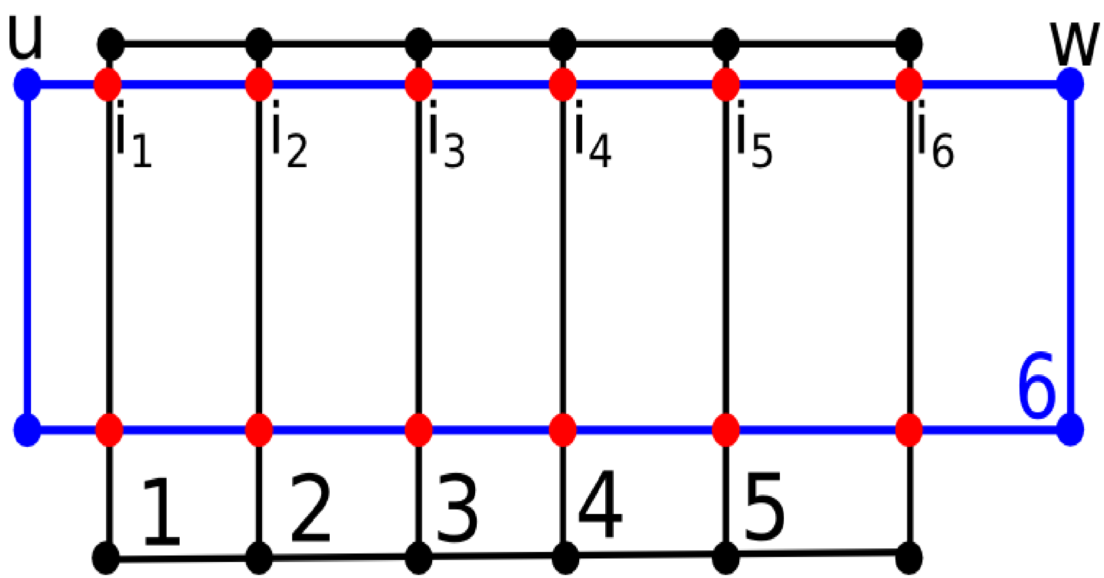
- No output.



Computing output polygons

3 – edge *e=(u,w)* with intersections.

- *e* is divided into segments.
- Segments classification → similar to the cases 1 and 2.



- (u,w) divided into 7 segments.
- 5 will be in output.

Computing output polygons

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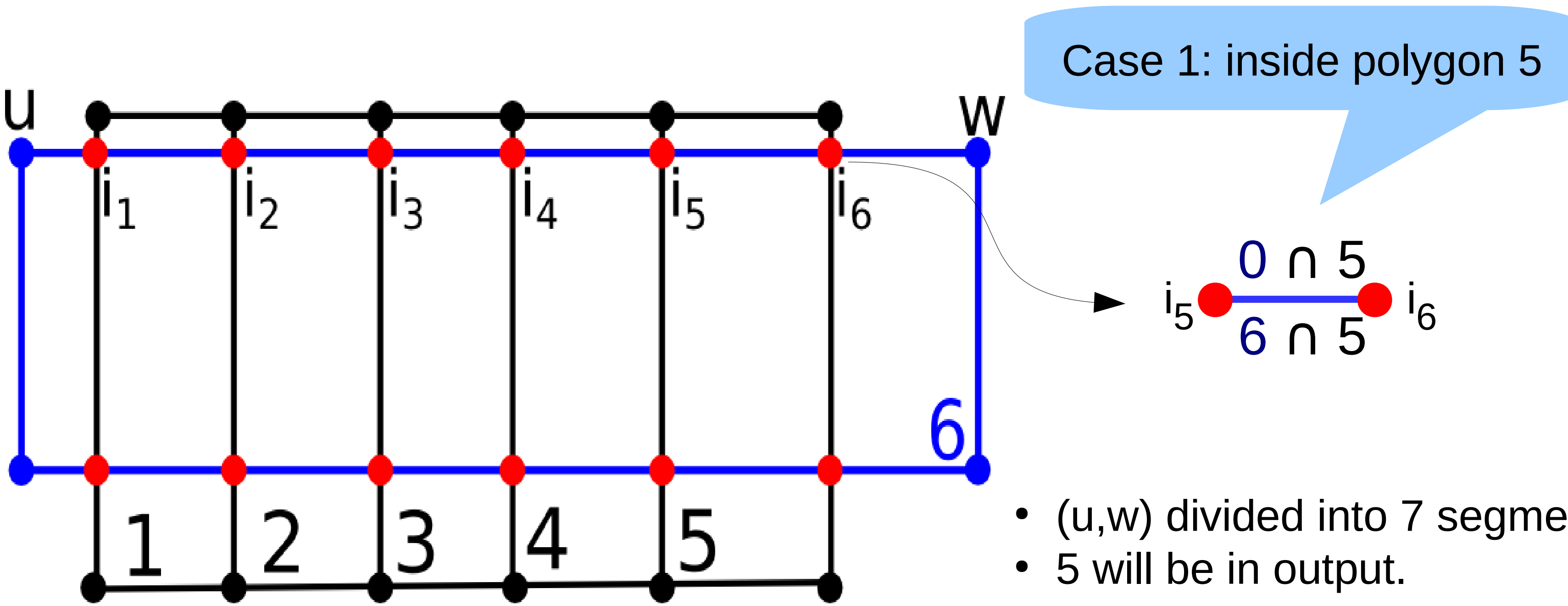

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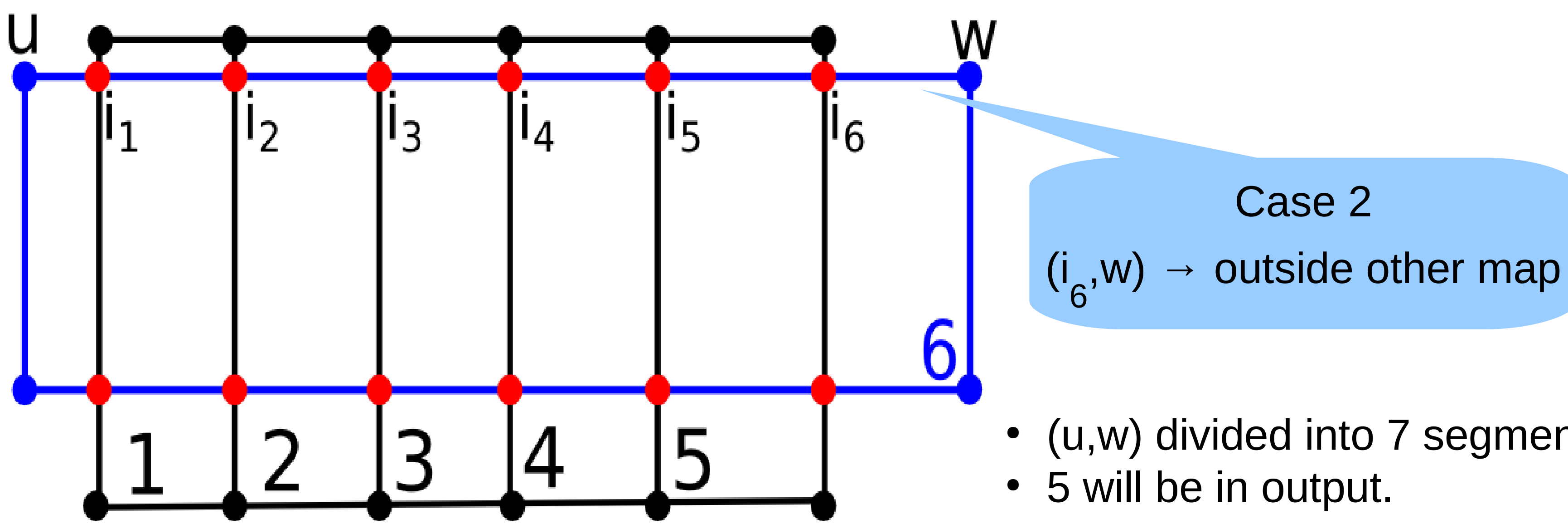
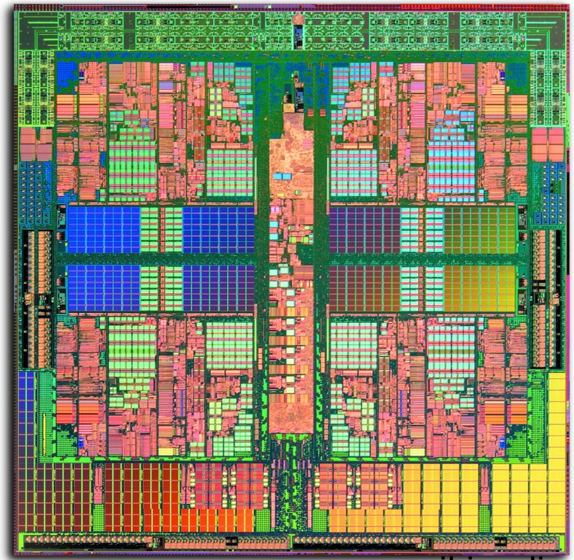


- (u,w) divided into 7 segments.
- 5 will be in output.

Parallel implementation

- This algorithm → few data dependency → very parallelizable.
  - Uniform grid creation: edges in parallel.
  - Locate vertices in polygons.
  - Compute intersections: cells in parallel.
  - Compute output edges: process input edges in parallel.

- Most of computers: multicore → OpenMP.



source: wikipedia

Implementation details

- Computation is performed using rational numbers → no roundoff errors.
- EPUG-OVERLAY implemented using GMPXX.
- Special cases: simulation of simplicity.

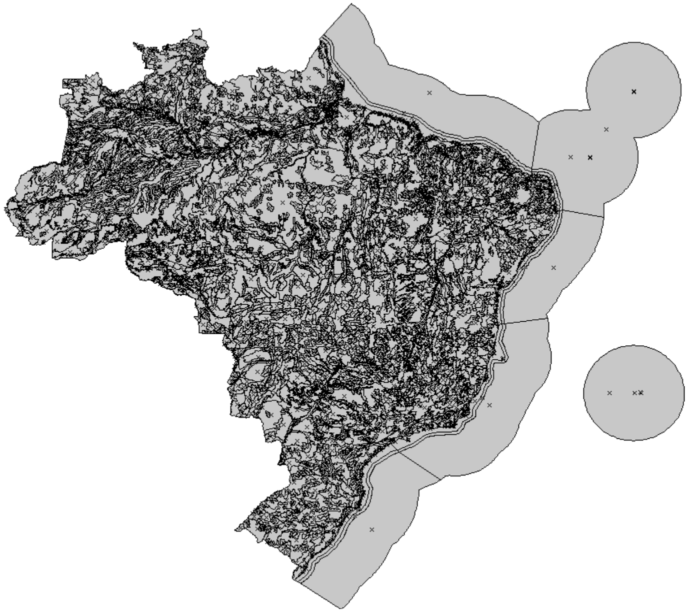
Experimental results

- EPUG-OVERLAY implemented in C++ .
- Tests:
  - Xeon E5-2687 → 16 cores / 32 threads.
  - 128 GiB of RAM.
  - Linux Mint 17

Experimental results

- 2 Brazilian and 4 North American datasets.
- Shapefiles converted to our format.

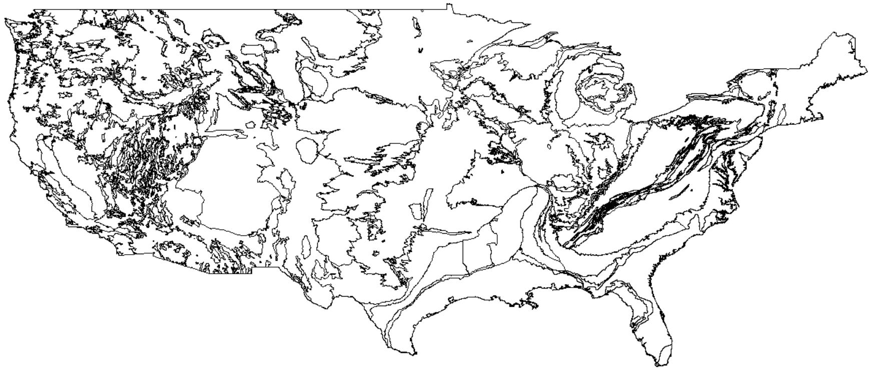
- BrCounty: 342,738 vertices, 2,959 faces
- BrSoil: 258,961 vertices, 5,567 faces.

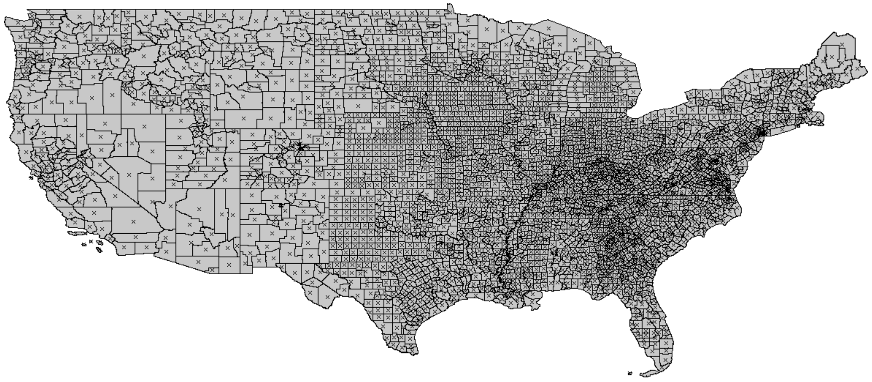


Experimental results

- 2 Brazilian and 2 North American datasets.
- Shapefiles converted to our format.

- UsAquifers: 358,551 vertices, 3,235 faces.
- UsCounty: 3,648,726 vertices, 3,552 faces.
- UsWaterBodies: 21,652,410 vertices, 219,831 faces.
- UsBlockBoundaries: 32,762,740 vertices, 518,837 faces.





Experimental results

- Processing time.
- First level grid: created s.t. the expected number of edges-edges tests per cell = 50.
- Second level grid: 40 x 40 cells, refined when #tests > 50

New results!

Maps:	*BrSoil* × *BrCounty*			*UsAq.* × *UsCounty*			*UsWBodies* × *UsBBound.*		
Grid size:	200×200			400×400			2000×2000		
	Time (sec.)		**Parallel**	**Time (sec.)**		**Parallel**	**Time (sec.)**		**Parallel**
Threads:	1	32	**speedup**	1	32	**speedup**	1	32	**speedup**
Read maps	1.0	1.0	1	5.3	5.5	1	73.1	74.5	1
Make grid	2.0	0.6	3	14.2	4.4	3	185.9	58.0	3
Refine 2-level grid	6.3	0.4	15	8.4	0.5	16	161.6	9.9	16
Intersect edges	1.0	0.1	8	2.6	0.3	8	505.5	30.9	16
Locate vertices	4.8	0.4	12	15.3	1.7	9	379.0	38.5	10
Comp. output faces	0.5	0.1	4	0.9	0.2	5	110.4	11.8	9
Write output	1.0	0.6	2	4.5	4.6	1	40.4	41.6	1
Total w/o I/O	14.6	1.6	9	41.4	7.1	6	1342.4	149.1	9
Total with I/O	16.6	3.6	5	51.2	17.2	3	1455.9	265.2	6

Experimental results

- Processing time.
- First level grid: created s.t. the expected number of edges-edges tests per cell = 5
- Second ce sts
- Good s

~ 200-300 thousand edges/vertices

Up to ~3 million edges/vertices

~ 20-30 million edges/vertices

Maps:	*BrSoil × BrCounty*			*UsAq. × UsCounty*			*UsWBodies × UsBBound.*		
Grid size:	200×200			400×400			2000×2000		
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Good speedup

Experimental results

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I/O

I/O

RPI - Rensselaer Polytechnic Institute

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Experimental results

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- First level grid: created s.t. the expected number of edges-edges tests per cell = 5…
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Mem. alloc.

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Grass (serial/not exact): 5321s

Experimental results

- Why not have 3, 4, 5 levels, … , quadtree?
- Uniform grid: simple and easily parallelizable.
- More levels: +memory and +time to create.

Maps overlaid	3-level grid				Quadtree	
	1^{st}	2^{nd} & 3^{rd}	Time (sec.)	Size (GB)	Time (sec.)	Size (GB)
BrSoil × *BrCounty*	200^2	40^2	54	1.1	70	1.7
UsAquifers × *UsCounty*	400^2	40^2	472	1.5	440	2.5
UsWBodies × *UsBBound.*	2000^2	40^2	290	43.7	8312	15.5

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More time than our entire algorithm!

	3-level grid				Quadtree	
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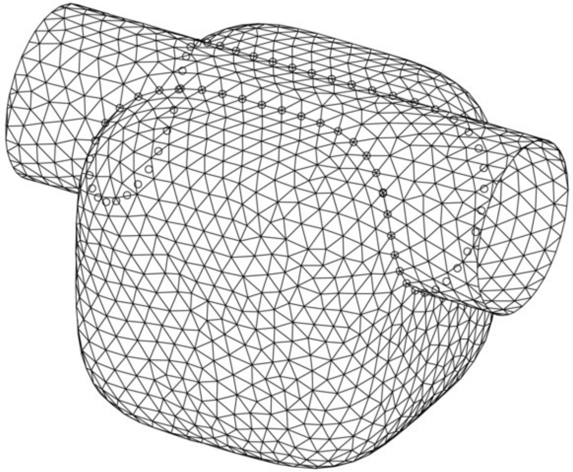
Next steps: 3D-EPUG-OVERLAY

- Work in progress.



source: wikipedia

- Will use similar techniques:
  - Rational numbers
  - “3D maps” represented by a set of triangles
  - Triangles: left/right objects
  - 3D uniform grid for intersection and point in polygon
  - Simulation of simplicity
  - Algorithm designed to be **parallel**

- EPUG-OVERLAY is efficient → 3D-EPUG0-OVERLAY will be.

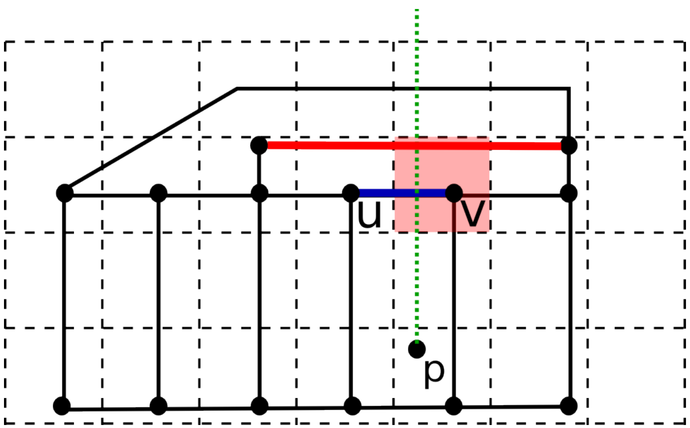
Conclusions

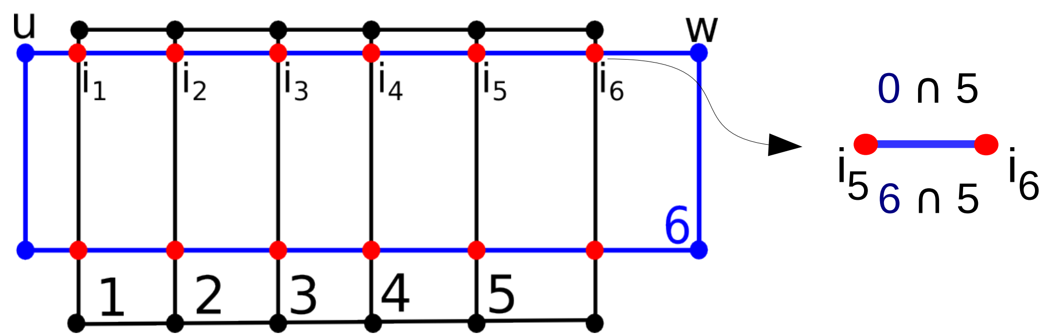
- EPUG-OVERLAY is an efficient method.
- Use precise arithmetic, but the performance is comparable with GRASS.
- Parallelizable algorithm → use computing power of modern computers.

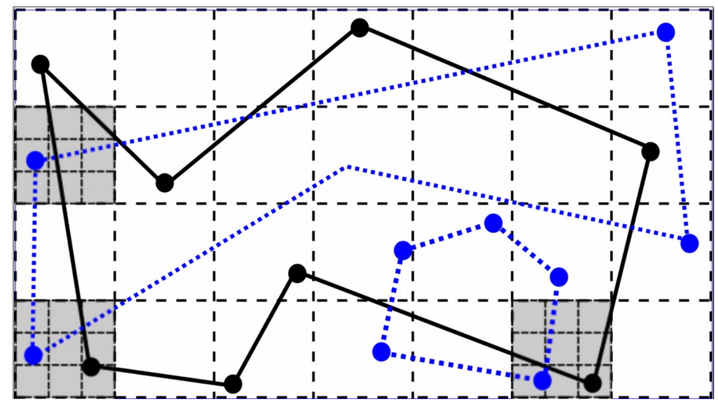
- Work in progress: 3D-EPUG-OVERLAY.

- Future work:
  - Compare the quality of the output.
  - Perform more theoretical analysis.

Thank you!







Acknowledgement:

Contact:

Salles V. G. de Magalhaes: vianas2@rpi.edu
W. Randolph Franklin: mail@wrfranklin.org

Experimental results

- The importance of the two-level uniform grid.
- UsWBodies x UsBBound.
- 1 level: 20,000 cells w/ 10,000+ pairs of edges
- 2 levels: 100 cells w/10,000+ pairs of edges!