# Restricted Bathymetric Tracklines Interpolation 

[Extended Abstract]

Wenli Li; W. Randolph Franklin, Salles V. G. Magalhães; Marcus V. A. Andrade ${ }^{\dagger}$

## 1. INTRODUCTION

In this paper, we propose a method to interpolate a small bathymetry with long and thin sand bars as distinctive features. Underwater terrain, or bathymetry, is measured by a single beam or multibeam echosounder attached under a boat [5]. Measurement points are distributed along the track of the boat and their positions and values constitute a trackline. Although full coverage is possible in multibeam surveys, tracklines are often sparse and thin, especially in single beam surveys, and it is expected to interpolate the bathymetry from tracklines. There are dozens of existing spatial interpolation methods with their strengths and weaknesses, and some are more widely used than others [1, 4]. One of the methods, overdetermined laplacian partial differentiation equations (ODETLAP), has been successfully used in bathymetry data interpolation and shown to be better than some of the popular methods $[2,3]$. However, it is more suited for large-scale bathymetry without significant linear or continuous features.

## 2. PROPOSED METHOD

We tried many different methods of interpolation for our data but the results are unsatisfactory. Given that the trackline in our data consists of largely parallel pieces, which is also true of many other bathymetric surveys, we propose to compute intermediate tracklines. The proposed method has two steps. The first step is to compute intermediate tracklines and the second step is to apply ODETLAP interpolation on the trackline and intermediate tracklines.

The main step is to compute intermediate tracklines. Assuming tracklines are nearly parallel to each other, the objective is to compute the intermediate trackline between a pair of neighboring tracklines. The first task is to determine the location of the intermediate trackline, or which points are on it. To do this, we intersect the tracklines with straight lines perpendicular to them (vertical lines for the data), and calculate the midpoint of the two intersection points of a straight line as the position of an intermediate trackline point c. Then we find pairs of points centered at and not too far from $c$ on the two tracklines for pattern matching, as shown in Figure 1 (left), and linearly interplate the value of $c$ from the pair of trackline points that best matches. To compute the matching of a pair of trackline points, we compute the sum of squared difference (SSD), as two vectors, of the values of two equal length trackline segments centered at the two points, as shown in Figure 1 (middle). The smaller the value of SSD, the better the two points match. Lastly, we want to

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Figure 1: Computing intermediate tracklines.


Figure 2: Tracklines and intermediate tracklines.
enforce the consistency of interpolations. We say two points on the intermediate trackline are interpolated consistently if the two line segments defined by the two pairs of trackline points for interpolation do not intersect each other. In Figure 1 (right), point $b$ is interpolated with the solid line segment before $c$ is interpolated, so that $c$ only has three dashed line segments to choose from, for consistency with $b$.

The next step is ODETLAP interpolation [6]. The method is to establish an overdetermined system of linear equations involving the value of every known or unknown point. There are two types of equations. For the value of each non-border point $z_{i, j}$, there is an equation $R\left(4 z_{i, j}-z_{i-1, j}-z_{i+1, j}-\right.$ $\left.z_{i, j-1}-z_{i, j+1}\right)=0$, which enforces smoothness of interpolated values over the entire terrain. $R$ is a constant scale factor setting the importance of the first type of equations. For the value of each known point $z_{i, j}$, there is an equation $z_{i, j}=h_{i, j}$, which enforces the value of each known point to be its actual value. The larger $R$ is, the smoother the interplation will be, while the smaller $R$ is, the more accurate the interpolated values of known points will be.

## 3. EXPERIMENTS

The experimental data, represented as a $599 \times 1084 \mathrm{DEM}$, is a single beam survey of a 400 by 700 meters area of tidal sand bars off the coast of Martha's Vineyard, Massachusetts. The value of each point is either a depth or a nodata value, representing a known or an unknown point. The range of depth values is from 1.447 to 6.642 meters under sea level. Figure 2 (left) shows the trackline, with the known points drawn larger than actual size for illustration. Figure 2 (right) shows the result of interpolating intermediate tracklines twice, once for the trackline and once for the result of the first interpolation, including the trackline and intermediate tracklines. The known and interpolated points are also drawn larger than actual size. For simplicity, thin marginal areas on the left and the right of the DEM is excluded from consideration.


Figure 3: Results of various interpolation methods.

Figure 3 shows the results of various methods of interpolation [1, 4], using relief shading to highlight artifact. The first method is nearest neighbor interpolation, where each unknown point is assigned the value of its nearest known points. The second method is natural neighbor interpolation, where each unknown point is assigned the weighted sum of the values of its nearest known points. The third method is inverse distance weighting (IDW), where each unknown point is assigned a weighted average of some or all of the known points. The result in the figure is computed with the power parameter $p=4$. The fourth method is interpolation by a triangulated irregular network (TIN). The z-tolerance is set at 0.01 m in the result and 13270 known points are used for the triangulation. The fifth method is ODETLAP interpolation with $R=0.1$. The maximum error of the known points is 0.02 m . The sixth method is the proposed method that computes intermediate tracklines and uses ODETLAP interpolation with $R=0.1$. The maximum error in ODETLAP is also 0.02 m . Of all the six methods, the first three are exact, while the last three are inexact.

Each of the results reflects in a sense the structure of the underlying bathymetry. For nearest neighbor interpolation, the result consists of patches of constant values and is thus not continuous. In particular, obvious lines formed by sudden change in value can be seen, where points are almost equidistant to two pieces of trackline and have very different values assigned to them. Actually, the separating lines are on the medial axis of the trackline. The result of natural neighbor interpolation is much smoother, in fact, too smooth between distant pieces of trackline so that features are almost lost. Besides, the method does not interpolate outside the convex hull of the known points. The result of IDW is closer to that of nearest neighbor, but is smoother. Separating lines are visible between trackline pieces and features are not connected. A TIN can often represent a terrain using much less points than a DEM while retaining the general shape. However, because all the known points are on a trackline, the triangulation consists mostly of long and thin triangles that are not representative of the shape of a terrain. The result of ODETLAP is very smooth, without very obvious traces of the trackline or separating lines, but suffers from the same problem of disconnected features between pieces of trackline. The result of the proposed method is quite different from
the others. After twice computing intermediate tracklines, the distance between neighboring tracklines is much closer, and features are able to be connected in the subsequent ODETLAP interpolation. However, another type of artifact appears. Sharp corners can be seen in some places due to the linear interpolation used for intermediate tracklines.

## 4. CONCLUSIONS AND FUTURE WORK

We have proposed a method for nearly-parallel bathymetric tracklines interpolation that computes intermediate tracklines and uses ODETLAP interpolation. Experimental results show the unique strength of the proposed method in connecting and recovering features that are not parallel to the tracklines. Two issues will be addressed in the future. First, using non-linear interpolation to avoid pointed corners, and second, designing a more general method for more complex cases like non-parallel or partially parallel tracklines. It is possible to verify the interpolated bathymetry of sand bars through satellite imagery in the same period.

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[^0]:    *Rensselaer Polytechnic Institute, Troy, NY, USA. Email: liw9@rpi.edu, mail@wrfranklin.org, vianas2@rpi.edu
    ${ }^{\dagger}$ Universidade Federal de Viçosa, Viçosa, Brazil. Email: marcus@ufv.br

