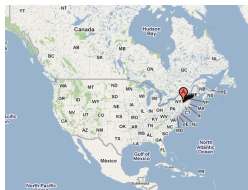


Towards a mathematics of terrain

W. Randolph Franklin

Rensselaer Polytechnic Institute
Troy, NY, USA



20th FWCG, Stony Brook, 30 Oct 2010

partially supported by NSF grant CMMI-0835762

Disclaimer and goals

Unlike the other talks, this is a report on work not done.

Goals: Math that

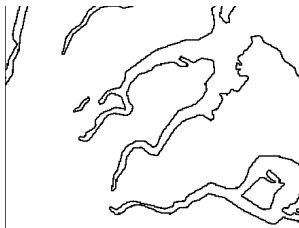
- allows the representation of only legal terrain (= height of land above geoid),
- minimizes what needs to be stated explicitly, and
- enforces global consistencies.

Why? To put compression and other ops on a logical foundation.

Terrain properties

Messy, not theoretically nice.

- Often discontinuous (C^{-1}).
- Many sharp local maxima.
- But very few local minima.
- Lateral symmetry breaking — major river systems.



Peninsulas or fjords?

- Different formation processes in different regions.
- Features do not superimpose linearly; two canyons cannot cross and add their elevations.
- C^∞ linear systems, e.g., Fourier series, are wrong.
- Structure that people can recognize even though hard to formalize; see Figure.
- Multiple related layers (elevation, slope, hydrology).

Current representations

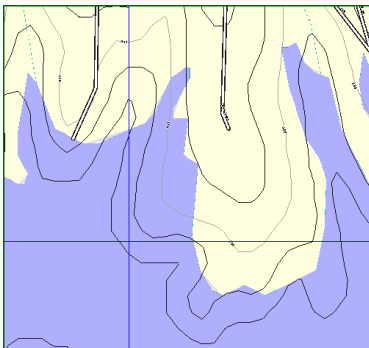
- Array of elevation posts.
- Triangular splines, linear or higher.
- Fourier series.
- Wavelets

Theory vs practice:

- Slope is derivative of elevation, but
- that amplifies errors, and
- lossy compression has errors, so
- maybe we want to store it explicitly.

Also, shoreline is a level set, but...

Inconsistencies between layers



Elevation contours crossing shoreline

Slope is important

- mobility
- erosion
- aircraft
- visibility
- recognition



Accurate elevations \nRightarrow accurate slopes

Ignoring errors, slope is simply $f'(x)$

But $\limsup_{i \rightarrow \infty} |(f_i(x) - f(x))| \rightarrow 0$, gives no guarantees about $\limsup_{i \rightarrow \infty} |(f'_i(x) - f'(x))|$

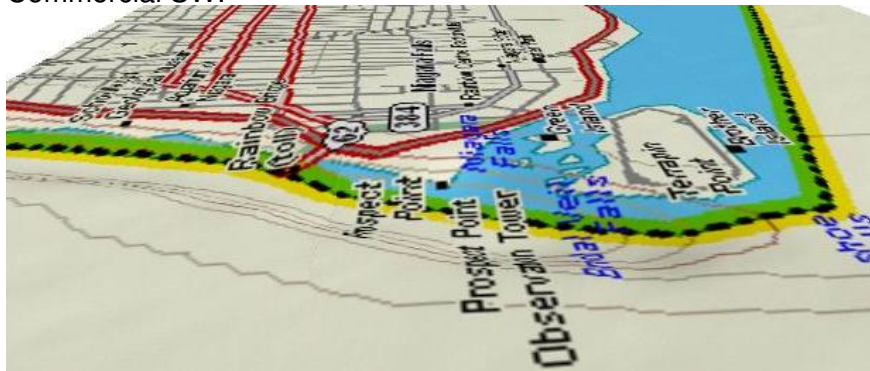
Consider two approximations to $y(x) = 0$



Elevation got better but slope got worse.

Bad commercial slope representation

Commercial SW:



Bad commercial slope representation



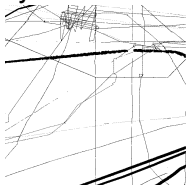
Commercial SW:

Photo:

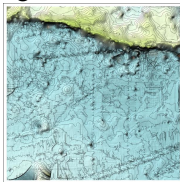


Sea floor bathymetry trackline fitting

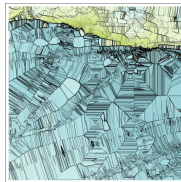
Problem: Trackline data is very unevenly spaced, leading to very bad surface fitting.



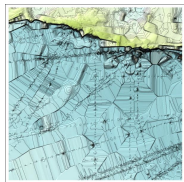
Bathymetry
Dataset



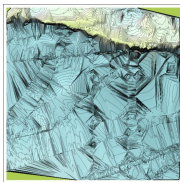
Kriging w.
ArcGIS



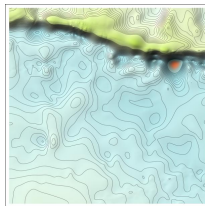
Voronoi
Polygons



Inverse Distance



2nd-order Spline
Interp



Soln: ODETLAP,
 $R = 100 \rightarrow 10$

Relevant terrain property

- Terrain is unlikely to have created artifacts exactly where the multibeam bathymetry later scanned it.
- How to work this fact into the math?

Path planning

Example of a common terrain operation. Cost depends on

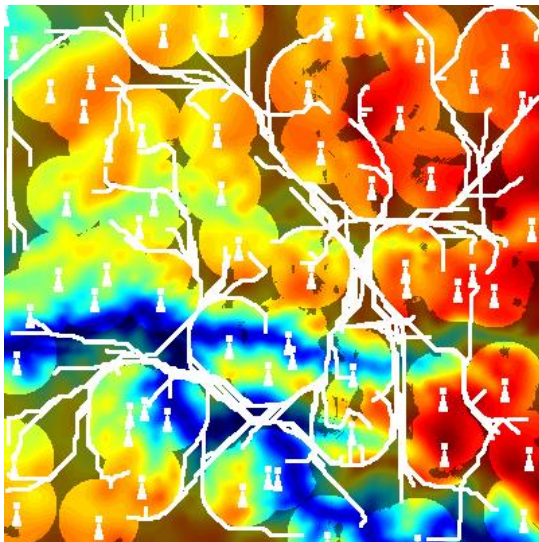
- distance
- uphill climb
- being seen

not a *metric*: $d(a, b) \neq d(b, a)$

not a *scalar field difference*: $d(a, b) \neq -d(b, a)$

$$c = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \cdot \left(1 + \max \left(0, \frac{\Delta z}{\sqrt{\Delta x^2 + \Delta y^2}} \right) \right) \cdot (1 + 100v)$$

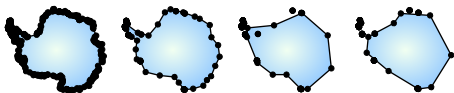
Smugglers and Border Guards



Math should match physics

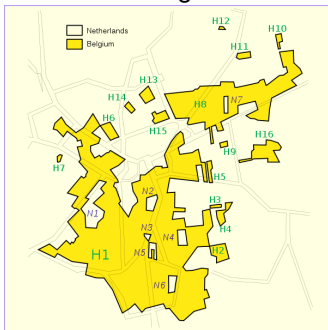
- Fourier series appropriate for small vibrations, not terrain.
- Truncating a series produces really bad terrain.
- Anything, like Morse complexes, assuming continuity is irrelevant.
- Fractal terrain is not terrain.
- Wavelets: how to enforce long-range consistency?
- Topology, by itself, is too weak.
- Terrain is not linear, not a sum of multiples of basis function.

Examples of rich structure



Line generalization

- group theory
- trig
- constructive solid geometry
- line generalization from level sets
- hydrology from terrain
- polygon properties from local geometry and topology.



Baarle-Nassau - Baarle-Hertog border

Terrain formation by scooping

- **Problem:** Determine the appropriate operators, somewhere inside the range from conceptually shallow (ignoring all the geology) to deep (simulating every molecule).
- **One solution: Scooping.** Carve terrain from a block using a scoop that starts at some point, and following some trajectory, digs ever deeper until falling off the edge of the earth.
- **Properties:** Creates natural river systems w cliffs w/o local minima.
- Every sequence of scoops forms a legal terrain.
- Progressive transmission is easy.

Terrain formation by features

- Represent terrain as a sequence of features — hills, rivers, etc ..
- plus a combining rule.
- This matches how people describe terrain.
- Progressive transmission.
- The intelligence is in the combining rule.

How compact is this rep? How to evaluate it?

Implications of a better rep

- Put earlier empirical work on a proper foundation.
- Formal analysis and design of compression.
- Maximum likelihood interpolation, w/o artifacts.
- Treat more sophisticated metrics, like suitability for operations like path planning, or recognizability.
- Close the loop to pre-computer descriptive geometry.