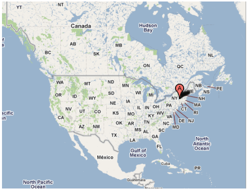
Algorithms for terrain and bathymetric sensor data

W. Randolph Franklin, Zhongyi Xie, Eddie Lau, You Li

Rensselaer Polytechnic Institute
Troy, NY, USA



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Intro

Three algorithic advances and a research topic in processing topographic and bathymetric sensor data:

- lossy terrain compression that maintains slope accuracy,
- bathymetric surface fitting to irregular tracklines,
- lossy compression of 5D environmental data, and
- terrain modeling to maintain hydrological validity.

Why? To attack several issues raised by the large amounts of data now available.

Eventual goal: A unified system.

Lossy terrain compression that maintains slope accuracy

- Accurate elevations $\not\Rightarrow$ accurate slopes
- Bad commercial slope representation.
- *Goal:* Compress and reconstruct terrain so that slope derived from reconstructed terrain is good.

Accurate elevations $\not\Rightarrow$ accurate slopes

- Ignoring errors, slope is simply $f'(x)$
- But $\limsup_{i\to\infty} |(f_i(x) - f(x)| \to 0$, gives no guarantees about $\limsup_{i\to\infty} \left|(f_i'(x) - f'(x)\right|$
- Consider two approximations to $y(x) = 0$

- Elevation got better but slope got worse.

Bad commercial slope representation

Commercial SW:

Bad commercial slope representation

Commercial SW: 

Photo:

ODETLAP –Overdetermined Laplacian Method

Fundamental representation for this work

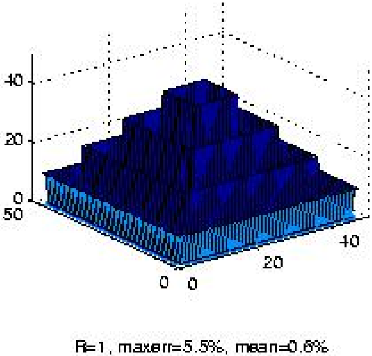
- Small set of posts $\Rightarrow$ complete matrix of posts
- Overdetermined linear system:
  - $z_{ij} = h_{ij}$ for known points,
  - $4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$ for all nonborder points.
  - Emphasize accuracy or smoothness by weighting the two types of equations differently.
- Original goal: fill contours to a grid w/o showing terraces; competing methods have these problems:
  - Information does not flow across contours $\rightarrow$ slopes discontinuous
  - If rays are fired from the test point to the first known point, then method is not conformal etc.

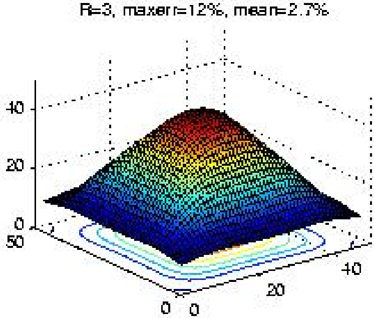
ODETLAP Advantages

Handles

- missing–data holes.
- incomplete contours,
- complete contours,
- kidney–bean contours,
- isolated points,
- inconsistent data.

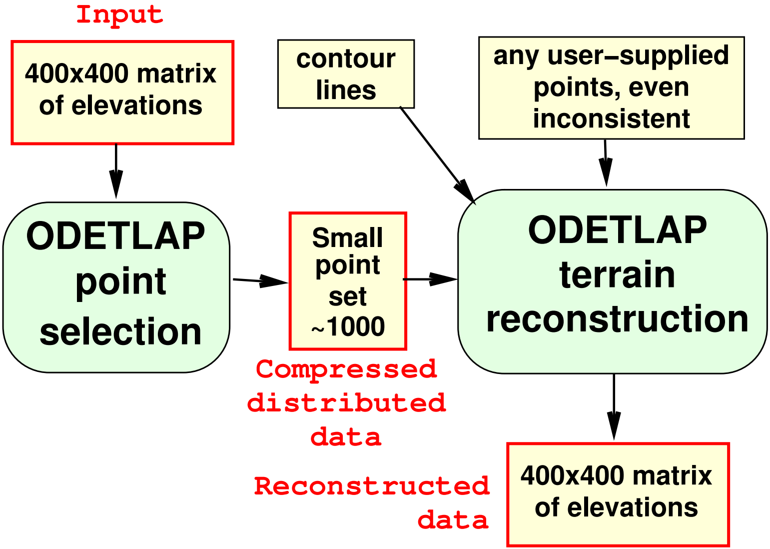
ODETLAP hard example



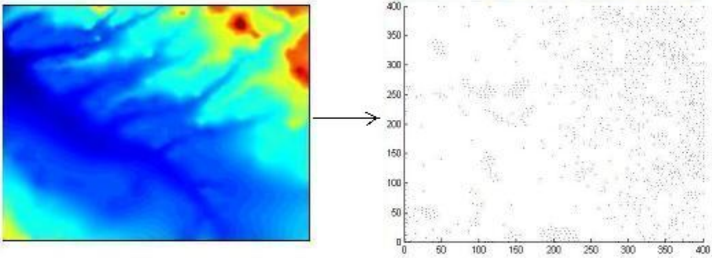


- input: contours with sharp corners
- output: smooth silhouette edges, inferred top

ODETLAP process



ODETLAP summary



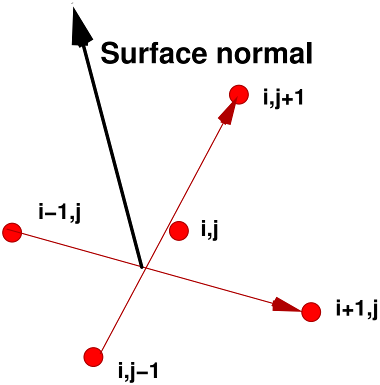
Original Surface
(320 KB)

Compressed Surface
(4071 Bytes)

Average Absolute Error = 2.451
Maximum Absolute Error = 25.822

Slope definition, accuracy

- Zevenbergen-Thorne $\left(\left(p_{i-1,j} - p_{i+1,j}\right) \times \left(p_{i,j-1} - p_{i,j+1}\right)\right)_z$
- p_{ij} not used



Limits of slope accuracy

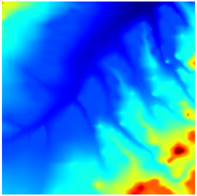
- 1m elevation resolution
- 30m post spacing
- slope precision: $\arctan\left(\frac{1}{30}\right) \approx 3\% \approx 2^\circ$

Info content

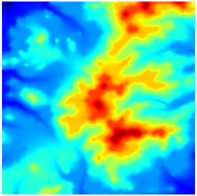
- Slope's autocorrelation distance is smaller than elevation's
- However, slope has less relative precision.

Level-II sample datasets

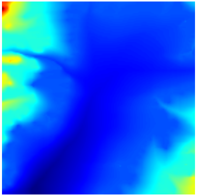
400 $\times$ 400 elevation matrices, *elevation range*



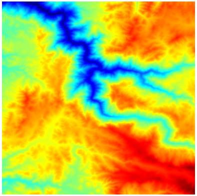
Hill1 *505m*



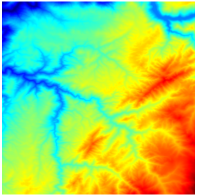
Hill2 *745m*



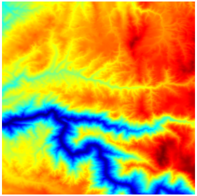
Hill3 *500m*



Mtn1 *1040m*



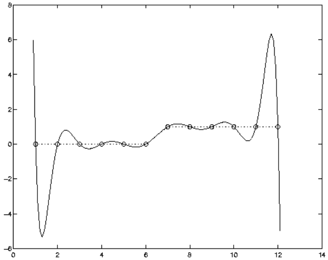
Mtn2 *953m*



Mtn3 *788m*

Idea 1: Pin down the elevation at sets of close points

- When inserting a point into known set, also insert some adjacent points
- *Thesis:* that will force the slope to be accurate there.
- Not really.
- *Analogy:* Lagrangian interpolation.



Keep trying.

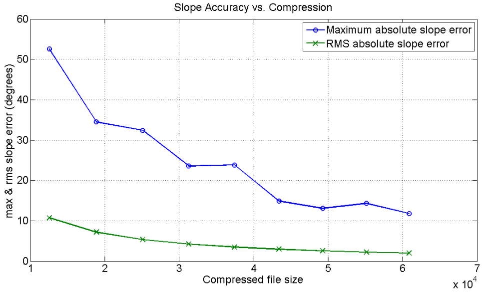
Idea 2: Extend ODETLAP

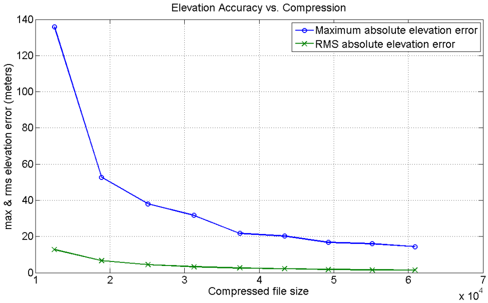
- Explicitly incorporate slope
- New overdetermined linear system:
  - unknowns: z_{ij}
  - known:
    - some h_{ij},
    - some $\Delta_x h_{ij} \triangleq h_{i-1,j} - h_{i+1,j}$,
    - some $\Delta_y h_{ij} \triangleq h_{i,j-1} - h_{i,j+1}$,
  - for all nonborder points:

    $$4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$$
  - for known h_{ij}: $z_{ij} = h_{ij}$
  - for known $\Delta_x h_{ij}$ and $\Delta_y h_{ij}$:

    $$z_{i-1,j} - z_{i+1,j} = \Delta_x h_{ij}$$
    $$z_{i,j-1} - z_{i,j+1} = \Delta_y h_{ij}$$

Preliminary results





Topics

Three algorithic advances and a research topic in processing topographic and bathymetric sensor data:

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- **bathymetric surface fitting to irregular tracklines,**
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Fitting bathymetric data is hard

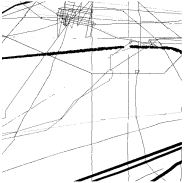
- MBB data is very unevenly spaced (dense in a swath along the ship tracklines, but then nonexistent for a long distance sideways),
- depth accuracy is a few percent, and
- insufficient data to infer features that are probably there.

Current methods often

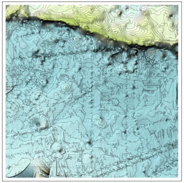
- have a specific distance wired into the formula,
- do not let information flow past data points, and so
- produce artifacts (e.g., abrupt slopes, acquisition footprint);
- show details that aren't justified.

Sea floor bathymetry trackline fitting

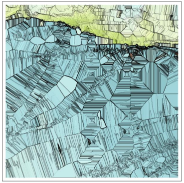
Problem: Trackline data is very unevenly spaced, leading to very bad surface fitting.



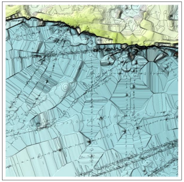
Bathymetry Dataset



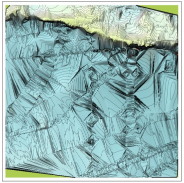
Kriging w. ArcGIS



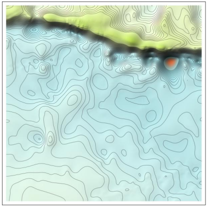
Voronoi Polygons



Inverse Distance



2nd-order Spline Interp



Soln: ODETLAP, $R = 100 \to 10$

Relevant terrain property

- Terrain is unlikely to have created artifacts exactly where the multibeam bathymetry later scanned it.
- How to work this fact into the math?

ODETLAP extension:

- vary R depending on distance to known points

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5D data compression

- Sensors, e.g., in World Ocean Atlas 2005, collecting multiple bands of environmental data –
- temperature, salinity, oxygen concentration,
- producing set of values over 5D grid (x, y, z, t, b).
- *Compress it!*
- little prior art.

Principles:

- Assume one band's large derivative at particular $(x, y, z, t) \Rightarrow$ likely for the other bands,
- Treat the data as one 5-D dataset, and
- Compress lossily since the data is imprecise.

Data compression technique

- extend ODETLAP to 3D, then 4D, 5D.
- *Major challenge:* Everything harder in higher dimensions.
- *To date:* compression ratios of 100:1 (mean error $< 1.5\%$).

	3D-ODETLAP			3D-SPIHT		
Variable	Mean Err(%)	Max Err(%)	Compr Ratio	Mean Err(%)	Max Err(%)	Compr Ratio
Salinity	0.0532	0.2174	77:1	0.0530	0.4946	11:1
Temperature	0.4993	2.0673	98:1	0.50	17.91	135:1
Dissolved O_2	0.9993	4.4145	100:1	1.002	24.9965	71:1
Apparent O_2 util.	0.9999	4.0170	85:1	0.9991	20.3609	81:1
Percent O_2 satur.	0.9985	4.5672	78:1	0.9969	20.3610	65:1
Phosphate	0.9993	4.5241	86:1	0.9978	15.6922	65:1
Nitrate	1.0242	4.6946	66:1	1.0006	18.5360	59:1
Silicate	0.9996	5.1437	91:1	1.0018	21.6457	81:1

ODETLAP's smaller compression error than SPIHT

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Maintaining hydrological validity

Given incomplete hydrography; fill in gaps.

- Presented on Wed
- Don’t work directly on partial hydrography.
- Compute deeper representation (terrain) from it.
- Derive hydrography.
- Result is guaranteed internally consistent.

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Disclaimer and goals

This section is a report on work not yet done.

Goals: Math that

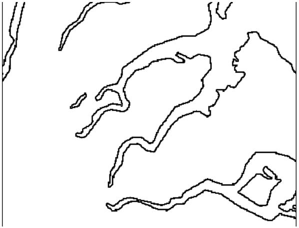
- allows the representation of only legal terrain (= height of land above geoid),
- minimizes what needs to be stated explicitly, and
- enforces global consistencies.

Why? To put compression and other ops on a logical foundation.

Terrain properties

Messy, not theoretically nice.

- Often discontinuous (C^{-1}).
- Many sharp local maxima.
- But very few local minima.
- Lateral symmetry breaking — major river systems.



Peninsulas or fjords?

- Different formation processes in different regions.
- Features do not superimpose linearly; two canyons cannot cross and add their elevations.
- C^{∞} linear systems, e.g,. Fourier series, are wrong.
- Structure that people can recognize even though hard to formalize; see Figure.
- Multiple related layers (elevation, slope, hydrology).

Current representations

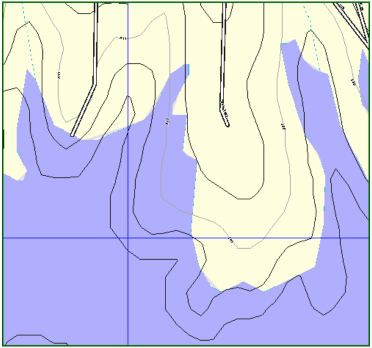
- Array of elevation posts.
- Triangular splines, linear or higher.
- Fourier series.
- Wavelets

Theory vs practice:

- Slope is derivative of elevation, but
- that amplifies errors, and
- lossy compression has errors, so
- maybe we want to store it explicitly.

Also, shoreline is a level set, but...

Inconsistencies between layers



Elevation contours crossing shoreline

Slope is important

- mobility
- erosion
- aircraft
- visibility
- recognition

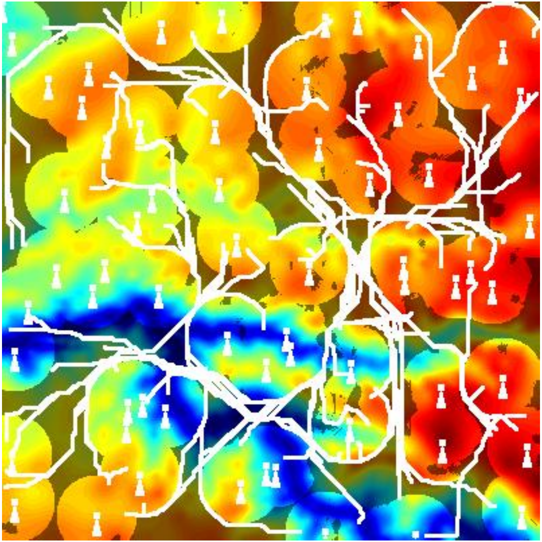
Path planning

Example of a common terrain operation. Cost depends on

- distance
- uphill climb
- being seen
- not a *metric:* $d(a,b) \neq d(b,a)$
- not a *scalar field difference:* $d(a,b) \neq -d(b,a)$

$$\mathcal{C} = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \cdot \left(1 + \max\left(0, \frac{\Delta z}{\sqrt{\Delta x^2 + \Delta y^2}}\right)\right) \cdot (1 + 100v)$$

Smugglers and Border Guards

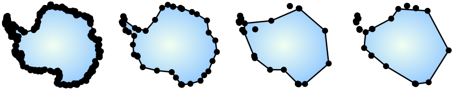


Math should match physics

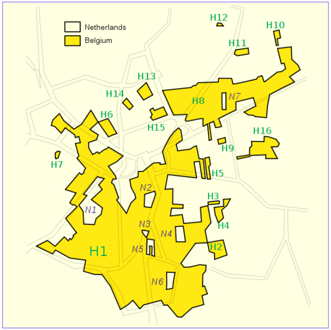
- Fourier series appropriate for small vibrations, not terrain.
- Truncating a series produces really bad terrain.
- Anything, like Morse complexes, assuming continuity is irrelevant.
- Fractal terrain is not terrain.
- Wavelets: how to enforce long-range consistency?
- Topology, by itself, is too weak.
- Terrain is not linear, not a sum of multiples of basis function.

Examples of rich structure

- group theory
- trig
- constructive solid geometry
- line generalization from level sets
- hydrology from terrain
- polygon properties from local geometry and topology.



Line generalization



Baarle-Nassau - Baarle-Hertog border

Terrain formation by scooping

- Problem: Determine the appropriate operators, somewhere inside the range from conceptually shallow (ignoring all the geology) to deep (simulating every molecule).
- One solution: **Scooping.** Carve terrain from a block using a scoop that starts at some point, and following some trajectory, digs ever deeper until falling off the edge of the earth.
- Properties: Creates natural river systems w cliffs w/o local minima.
- Every sequence of scoops forms a legal terrain.
- Progressive transmission is easy.

Terrain formation by features

- Represent terrain as a sequence of features — hills, rivers, etc ..
- plus a combining rule.
- This matches how people describe terrain.
- Progressive transmission.
- The intelligence is in the combining rule.

How compact is this rep? How to evaluate it?

Implications of a better rep

- Put earlier empirical work on a proper foundation.
- Formal analysis and design of compression.
- Maximum likelihood interpolation, w/o artifacts.
- Treat more sophisticated metrics, like suitability for operations like path planning, or recognizability.
- Close the loop to pre-computer descriptive geometry.