Evaluating Hydrology Preservation of Simplified Terrain Representations

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Abstract. We present a metric based on the potential energy of water flow to determine the error introduced by terrain simplification algorithms. Typically, terrain compression algorithms seek to minimize RMS (root mean square) and maximum error. These metrics fail to capture whether a reconstructed terrain preserves the drainage network. A quantitative measurement of how accurately a drainage network captures the hydrology is important for determining the effectiveness of a terrain simplification technique. Having a measurement for testing and comparing different models has the potential to be widely used in numerous applications (floods, erosion, pollutants, etc). In this paper, we first define a metric that maps the reconstructed drainage network onto the original terrain and computes the amount of energy needed for the water to flow. Next, a novel terrain simplification algorithm is presented that uses a targeted compression to preserve the important hydrology features. This method and other simplification schemes are then evaluated using the potential energy error metric to determine how much hydrology information is lost using the different compression techniques.

Key Words: Hydrology, GIS Metric, Terrain Compression, Drainage Network, Ridge Network

1 Introduction

Terrain data is being sampled at ever increasing resolutions over larger geographic areas requiring special compression techniques to manipulate the data. Typically the effectiveness of a terrain compression technique is how well it minimizes the root mean square or the maximum error between the original terrain and the reconstructed geometry [7]. This metric is not always the best choice for preserving hydrological information, since channels and ridges, essential for the calculation of drainage networks [13], might be lost. For example, a scheme which naively interpolates the terrain between two points on opposite sides of a river can flatten the terrain and block flow.

Often measuring the amount of water flow occurs by taking ground truth measurements, where hydrology statistics are determined by direct measurement. This can be expensive, time consuming and requires accessing remote locations. Rapid technological advances are making it possible to have accurate, high resolution elevation data. This provides for a more accurate simulation of hydrology, in ways that were once impractical. In order for this to happen, it is essential that the scientific community has the tools available that can store and manipulate large terrain datasets [1]. Accurate hydrological simulations could allow better understanding of regions at greatest risk of flooding, help minimize the threat of natural disasters and to track and predict the flow of pollutants. This work could also be applied to other flow based models. For instance, instead of water, it could be used to understand threat areas due to volcanic activity. Also, it could be applied to high resolution data for segmentation based on the watersheds.

2 Prior Art

Past work has been done for defining a metric for comparing how well a computed drainage compares to the real world drainage [14]. Most of the time, real world flow measurements are unavailable and flow simulations have to be used to make predictions. Numerous methods have been developed for estimating a drainage network from a specified segment of terrain. The D8 model can assign flow in one of the eight possible directions. In the SFD (single flow direction) version of the D8 model the entire amount of flow from each cell is entirely distributed to the lowest adjacent neighbor. This is not the case in the MFD (multi-flow direction) version the flow is fractionally distributed to all the lower adjacent neighbors.

A slightly more sophisticated MFD model is the D∞ model. As the name indicates, flow can travel in an infinite number of directions and is not limited to eight directions. The amount of water leaving each cell is distributed to one or more adjacent cells based on the steepest downward gradient [11].

Another implementation for finding drainage networks are digital elevation model networks or the DEMON model [4]. Instead of modeling flow as a point source that flows to an adjacent neighbor, DEMON captures the flow by contributing and dispersal areas. The motivation for using a method such as DEMON is that the representation allows for flow width to vary over nonplanar topography. However, this can introduce loops and inconsistencies in the hydrology.

Based on the fact that elevation data is only an approximation for the actual terrain, some methods allow for water to flow uphill until spilling over an edge. These flooding methods determine spill points out of every basin. In the Terraflow approach [5, 12], the path of least energy is used to flow uphill until reaching the spill point. The flow runs uphill in situations when there is not an adjacent lower elevation. These methods often keep expanding the drainage networks until they flow off the edge of the terrain. This is because it is assumed that the initial input DEM is prone to collection and sampling errors that cause unrealistic depressions. The main benefits of Terraflow are the ability to avoid dataset issues, obtain long continuous river flow and scalability on massive datasets. The main disadvantages are that this approach may miss realistic drainage basins and poorer performance on non-massive datasets.

Typically for any of the method listed above, the inputs are a DEM (Digital Elevation Model) and a flow accumulation threshold. The outputs are a flow direction grid and a flow accumulation grid. The flow direction grid specifies the direction of flow. The flow accumulation grid is an integer corresponding to the amount of flow and a cell is considered part of the drainage network if its flow accumulation threshold is larger then the threshold value given as an input.

3 Overview

The contributions of our research are:

1. A new metric for measuring the amount of hydrology error introduced by a terrain simplification algorithm. The drainage network is computed on the reconstructed terrain and then is mapped onto the original terrain. The amount of potential energy error is computed. Flow traveling uphill will increase the error, while flow traveling downhill will lower the amount of error.
2. Efficient drainage network computation based on a system of linear equations. The resulting drainage often contains longer and more realistic drainage networks then ArcGIS [10] which is typically regarded as the industry standard.
3. Simple, fast and effective computation of the ridge network. Inverting the terrain and running the drainage network provides an approximation of the ridge network. Often compression techniques smooth out ridges. Having an accurate representation of the ridge network can assist compression algorithms and also has applications in siting, path planning and hydrology.
4. Introduction of a new compression method that is hydrology-aware. In other words, specifically targeting the compression technique to minimize the amount of drainage network error.

4 Drainage Network Error Metric

Standard metrics for evaluating the effectiveness of terrain simplification algorithms use root mean squared (RMS) and maximum error. These measurements are ineffective when evaluating the loss of drainage network structure. Therefore, one of the main purposes of our research is to introduce a metric geared towards measuring this error.

It is important to remember that the goal of our hydrology metric is not to compare the reconstructed hydrology against an absolute truth. Hydrology computed on a digital representation may have significant errors due to sampling and data collection inaccuracies. Therefore, our hydrology metric does not compare the reconstructed drainage network versus the original drainage network directly, as with ground-truth methods. Instead, the hydrology metric takes the flow direction grid and the flow accumulation grid computed on the reconstructed terrain and maps it onto the original, uncompressed DEM(Figure 1).

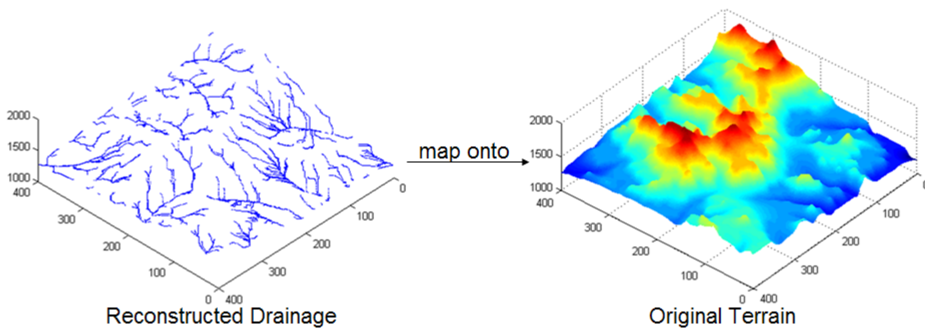

Fig. 1. To compute the potential energy error, the drainage is computed on the reconstructed terrain. This drainage network is then mapped onto the original terrain. The amount of water flowing uphill and downhill influences the metric. The highest elevations are visualized in dark red and the lowest elevations are dark blue.

To compute the accuracy of the drainage network, the gradient, amount of flow contributing cells and whether the flow is traveling uphill or downhill are taken into account. The total downward energy and upward energy is computed as a summation of the gradient $|(E_i - E_r)|$, where E_i is the original elevation matrix and E_r is the receiving elevation matrix where each cell contains the elevation of the adjacent cell in E_i that is receiving the water flow. The gradient is weighted by the amount of flow (variable W). Variable $EnergyDown$ is the sum of cells in the matrix where the flow travels downhill. Similarily, $EnergyUP$ is the summation of cells where the flow travels uphill. The final $Error$ is determined as the ratio of the total upward energy divided by the total downward energy.

$$EnergyDown = \sum (E_i - E_r) * W_i$$

$$EnergyUP = \sum (E_r - E_i) * W_i$$

$$Error = \frac{EnergyUP}{EnergyDown}$$

In order to compute the energy error metric the flow is computed on the reconstructed DEM. The error is determined by comparing the flow direction matrix computed on the reconstructed geometry with the elevation matrix from the original DEM. A perfect match would have an energy matrix equal to zero. This would occur if the flow never went uphill, which is the case when using the flow direction grid from the original terrain. Therefore, the closer the metric is to zero, the more accurate the reconstructed drainage network.

5 Ridge-River Drainage Calculation

In this work, the drainage network is computed using a standard D8 model [11] based on steepest descent flow. In this implementation each cell flows to the lowest adjacent neighbor and flow is forbidden from traveling uphill. The method is executed on both the original and inverted terrain, and this can be done in parallel. The inverted terrain is derived from the original elevation matrix using the equation below:

$$I_e = Max(E) - E + Min(E) \tag{1}$$

where E is the original elevation matrix and I_e is the inverted elevation matrix. The drainage network is computed using E to determine the drainage network and I_e to determine the ridge network. The two networks are combined together and throughout this paper they will be refered to as the ridge-river network[9], as seen in Figure 2. The process for computing each network is identical except

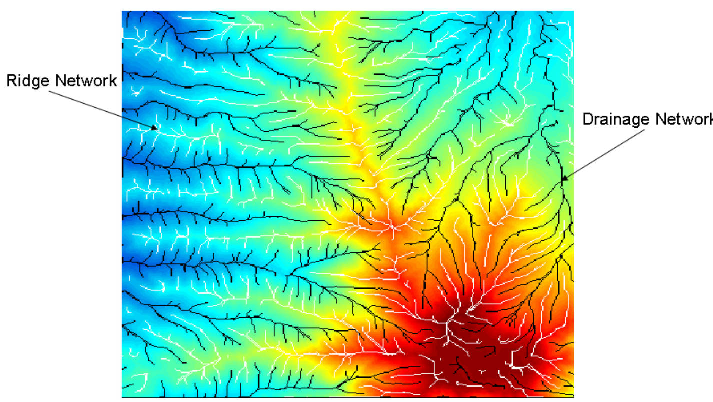

Fig. 2. The ridge-river network, with rivers in black and ridges in white. The highest elevations are visualized in dark red and the lowest elevations are dark blue.

for the previously defined elevation inversion described above. Figure 3 shows our drainage network computation compared to ArcGIS developed by ESRI. Our implementation results in less fragmented and more realistic river networks.

The output of the initial drainage computation is a flow accumulation grid, where each cell contains an integer corresponding to how many other cells contribute flow to that point. Cells above a predefined threshold are considered significant and are added to the drainage and ridge network. It is not necessary to store all these cells since they are clustered together and therefore add little value to a point selection compression technique. The Douglas-Peucker [6]

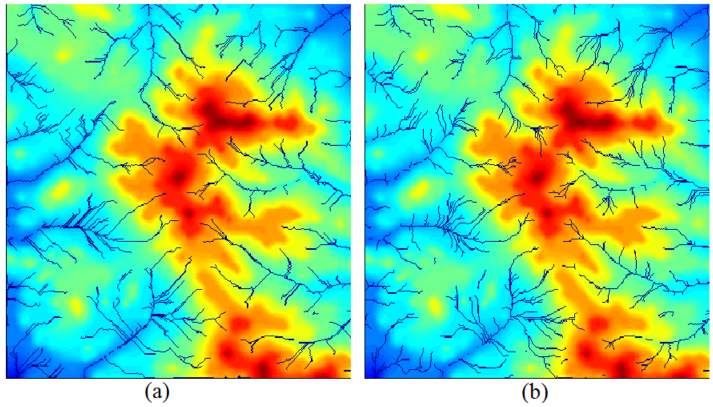

Fig. 3. (a) Our method for computing the drainage networks compared to (b) ArcGIS. Notice how our method is less fragmented then ArcGIS.

algorithm is used to reduce the number of points required to represent each river segment. The refined points can be stored and further compressed. To reconstruct the terrain we use an implementation of Over-determined Laplacian Partial Differential Equations (ODETLAP) [7] where each point is considered to be the average of its four neighbors, with a subset of the ridge-river points being known.

Different from other methods that use flooding [1], our method computes flow using a system of linear equations $Ax = b$ where x is an unknown N^2 length vector equal to the amount of water accumulation at each cell and b is the initial flow or "rain" at each cell, usually equal to 1. Matrix A is a $N^2 \times N^2$ sparse matrix: the identity matrix with additional non-zero entries to represent flow between neighboring cells. For instance, if cell X_1 receives flow from cell X_2 and X_5, row 1 in matrix A will contain non-zero elements in columns 1, 2 and 5. Therefore the number of non-zero entries in matrix A is bounded by $2N^2$, where N is the size of the $N \times N$ DEM. The upper bound of $2N^2$ is determined since there will be N^2 non-zero entries to load the identity matrix. All other non-zero entries represent flow from one cell to one other cell. There can be at most N^2 additional non-zero elements, since each cell can flow in only one direction. Taking advantage of the sparse nature of matrix A the linear system can be solved efficiently. In Figure 4 we show the compute time to initialize and solve the linear system corresponding to the matrix size.

An important problem that needs to be addressed is the occurrence of plateaus, which are regions where the flow direction can not be determined based on steepest decent flow. To deal with these cases, the plateaus are first identified using a very fast variant of the Union-Find algorithm developed by Franklin and Landis [8]. The input is a $3N-2$ by $3N-2$ binary matrix and the output contains a list of components, with each component representing one plateau.

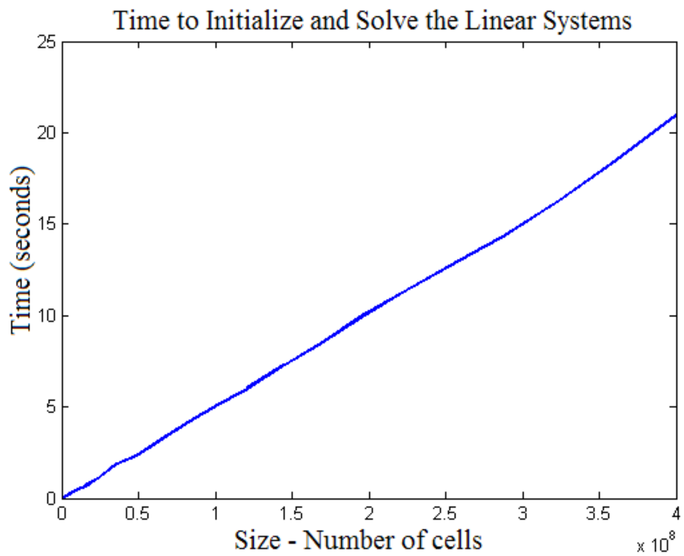

Fig. 4. Time to initialize and solve the sparse linear system for varying size datasets.

Figure 5 shows the plateaus and initial drainage network. Once identifying the flat areas, the cell directions are set using a similar strategy to Terraflow [12], where a breadth-first search assigns directions towards the root or spill point. Spill points are identified as cells in a flat component that contain a nonzero direction. In other words, a cell in the component that has an adjacent cell with a smaller elevation. Flat areas that have no spill points are determined to be sinks. The directions of every cell in a sink are assigned to flow to a single point.

After assigning directions to every plateau and sink, the final flow stage can be computed. The linear system of equations is modified to include the directions assigned to the plateaus and sinks. The flow is recomputed and the final flow accumulation grid and flow direction matrix is determined. Figure 5 shows our final flow computation with the drainage network and watersheds.

The major benefits of this approach is simplicity, scalability and it is consistent (there is never a flow loop). However a significant disadvantage is that this method does not account for flow sampling and dataset inaccuracies that often unrealistically block flow.

6 Terrain Simplification to Preserve the Hydrology

In Figure 6, we show a flow chart describing the ridge-river terrain simplification technique for compressing and uncompressing the hydrology structure of a terrain. Inputs are shown in boxes and programs are shown in circles. The method

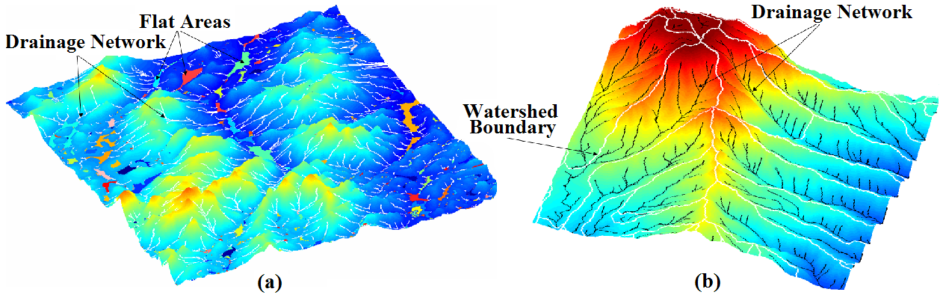

Fig. 5. (a) Visualizes flat regions. (b) Is an example of our computed drainage network.

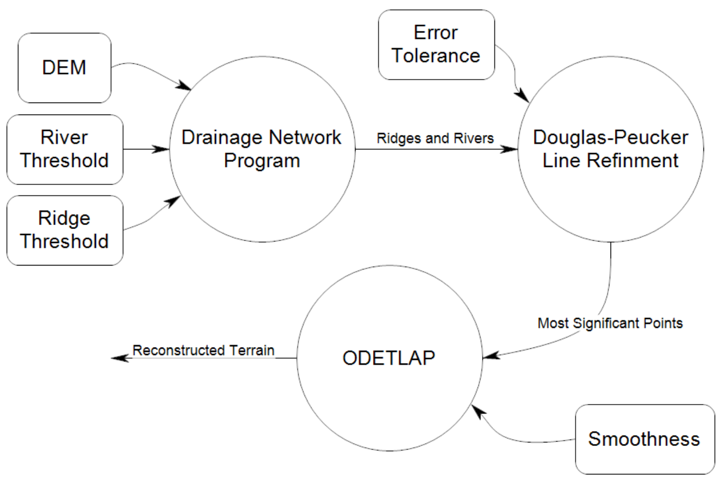

Fig. 6. Flow chart of the ridge-river technique. Inputs are in boxes and programs in circles.

is based on ODETLAP which reconstructs an approximation of a terrain using a set of points. The recovered terrain achieves a more accurate representation as more points are included.

The basic idea the of ODETLAP point selection is to include the most important points that lie on the river and ridge networks, which are determined by the drainage network program. Since these points are clustered together and redundant, we use the Douglas-Peuker curve compression algorithm to reduce the number of points required to represent each ridge/river segment. This line simplification uses a error tolerance that defines the maximum a simplified line can deviate from the original. These refined pointes are used to represent the terrain.

6.1 Approximating Terrain using Over-determined Laplacian PDEs

To reconstruct the terrain from a subset of the original elevation data, we use Over-determined Laplacian Differential Equations (ODETLAP). The input to this method is a compressed subset of points and the output is the reconstructed surface geometry. The Laplacian PDE is extended by adding a new equation to form an over-determined system so that we can control the relative importance of smoothness versus accuracy in the reconstruction. Benefits of the method include the ability to process isolated, scattered elevation points and the fact that reconstructed surface could generate local maxima, which is not possible in the original Laplacian PDE by the maximum principle.

ODETLAP can process not only continuous contour lines but isolated points as well. The surface produced tends to be smooth while preserving high accuracy to the known points. Local maxima are also well preserved. Alternate methods generally sub-sample contours due to limited processing capacity, or ignore isolated points.

Since we are working on single value terrestrial elevation matrix, we have the Laplacian equation for every unknown non-border point.

$$4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} \tag{2}$$

In terrain modeling this equation has the following limitations:

- The solution of Laplace's equation never has a relative maximum or minimum in the interior of the solution domain, this is called the maximum principle, so local maxima are never generated.
- The generated surface may droop if a set of nested contours is interpolated

To avoid these limitations, an over-determined version of the Laplacian equation is defined as follows: apply the equation (2) to every non-border point, both known and unknown, and a new equation is added for a set S of known points:

$$z_{ij} = h_{ij} \tag{3}$$

Where h_{ij} stands for the known elevations of points in S and z_{ij} is the computed elevation for every point, like in equation (2). The system of linear equations is over-determined, i.e., the number of equations exceeds the number of unknown variables, so instead of solving it for an exact solution (which is now impossible), an approximated solution is obtained by setting up a smoothness parameter R that determines the relative importance of accuracy versus smoothness.

6.2 ODETLAP Point Selection

In prior work [15], determining which points to input into ODETLAP was based on certain geometric algorithm including Triangulate Irregular Network, Visibility test, Level Set Component that discovers important points which reflect the terrain structure and use our extended Laplacian PDE to approximate the terrain from these points. In this paper the goal is not only to preserve overall terrain structure, but also to ensure that important hydrology features are preserved as well. Our experiments have shown that points on the ridge network and drainage network are the most effective in capturing the hydrology. The ridge-river technique computes both the rivers and ridges, and then simplifies the line network to capture the most significant points.

The drainage and ridge networks are simplified using the Douglas-Peucker[6] line refinement algorithm. This algorithm selects the most significant points need to reconstruct a line within a given error tolerance. This tolerance specifies the maximum distance the line can deviate from the original. The higher the tolerance, the fewer points required and the greater the difference between the original network and the reconstructed network. The output from the Douglas-Peucker algorithm is an ordered list of the most significant points needed to reconstruct the line. These points represent our compressed version of the hydrology. As Figure 7 illustrates, when the tolerance is set appropriately there is a significant reduction in number of points with the difference in the reconstructed lines being negligible.

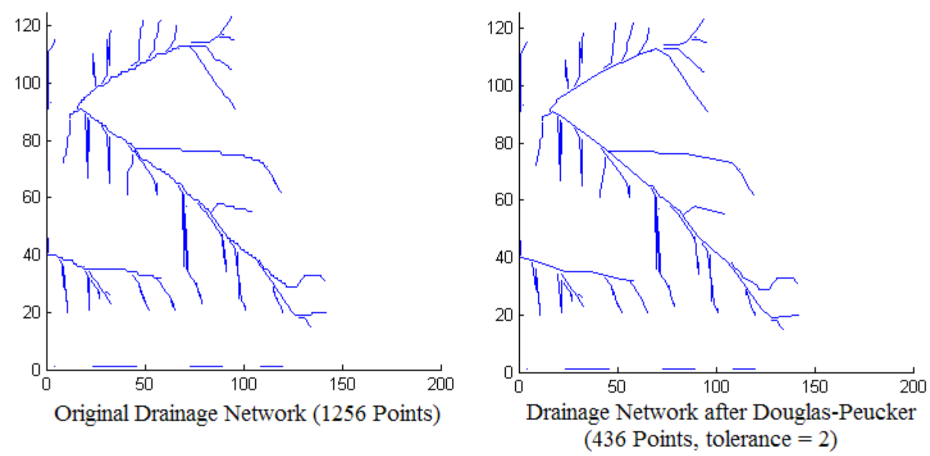

Fig. 7. Simplifying the original drainage network using Douglas-Peucker. The refined line network is reduced by a factor of 3 with little visible difference.

Our compressed version of the hydrology exists as a subset of elevations from the original DEM, plus the points along the reconstructed line using the Bresenham line rasterization algorithm[2]. All the points are selected along drainage significant features. The reasoning is that we want to capture the most significant points that preserve the hydrology.

6.3 Recovering Terrain Using a ODETLAP Hydrology Customization

To more accurately capture the structure of the hydrology, the ODETLAP equations are modified for points selected on the ridge-river network. This drastically reduces the amount of error introduced, as shown in Figure 8. The modification to equation 2 takes into account our observation that points on the drainage network have lower elevation values than the average of their neighbors:

$$4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} - D_R \tag{4}$$

where D_r stands for decrement for the rivers, this variable is an integer corresponding the number of meters the rivers lie below the average of the 4 neighbors. Similarly, ridge network points are higher then the average of their four neighbors, thus for selected ridge network points, the equation becomes:

$$4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} + I_R \tag{5}$$

where I_R is an integer corresponding to the increment for the ridges. Experimentation has shown that setting $D_R = I_R = 2$ has been effective. In future work we plan to study how varying this parameter affects the results and investigate ways to automatically select an optimal value.

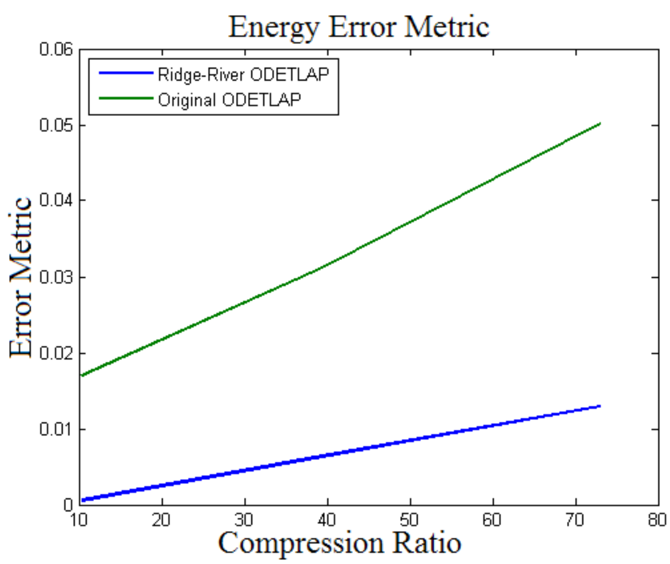

Fig. 8. Modifying the ODETLAP equations to better represent ridges and rivers has a drastic decrease in the amount of hydrology error. Both plotted lines above use the same set of points.

7 Results

The primary focus has been to describe a metric that accurately captures the amount of error introduced into a reconstructed drainage network. Using this metric, we have been developing an algorithm for achieving high compression ratios without significantly altering the hydrology. The current effectiveness of this approach is shown in Table 1. We compare our new compression technique to a Triangulated Irregular Network (TIN) and JPEG2000 [3] image compression. JPEG2000 obtains a low percentage of cells that flow uphill, which correlates to a fairly low hydrology error. The ridge-river technique is effective in achieving high compression ratios with a fairly low error, however, it currently does not consistently beat JPEG2000. We strongly believe that small modifications to the current ridge-river method will allow us to achieve a significantly better hydrology error. We are investigating further modifications to the ODETLAP equations, and to automatically select optimal parameters.

		Ridge-River		JPEG2000		TIN	
DATASET	Compr. Ratio	% Uphill	ERROR	% Uphill	ERROR	% Uphill	ERROR
hill1	13	2.05%	0.0023	0.12%	0.0020	0.79%	0.0432
	32	3.16%	0.1149	0.18%	0.0030	1.11%	0.0502
	54	2.46%	0.2316	0.24%	0.0082	1.33%	0.0600
hill2	14	0.85%	0.0005	0.21%	0.0010	1.25%	0.0333
	37	1.21%	0.0063	0.31%	0.0017	1.80%	0.0304
	60	1.39%	0.0129	0.46%	0.0047	2.43%	0.0421
hill3	11	2.65%	0.0026	0.10%	0.0059	0.76%	0.0311
	27	4.33%	0.0075	0.11%	0.0051	0.77%	0.0434
	47	2.70%	0.0100	0.13%	0.0161	0.85%	0.0405
mtn1	16	3.75%	0.0267	0.41%	0.0026	3.96%	0.0563
	39	4.96%	0.0530	0.80%	0.0036	5.11%	0.0583
	60	5.91%	0.0611	1.33%	0.0067	6.28%	0.0667
mtn2	16	3.93%	0.0769	0.40%	0.0033	4.42%	0.0748
	38	5.15%	0.1169	0.75%	0.0033	5.72%	0.0874
	59	6.21%	0.1377	1.32%	0.0067	7.09%	0.0904
mtn3	15	3.10%	0.0254	0.40%	0.0015	4.16%	0.0592
	39	4.33%	0.0493	0.78%	0.0027	5.63%	0.0624
	61	5.13%	0.0639	1.40%	0.0050	6.63%	0.0650

Table 1. The amount of potential energy error for six 400 by 400 datasets sampled at 30m resolution. The percentage of flow traveling uphill is also shown, along with the compression ratio of each dataset.

Visual inspection of the reconstructed drainage networks correspond to the measurement determined by the potential energy metric. This is observed in Figure 9, where the higher error correlates to fragmented and uphill drainage networks. The modular design of our terrain simplification approach allows substituting different algorithms in place of the ones focused on in this paper. For

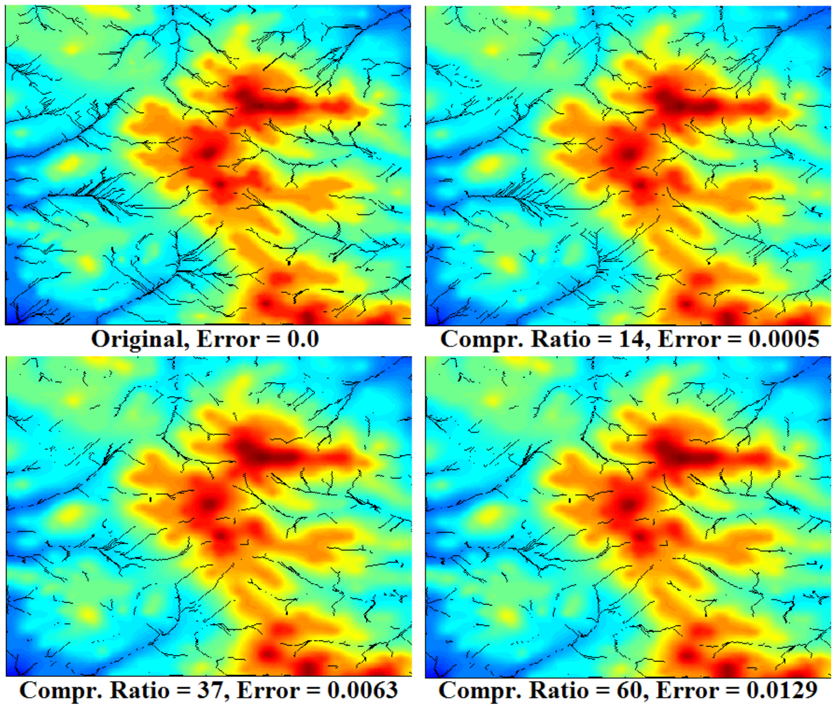

Fig. 9. The images show the a 400×400 hill2 dataset sampled at 30m resolution and compressed using the ridge-river technique. The color regions represent the elevations with blue being low and red corresponding to high elevation. The black regions shows the significant drainage network above the threshold of 100. The higher potential energy error metric correlates with a visible difference in the drainage network. Notice how the high error corresponds to short fragmented drainage networks.

instance, Terraflow or ArcGIS could be used to compute the ridge-river network. Also, a different line simplification technique could be used instead of Douglas-Peucker. This allows modification to fit the specific objectives of the user and application.

Points on the ridges of the terrain, as well as the rivers are important for preserving the hydrology. Rather than use an existing algorithm we discovered that inverting the terrain and running the drainage network provides a quick, effective method for approximating the ridge network. This approach can be done with any drainage network program. The ridges are important in terrain compression for extracting and exploiting terrain structure, but also have other GIS applications such as visibility siting, hydrology and edge detection.

8 Conclusion

The potential energy metric introduced in this paper provides a quantitative measurement of the amount of error introduced by a terrain simplification technique. This value is reflective of the visible examination of the drainage networks, with higher error corresponding to fragmented and unrealistic flow directions (flow traveling uphill).

The original DEM is an approximation of the real world terrain surface and not the complete truth, due to dataset and sampling errors. Flow can travel in different directions then the original drainage network, yet contain a low error metric if the flow directions are reasonable. Standard metrics such as root mean squared error and maximum error are ineffective in evaluating the amount of error introduced, as they do not take into account important hydrology features.

As more terrain is being sampled at ever increasing resolutions, it becomes more important to be able store and manipulate large elevation datasets and evaluate the amount of error introduced by lossy compression. However, current techniques for compressing these datasets lose important information that is essential for running operations on the reconstructed geometry with reliable results.

Understanding how compression affects important terrain structure, such as hydrology, allows the GIS community to understand how a compression technique affects the drainage accuracy of the reconstructed terrain. Our targeted compression technique has the goal of minimizing the amount of potential energy error, thus allowing for high compression ratios with minimal loss of hydrology information. The net result of this work is a compression scheme and error evaluation metric with applications including flooding, erosion, sanitation, and environmental protection.

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