



# Approximating Terrain with Over-Determined Laplacian PDEs

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## Abstract:

We extend Laplacian PDE by adding a new equation to form an over-determined system (ODETLAP) which can be used to approximate the whole terrain from a few isolated points. We compress the terrain by selecting a few points which could later be lossily 'decompressed' using ODETLAP. Points selection algorithms include TIN, Visibility test, Level Set Components and Regular Selection.

## ODETLAP:

Two sets of equations make the system over-determined:

Laplacian Equation: every non-border point is the average of its neighbors:

$$4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}$$

New equation: Some points are already known:

$$z_{ij} = h_{ij}$$

Use a smoothness parameter R to interpolate the two, R reflect the relative importance of accuracy vs. smoothness.

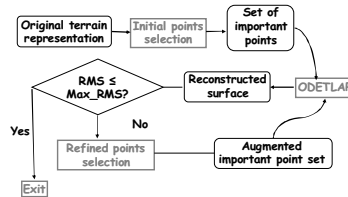
Adds capabilities to the classical system

- Local maxima inference
- Inconsistent data conflation

## Encoding ODETLAP's output:

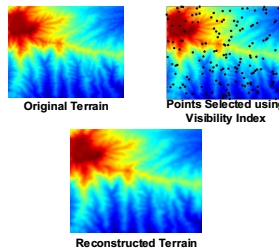
Code (x, y) separately from z:

- Run-length encode the bitmap;
- Delta code (z), then use bzip2.
- Approach information theoretic within 20%



## ODETLAP Point Selection:

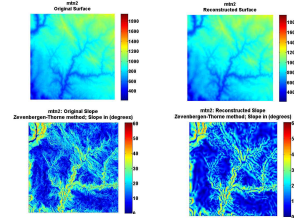
1. Incremental TIN to find most important points, then greedy insertion of worst points (Allows progressive transmission)
2. Regular grid of points (more points that compress better) (More compact)
3. Visibility Index to find points that represent the structure of the terrain



## 1. Compression Results (TIN + Greedy ODETLAP)

Data	Size, bytes	Compression ratio	RMS Elev Error, m	RMS Slope Error, deg
hill1	2984B	107:1	8.49	2.81
hill2	5358B	60:1	9.93	5
hill3	1739B	184:1	8.31	1.65
mtn1	9744B	33:1	9.48	8.34
mtn2	9670B	33:1	9.55	8.36
mtn3	9895B	32:1	9.68	7.87

## 2. TIN + Greedy Elevation Comparison: Mtn2 Dataset (Compressed Size: 7641 bytes)



## 3. Accuracy vs. Size

