Tradeoffs when Multiple Observer Siting on Large Terrain Cells

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Summary. This paper demonstrates a toolkit for multiple observer siting to maximize their joint viewshed, on high-resolution gridded terrains, up to 2402×2402, with the viewsheds' radii of up to 1000. It shows that approximate (rather than exact) visibility indexes of observers are sufficient for siting multiple observers. It also shows that, when selecting potential observers, geographic dispersion is more important than maximum estimated visibility, and it quantifies this. Applications of optimal multiple observer siting include radio towers, terrain observation, and mitigation of environmental visual nuisances.

Key words: terrain visibility, viewshed, line of sight, siting, multiple observers, intervisibility

1 Introduction

Consider a terrain elevation database, and an observer, $\mathcal{O}$. Define the *viewshed* as the terrain visible from $\mathcal{O}$ within some radius of interest, R, of $\mathcal{O}$. The observer might be situated at a certain height, $\mathcal{H}$, above ground level, and might also be looking for targets also at height $\mathcal{H}$ above the local ground. Also, define the *visibility index* of $\mathcal{O}$ as the fraction of the points within R of $\mathcal{O}$ that are visible from $\mathcal{O}$. This paper goes beyond merely computing viewsheds of individual observers. It combines a fast viewshed algorithm with an approximate visibility index algorithm, to site multiple observers so as to jointly cover as much terrain as possible.

The multiple observers case is particularly interesting and complex, and has many applications. A cell phone provider wishes to install multiple towers so that at least one tower is visible (in a radio sense) from every place a customer's cellphone might be. Here, the identities of the observers of highest visibility index are of more interest than their exact visibility indices, or than the visibility indices of all observers. One novel future application of siting radio transmitters will occur when the moon is settled. The moon has no ionosphere to reflect signals, and no stable satellite orbits. The choices for long-range communication would seem to include either a lot of fiber optic cable or many relay towers. That solution is the multiple observer visibility problem.

As another example, a military planner needs to put observers so that there is nowhere to hide that is not visible from at least one. This leads to a corollary application, where the other side's planner may want to analyze the first side's observers to find places to hide. In this case, the problem is to optimize the targets' locations, instead of the observers'.

Again, a planner for a scenic area may consider each place where a tourist might be to be an observer, and then want to locate ugly infrastructure, such as work yards, at relatively hidden sites. S/he may wish site a forest clearcut to be invisible to observers driving on a highway sited to give a good view. Finally, an architect may be trying to site a new house while following the planning board's instruction that, "You can have a view, but you can't be the view."

Our programs may easily produce a set of observers with *intervisibility*, i.e., their views of each other form a connected graph, but we do not impose that constraint in the experiments reported here.

In contrast to many other researchers, we consider that speed of execution on large datasets is important. Many prototype implementations, demonstrated on small datasets, do not scale up well. That may happen either because of the size and complexity of the data structures used, or because of the asymptotic time behavior. For instance, even an execution time proportional to $N \log(N)$, where N is the size of the input, is problematic for $N = 10^6$. In that case, the $\log(N)$ increases the time by a factor of 20. Some preliminary published algorithms may even be exponential if performing a naive search. Therefore, we strive for the best time possible.

In addition, large datasets may contain cases, which did not occur in the small test sets, that require tedious special programming by the designer. In a perfect software development process, all such cases would have been theoretically analyzed *a priori*, and treated. However, in the real world, testing on the largest available datasets increases our confidence in the program's correctness.

Next, a large enough quantitative increase in execution speed leads to a qualitative increase in what we can do. Only if visibility can be computed efficiently, can it be used in a subroutine that is called many times, perhaps as as part of a search, to optimize the number of observers. This becomes more important when a more realistic function is being optimized, such as the total cost. E.g., for radio towers, there may be a tradeoff between a few tall and expensive towers, and many short and cheap ones. Alternatively, certain tower locations may be more expensive because of the need to build a road. We may even wish to add redundancy so that every possible target is visible from at least two observers. In all these cases, where a massive search of the solution space is required, success depends on each query being as fast as possible.

Finally, although the size of available data is growing quickly, it is not necessarily true that available computing power is keeping pace. There is a military need to offload computations to small portable devices, such as a Personal Digital Assistant (PDA). A PDA's computation power is limited by its battery, since, approximately, for a given silicon technology, each elemental computation consumes a fixed amount of energy. Batteries are not getting better very quickly; increasing the processor's cycle speed just runs down the battery faster.

There is also a compounding effect between efficient time and efficient space. Smaller data structures fit into cache better, and so page less, which reduces time. The point of all this is that efficient software is at least as important now as ever.

The terrain data structure used here is either a 1201×1201 matrix of elevations, such as from a USGS level-1 Digital Elevation Model cell, or a 2402×2402 extract from the National Elevation Data Set. The relative advantages and disadvantages of this data structure versus a triangulation are well known, and still debated; the competition improves both alternatives. This current paper utilizes the simplicity of the elevation matrix, which leads to greater speed and small size, which allows larger data sets to be processed.

For distances much smaller than the earth's radius, the terrain elevation array can be corrected for the earth's curvature, as follows. For each target at a distance D from the observer, subtract $D^2/(2E)$ from its elevation, where E is the earth's radius. The relative error of this approximation is $(D/(2E))^2$. It is sufficient to process any cell once, with an observer in the center. The correction need not changed for different observers in the cell, unless a neighboring cell is being adjoined. Therefore, since it can be easily corrected for in a preprocessing step, our visibility determination programs ignores the earth's curvature.

The radius of interest, R, out to which we calculate visibility, has no relation to the distance to the horizon, but is determined by the technology used by the observer. E.g., if the observer is a radio communications transmitter, doubling R causes the required transmitter power to quadruple. If the observer is a searchlight, then its required power is proportional to R^4.

In order to simplify the problem under study enough to make some progress, this work also ignores factors such as vegetation that need to be handled in the real world. The assumption is that it's possible, and a better strategy, to incorporate them only later.

This paper extends the earlier visibility work in [9] and [11], which also survey the terrain visibility literature. The terrain siting problem was identified as far back as 1982 by Nagy, [2]. Other notable pioneer work on visibility includes [5, 18, 23]. [24] studied visibility, and provided the Lake Champlain W data used in this paper. [22] presented new algorithms and implementations of the visibility index, and devised the efficient viewshed algorithm that we use. One application of visibility is a more sophisticated evaluation of lossy compression methods, [1]. [3, 4, 19] analyze the effect of terrain errors on the computed viewshed. [6] proposes modified definitions of visibility for certain applications. [17] explores several heuristics for siting multiple observers, and reports on the experimental tradeoffs that were observed. [25] discusses many line-of-sight issues. For more details on the results in this paper, see [13, 26]. An extended abstract was published in [7]. Lack of space here prevents the presentation of our experiments on the effect of lowered resolution on the quality of the siting.

The results reported here are part of a long project that may be called *Geospatial Mathematics*. Our aim is to understand and to represent the earth's terrain elevation. Previous results have included these:

1. a Triangulated Irregular Network (TIN) program that can completely tin a 10801×10801 block of 3×3 level-2 DTEDs, [8, 10, 21],
2. Lossy and lossless compression of gridded elevation databases, [12], and
3. Interpolation from contours to an elevation grid, [15, 14, 16].

2 Siting Toolkit

This toolkit, whose purpose is to select a set of observers to cover a terrain cell, consists of four core C++ programs, supplemented with zsh shell scripts, Makefiles, and assorted auxiliary programs, all running in SuSE Linux. The impact of this toolkit resides in its efficient processing of large datasets.

1. VIX calculates approximate visibility indices of every point in a cell. VIX takes several user parameters: R, the radius of interest, H, the observer and target height, and T, a sample size. VIX reads an elevation cell. For each point in the cell in turn, VIX considers that point as an observer, picks T random targets uniformly and independently randomly distributed within R of the point, and computes what fraction are visible. That fraction is this point's estimated visibility index.
2. FINDMAX selects a manageable subset, called the top observers, of the most visible tentative observers from VIX's output. This is somewhat subtle since there may be a small region containing all points of very high visibility. A lake surrounded by mountains would be such a case. Since multiple close observers are redundant, we force the tentative observers to be spread out as follows.
   a) Divide the cell into smaller blocks of points. If necessary, first perturb the given block size so that all the blocks are the same size, ± 1.
   b) In each block, find the K points of highest approximate visibility index, for some reasonable K, e.g., 3. If there were more than K points with equally high visibility index, then select K at random, to prevent a bias towards selecting points all on one side of the block.
3. VIEWSHED finds the viewshed of a given observer at height H out to radius, R. The procedure, which is an improvement over [11], goes as follows.

   a) Define a square of side $2R$ centered on the observer.
   b) Consider, in turn, each point around the perimeter of the square to be a target.
   c) Run a sight line out from the observer to each target calculating which points adjacent to the line, along its length, are visible, while remembering that both the observer and target are probably above ground level.
   d) If the target is outside the cell, because R is large or the observer is close to the edge, then stop processing the sight line at the edge of the cell.

   Various nastily subtle implementation details are omitted. The above procedure, due to [22], is an approximation, but so is representing the data as an elevation grid, and this method probably extracts most of the information inherent in the data. There are combinatorial concepts, such as Davenport-Schintzel sequences, which present asymptotic worst-case theoretical methods.

4. SITE takes a list of viewsheds and finds a quasi-minimal set that covers the terrain cell as thoroughly as possible. The method is a simple greedy algorithm. At each step, the new tentative observer whose viewshed will increase the cumulative viewshed by the largest area is included, as follows.
   a) Calculate the viewshed, $\mathcal{V}_i$, of each tentative observer $\mathcal{O}_i$. $\mathcal{V}_i$ is a bitmap.
   b) Let $\mathcal{C}$ be the cumulative viewshed, or set of points visible by at least one selected observer. Initially, $\mathcal{C}$ is empty.
   c) Repeat the following until it is not possible to increase $area(\mathcal{C})$, either because all the tentative observers have been included, or (more likely) because none of the unused tentative observers would increase $area(\mathcal{C})$.
      i. For each $\mathcal{O}_i$, calculate $area(\mathcal{C} \cup \mathcal{V}_i)$.
      ii. Select the tentative observer that increases the cumulative area the most, and update $\mathcal{C}$. Not all the tentative observers need be tested every time, since a tentative observer cannot add more area this time than it would have added last time, had it been selected. Indeed, suppose that the best new observer found so far in this step would add new area A. However we haven't checked all the tentative new observers yet in this loop, so we continue. For each further tentative observer in this execution of the loop, if it would have added less than A last time, then do not even try it this time.

In all the experiments described in the following sections, all the programs listed above are run in sequence. In each experiment, the parameters affecting one program are varied, and the results observed.

3 Vix and Findmax Experiments

Our goal here was to optimize VIX and FINDMAX, and to achieve a good balance between speed and quality. We used six test maps. Five of those maps were level-1 DEM maps, with 1201×1201 postings and a vertical resolution of 1 meter. The maps were chosen to represent different types of terrain, from flat planes to rough mountainous areas. Table 1 describes them, and Fig. 1 shows them.

Table 1. Elevation Statistical Values for the Level-1 DEM Maps

Name	Mean	Min	Max	Range	St dev
Aberdeen east	420.5	379	683	304	36.5
Baker east	1260.9	546	2521	1975	376.9
Gadsden east	257.6	118	549	431	73.7
Hailey east	1974.1	954	3600	2646	516.3
Lake Champlain west	272.5	15	1591	1576	247.8

The sixth map is a National Elevation Data Set (NED) downloaded from the USGS "Seamless Data Distribution System". From the original 7.5-minute map with bounds $(41.2822, 42.4899), (-123.8700, -122.6882)$, the first 2402 rows and columns were extracted. This map is from a rough mountainous region, and was chosen to test our programs on a larger higher resolution map, since some siting programs might have difficulties here. Table 2 gives its statistics.

Table 2. Elevation Statistics of the Large NED Map

Name	Mean	Min	Max	Range	St dev
California	706.9	205.9	2211.3	2005.4	2946.8

3.1 Testing VIX

These experiments tested the effect of varying T, the number of random targets used by VIX to estimate the visibility index of each observer. A higher T produces more accurate estimates but takes longer. Note that precise estimates of visibility indexes are unnecessary since they are used only to produce an initial set of potential observers, called the top observers. Actual observers are selected from this set according by how much they increase the cumulative viewshed.

We performed these tests with various values of R and H, on various datasets, The experiment consisted of five different test runs for all maps and an additional sixth test run for the larger map, as shown in Table 3. Each test

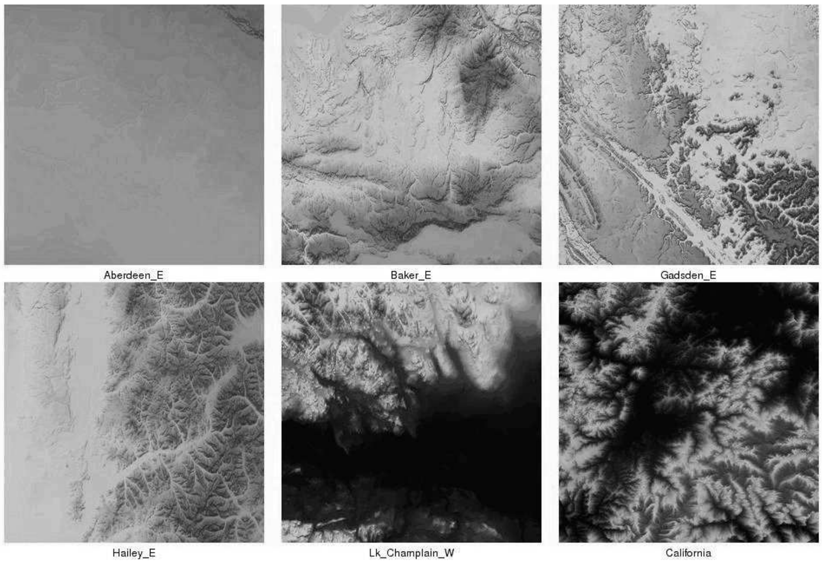

Fig. 1. The Test Cells

run contained 10 different test cases, listed in Table 4. $T = 0$ gives a random selection of observers since all observers have an equal visibility index of zero.

Table 3. Parameter Values for the Different Test Runs of the Experiment (Italicized Case Only for the California Dataset)

Parameter	Test runs					
Radius of interest R	100	100	100	80	300	*1000*
Observer and target height H	5	10	50	10	10	*10*

Table 4. Parameter Values for the Different Test Cases of the Experiment

Parameter	Test cases									
Sample size T	0	2	5	8	12	15	20	30	50	200

Each test case was executed 20 times for the 1201×1201 maps and 5 times for the 2402×2402 map. Each time enough observers were selected to cover 80% of the terrain. (Findmax used a block size of 100 and 1008 top observers.) The mean number of observers over the 20 runs was reported.

Figure 2 shows results for $R = 300$ and $H = 10$. The results were normalized to make the output from the experiments with no random tests to

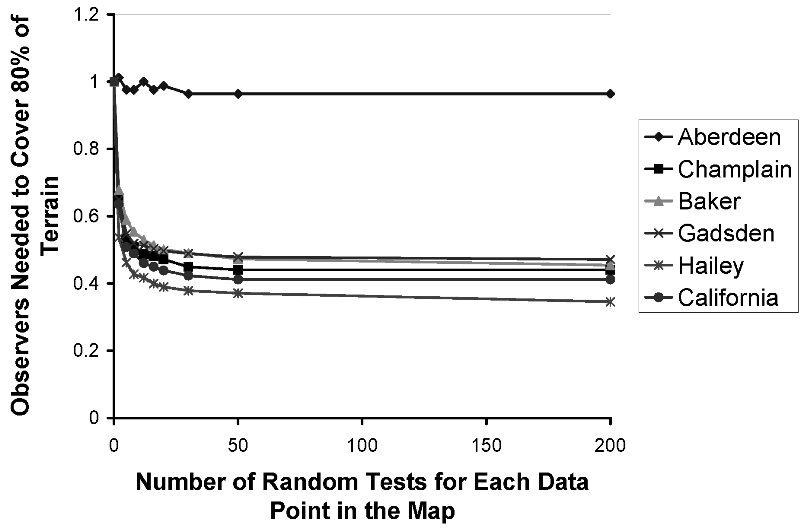

Fig. 2. Effect of Varying the Number of Tests per Observer on the Number of Observers Needed to Cover 80% of the Cell, for $R = 300, = 10$

be 1. That is, 1 is the result that can be achieved by randomly choosing top observers for SITE. Every value higher than one is worse than random, while every value lower then one is better. Figure 5 shows the Baker test case in more detail.

3.2 Testing FINDMAX

The purpose of the FINDMAX experiment was to evaluate the influence of FINDMAX on the final result of the siting observers problem. The two parameters evaluated were the number of top observers and the block size. The number of top observers specifies how many observers should be returned by FINDMAX. A larger number slows SITE because there are more observers to choose from, but may lead to SITE finally needing fewer observers. Therefore we want to keep this number as low as possible. It is computationally cheaper to increase the sample set in VIX than to increase the number of top observers. The block size specifies how much the top observers returned by FINDMAX are forced to spread out. A smaller number increases the number of blocks on a map and therefore reduces the number of top observers from a given block. This parameter has no influence on the computational speed.

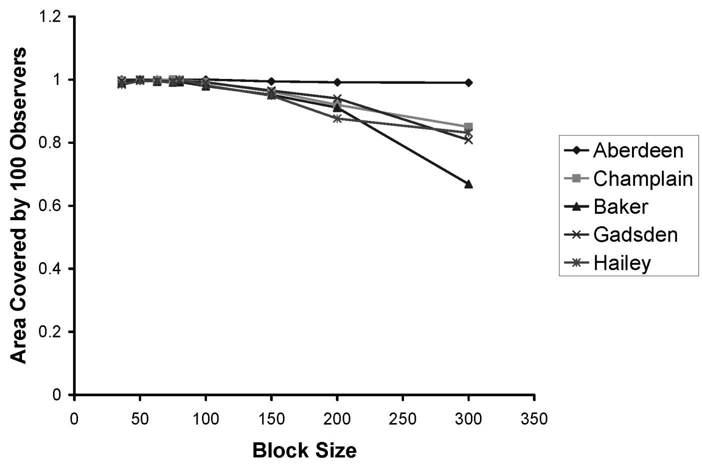

Fig. 3. Effect of Block Size on the Area Covered by 100 Observers, for Various 1201 × 1201 Cells

Test procedure

The experiment for the number of top observers consisted of 9 different test cases. It was only conducted on the level-1 DEM maps. During the experiment the values for the number of top observers ranged from 576 to 10080. In all the test runs a block size of 100 was chosen, resulting in 144 blocks. 576 top observers produced 4 observers per block; 10080 top observers produced 70 observers per block. All different values for the number of top observers are given in Table 5. together with their resulting number of observers per block.

The experiment for the block size was different for level-1 DEM maps than for the larger map. In the case of the level-1 DEM maps there were 9 different test cases with values for block size ranging from 36 to 300. This resulted in having between 1 and 1089 blocks per map. The number of top observers was chosen to be 1000.

The actual number depends on the number of blocks since each block needs the same number of top observers. In case of the larger maps there are 8 different test cases with values for block size ranging from 80 to 2402. This results in having between 1 to 900 blocks per map. The number of top observers was chosen to be 2000. The actual number depends on the number

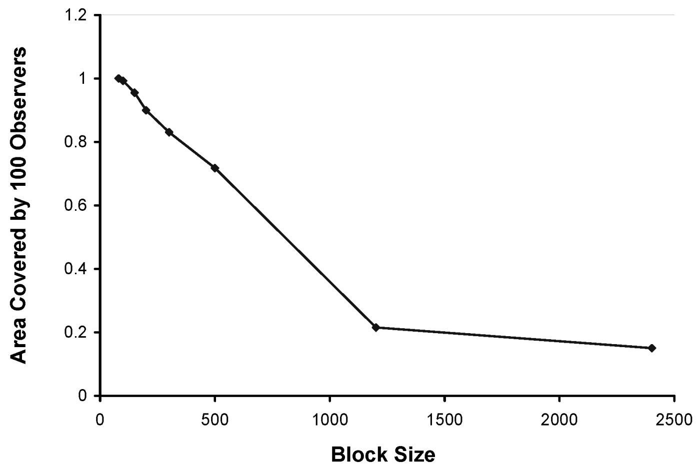

Fig. 4. Effect of Block Size on the Area Covered by 100 Observers, for the Large Cell

of blocks since each block needs the same number of top observers. All the different settings are given in Table 5.

Evaluation

In the sample size experiment, each test case was executed 20 times, with the entire application run each time until the site program was able to cover 80% of the terrain. VIX used $R = 100$, $H = 10$, and $T = 20$. The resulting number of observers needed to cover the 80% was noted, and the arithmetic mean from the results of the same test case calculated.

In the block size experiment, each test case was executed 20 times for the level-1 DEM maps and 5 times for the larger test. The evaluation of the results is slightly different. The site program ran until 100 (400 for the larger map) observers were sited. The parameters used for VIX were $R = 100$, $H = 10$, and $T = 20$. The amount of terrain visible by the final observers was then noted. The reason for changing the evaluation method was due to the problem that in some test cases we were not able to cover 80% of the cell.

Figure 3 shows for different maps how much terrain can be seen by 100 observers. For all data sets the parameters used were $R = 100$ and $H = 10$. The results are normalized by 1. For each map the best result achieved by any

Table 5. The parameters for block size and top observers are given for the different test cases. The values in the "Blocks" column represent the actual number of blocks used by FINDMAX given the size of the map and the parameters for block size and top observers. The values in the "obs/block" column represent the number of top observers that FINDMAX calculates for each block.

Experiment	**Parameters & Numbers**	**Test Cases**								
Top Observers	Block Size	100	100	100	100	100	100	100	100	100
	Top Observers	576	864	1008	1296	1584	2016	3024	5040	10080
	Blocks	144	144	144	144	144	144	144	144	144
	Obs/Block	4	6	7	9	11	14	21	35	70
Block Size	Block Size	36	50	63	75	80	100	150	200	300
	Top Observers	1089	1152	1083	1024	1125	1008	1024	1008	1008
	Blocks	1089	576	361	256	225	144	64	36	16
	Obs/Block	1	2	3	4	5	7	16	28	63
Block Size	Block Size	80	100	150	200	300	500	1201	2402	
	Top Observers	2700	2304	2048	2016	2048	2000	2000	2000	
	Blocks	900	576	256	144	64	25	4	1	
	Obs/Block	3	4	8	14	32	80	500	2000	

value for the block size was considered to be 1. The results of the experiments using different values for the block size were scaled accordingly. Therefore the highest value that can be achieved is 1. Everything below one is worse.

Figure 4 shows for the larger map how much terrain can be seen by 100 observers. For the data sets the parameters used were $R = 100$ and $H = 10$. The results are normalized by 1. The best result achieved by any value for the block size was considered to be 1. The results of the experiments using different values for the block size were scaled accordingly. Therefore the highest value that can be achieved is 1. Everything below 1 is worse.

Figure 6 shows for different maps how many observers are needed to cover 80% of the data. For all data sets the parameters used were 100 for the radius of interest and 10 for the observer and target height. The results are normalized by 1.The results of the experiments that were achieved by computing 576 top observers was considered to be 1. Lower values are worse.

4 Conclusions

4.1 VIX Experiment

- A sample size of 20 to 30 random tests for VIX is a good balance between the quality of the result and the computational speed. Surprisingly this value is good for a wide range of parameters and terrain types.
- VIX improved the result on the level-1 DEM maps in the best case by reducing the amount of observers needed to 39% compared to randomly selecting top observers. The largest improvements were achieved for large

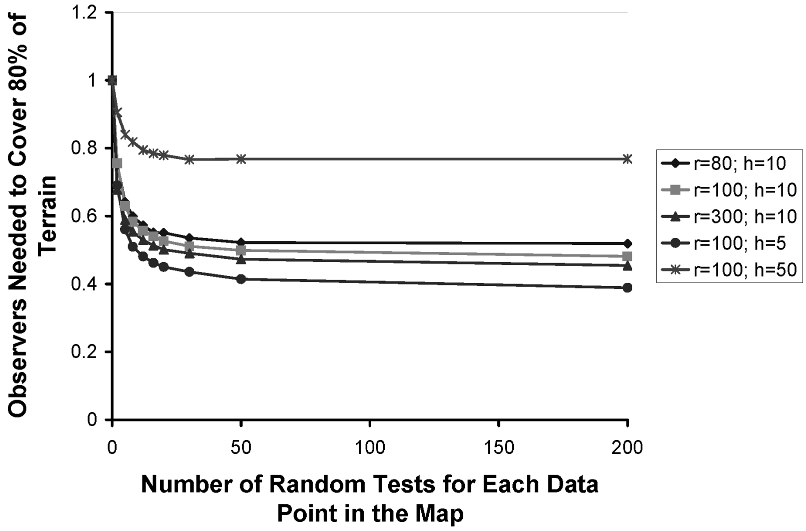

Fig. 5. Effect of Varying the Number of Tests per Observer on the Number of Observers Needed to Cover 80% of the Baker East Cell, for Various R and H

or rough terrain for large R or low H. The smallest improvement was achieved on flat terrain.

- On the larger map the improvement of VIX was even bigger. Possible explanations are that this terrain is the roughest, and that there were fewer top observers per data point than in the smaller maps.

4.2 FINDMAX Experiment

- The block size should be chosen to be small, i.e., 2 to 5 observers per block. When covering a larger fraction of the terrain, more blocks with a smaller number of observers per block is important.
- Increasing the number of top observers in FINDMAX increases the quality of the result, but requires much more time. It is cheaper to increase the number of random tests in VIX, but there is a limitation for what can be achieved by increasing the number of random tests. The best results in the entire experiment were achieved with 10000 top observers. This might not be obvious when comparing the graph of the results from the VIX experiments with the results from the FINDMAX experiments. However, during the FINDMAX experiments a relatively large number of random tests was chosen. Therefore the visibility index for FINDMAX was of a high resolution.

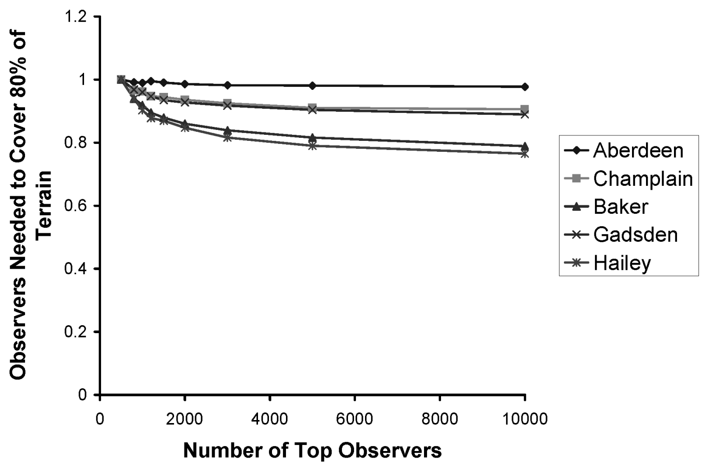

Fig. 6. Effect of Varying the Number of Top Observers Returned by FINDMAX on the Number of Observers Needed to Cover 80% of the Cell, for Various 1201 × 1201 Cells

5 The Future

The various tradeoffs mentioned above and the above experiments illuminate a great opportunity. They tell us that shortcuts are possible in siting observers, which will produce just as good results in much less time.

Another area for investigation is the connectivity of either the viewshed, or its complement. Indeed, it may be sufficient for us to divide the cell into many separated small hidden regions, which could be identified using the fast connected component program described in [20].

There is also the perennial question of how much information content there is in the output, since the input dataset is imprecise, and is sampled only at certain points. A most useful, but quite difficult, problem is to determine what, if anything, we know with certainty about the viewsheds and observers for some cell. For example, given a set of observers, are there some regions in the cell that we know are definitely visible, or definitely hidden? We have earlier demonstrated an example where the choice of interpolation algorithms for the elevation between adjacent posts affected the visibility of one half of all the targets in the cell.

This problem of inadequate data is also told by soldiers undergoing training in the field. Someone working with only maps of the training site will lose to someone with actual experience on the ground there.

Finally, the proper theoretical approach to this problem would start with a formal model of random terrain. Then we could at least start to ask questions about the number of observers theoretically needed, as a function of the parameters. Until that happens, continued experiments will be needed.

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