Terrain Representation Using Tessellation of Irregular Planar Tiles

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Executive Summary

A novel terrain representation based on tessellation of irregular planar tiles is being developed. We start with a regular square tessellation. Each square tile is used as a generator for a potential irregular planar tile. From all of the potential irregular tiles those which contribute most to the terrain coverage are picked and used for the tessellation.

Introduction

The algorithm developed can be used to represents digital terrain elevation maps with certain quality guarantees. To achieve this goal a representation based on irregularly shaped planar tiles is being used. We use an objective function to evaluate the fitness of the planar tiles used in the tessellation. The quality guarantee is a parameter of the objective function and bounds the maximum absolute error of the representation. The maximum absolute error is defined as the maximum absolute difference between the representation and the original terrain.

Methodology

We start by building a regular square tessellation of the original terrain. The typical size of a square tile is 2x2. Each of the square tiles contains 4 elevation values. Those are modeled using the following linear model:

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix}$$

While x and y are the given variables, z models the predicted variable: the elevation. Multiple linear regression is used to solve for the model parameters: a, b and c. The resulting parameters are the coefficients of the best fitting (in the least squares sense) plane equation. The plane equation $\tilde{z}(x,y) = ax + by + c$ is evaluated over the whole terrain using our objective function:

$$\forall(x,y) \quad z(x,y) - \tilde{z}(x,y) \leq Max\ Error\ Guarantee$$

The *(x, y)* tuples which satisfy the objective function make up a potential irregular planar tile. Each 2x2 square results in a potential planar tile.

In the next step the set of potential planar tiles is subjected to the picking procedure, which selects a subset of planar tiles to build a tessellation. Since the solution space is exponential, in a set with N members there are 2^N possible subsets, we use a greedy heuristic which usually gives an excellent solution:

Greedy-Subset Heuristic

```
Subset = empty
while Coverage (Subset) < Whole Terrain
        maxCoverage = 0
        for i = available Tiles
```

```
        t = Subset U i
        if Coverage (t) > maxCoverage
                maxCoverage = Coverage (t)
                bestTile = i
    Subset = Subset U bestTile
```

The result of the heuristic is a subset of irregular planar tiles, which tessellate the terrain subject to our objective function.

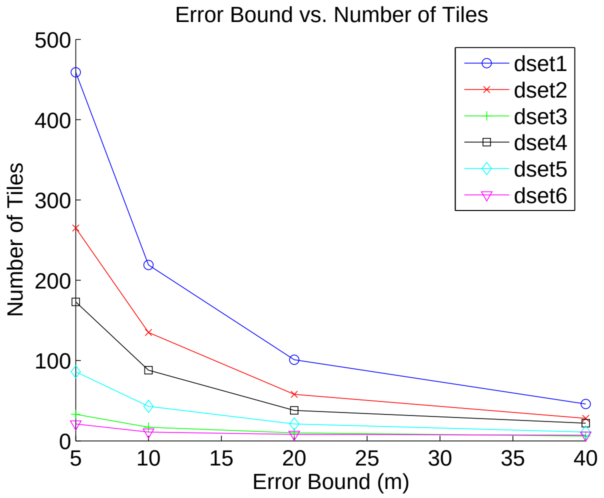
Data Structures

We use an indexed bitmap to keep the irregular tessellation. We have tried modern image compression methods like lossless JPEG 2000 on the resulting bitmaps without success. Thus we implement Huffman compression which assigns shorter codes for the larger tiles in the tessellation. We keep the planar coefficients of the irregular tiles in an indexed table and we have devised a method which aggressively compresses them with very little error on the resulting terrain.

Results

We use six different exemplary datasets all containing 300x300 elevations with varying terrain complexity. The first three of the datasets have horizontal resolution of approximately 90 m; the last three have a resolution of 30 m. The number of tiles drops exponentially with the increase in the error bound.

	Resolution	Min Elev.	Max Elev.
Dset1	90m	278m	1591m
Dset2	90m	122m	1076m
Dset3	90m	29m	262m
Dset4	30m	1097m	2014m
Dset5	30m	1085m	1810m
Dset6	30m	1398m	1627m



Conclusion

We have a novel method of terrain representation, which extends our previous work in [1], where a method based on regular tile tessellation is presented. The same problem of lossy terrain representation was studied in [2] using a very different perspective. Our method is very different from the level set method approach which is based on contour lines [3].

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References

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