C3 9/8/20 P1

COMPLEX VECTOR SPACE

4D
$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

2D
$$\begin{bmatrix} 1+2i \\ 3+4i \end{bmatrix} + \begin{bmatrix} 5+6i \\ 7+8i \end{bmatrix} = \begin{bmatrix} 6+8i \\ 10+12i \end{bmatrix}$$

$$7\begin{bmatrix} 1+2i \\ 3+4i \end{bmatrix} = \begin{bmatrix} 7+14i \\ 21+28i \end{bmatrix}$$

MATRICES ARE A COMPLEX VECTOR SPACE

$$\begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix}$$

4D VECTOR SPACE

POLYNOMIALS OF DEGREE n

x^2+2x+1 $\qquad$ $10x^2+20x+10$

$2x^2+3x+4$

$3x^2+5x+5$

$\mathbb{C}^{2\times 2}$ $\qquad$ 2x2 COMPLEX MATRICES

$$\begin{bmatrix} 1 & i \\ -i & 2 \end{bmatrix}$$

TRANSPOSE

$$\begin{bmatrix} 1 & -i \\ i & 2 \end{bmatrix}$$

CONJUGATE

$$\begin{bmatrix} 1 & -i \\ i & 2 \end{bmatrix}$$

ADJOINT

$$\begin{bmatrix} 1 & i \\ -i & 2 \end{bmatrix}$$

MULTIPLICATION

$\rightarrow$ ALGEBRA

STATE OF QUANTUM SYSTEM
IS A COMPLEX VECTOR SPACE

WE CAN COMBINE TWO
C.V.S. TO MAKE A
NEW ONE

1. DIRECT SUM
ORDERED PAIR.

V_1: 1-D REAL NUMBERS
eg. 3, 5, 20

V_2 SAME

COMBO: (v_1, v_2) eg $(3, 4)$

4

PROVE IT'S A VECTOR SPACE

ADD: $(v_1, v_2) + (v_1', v_2')$

$= (v_1 + v_1', v_2, v_2')$

BASIS OF A VECTOR SPACE

<u>N-DIM</u> SET OF N BASIS VECTORS

$B = b_1 \ldots b_n$

EVERY VECTOR IS A LINEAR COMBO OF B.

<u>~~2D~~ 2D</u> NATURAL $(1,0)$, $(0,1)$

$(2,3) = 2(1,0) + 3(0,1)$

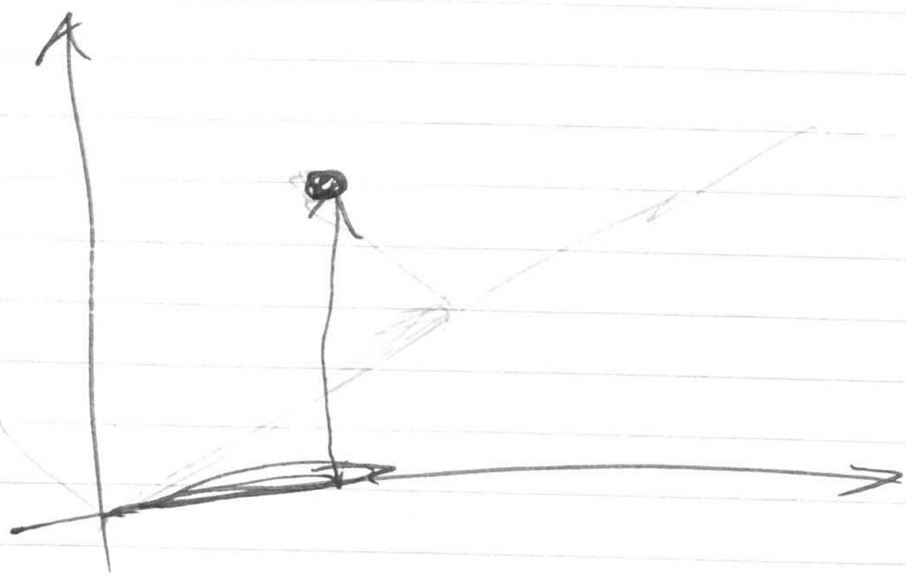
HERE'S ANOTHER BASIS SET

$(1,1)$, $(1,-1)$

$(2,3) = a(1,1) + b(1,-1)$

$2 = a+b$ $\quad a = \frac{5}{2} \quad b = -\frac{1}{2}$
$3 = a-b$

$(2,3) = \frac{5}{2}(1,1) - \frac{1}{2}(1,-1)$



HADAMARD

$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

BOOK MATRIX MULT ~~A~~

2ND WAY TO COMBINE VECTOR SPACE

TENSOR PRODUCT

SECT 2.7 P66

2 2D VECTOR SPACES

VECTOR FROM 1ST $\begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$

VECTOR FROM 2ND $\begin{pmatrix} b_0 \\ b_1 \end{pmatrix}$

TENSOR $\begin{pmatrix} a_0 b_0 \\ a_0 b_1 \\ a_1 b_0 \\ a_1 b_1 \end{pmatrix}$

7

PROVE IT'S ~~AFC~~ A VECTOR SPACE.

YOU CAN ADD

SCALE.

~~DIML~~ $\mathrm{DIM}(V_1) = m$

$V_2 \quad n$

$\mathrm{DIM}(V_1 \otimes V_2) = mn$

HOW TO CREATE A TENSOR PRODUCT VECTOR?

1. COMBINE 2 VECTS FROM COMPONENT SPACES
2. ~~ADD~~

ETC.

RESULT MIGHT NOT BE SEPARABLE.

9

8

BOOK p70 EX 2.7.2

$C^2 \otimes C^3 \to C^6$

$$\begin{bmatrix} 8 \\ 12 \\ 6 \\ 12 \\ 18 \\ 9 \end{bmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \otimes \begin{pmatrix} 4 \\ 6 \\ 3 \end{pmatrix}$$

SEPARABLE

$$\begin{pmatrix} 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 18 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \\ e \end{pmatrix}$$

NOT SEP

$\to$ ENTANGLED

NO SOLUTION.

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 6 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

CHAPTER 3 TEASER

(MARKOV MODELS)

VERSION 1 DETERMINISTIC

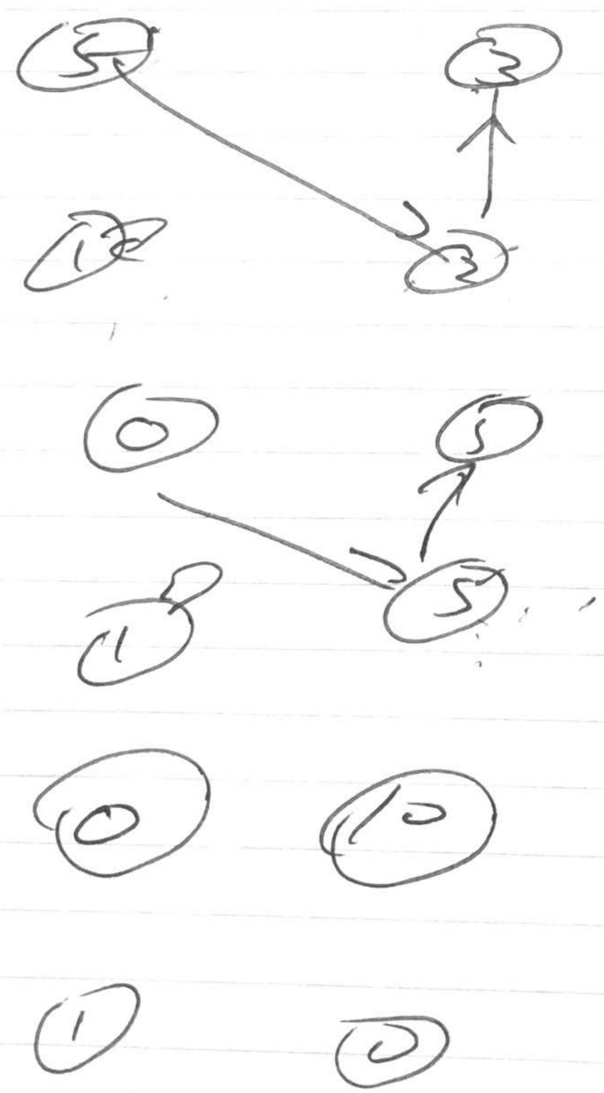
POTS WITH MARBLES

AT EACH STEP, THEY MOVE

T_0

T_1

T_2

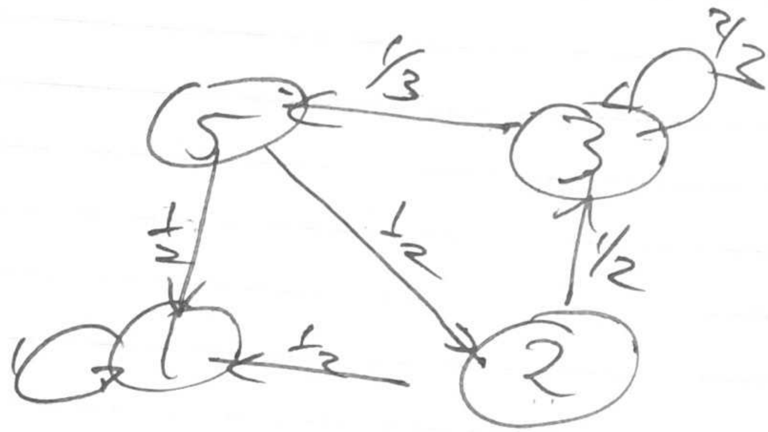


STATE $\begin{pmatrix}5\\3\\1\\2\end{pmatrix}$

TRANSITION MATRIX

1/2 PROBABILISTIC

T_0



T_1

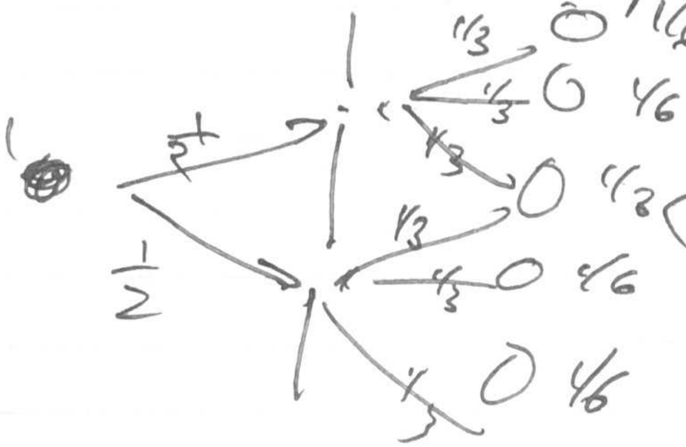
~~THIS IS WRONG DUNNO WHERE.~~

THIS IS MATRIX MULT ALSO.

P226

CLASSICAL

2 SLIT EXAMPLE



$\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

PROBABILITIES ADD + GET BIGGER

REAL NUMBERS.

IF PROBABILITIES ARE COMPLEX THEN THEY MIGHT CANCEL AND GET SMALLER.

SEE SECT 3.3