1a $\iint (x^2+2xy+y^2)\,dx\,dy$

$= \int \left(\frac{x^3}{3} + x^2y + y^2\right)\Big|_0^1 \,dy$

$= \int \left(\frac{1}{3} + y + y^2\right) dy$

$= \frac{y}{3} + \frac{y^2}{2} + \frac{y^3}{3} \Big|_0^1$

$= 7/6$

$c = \frac{6}{7}$

1b $F(x,y) = \frac{6}{7} \int_0^x \int_0^y \left(x_0^2 + 2x_0y_0 + y_0^2\right) dx\,dy$

$= \frac{6}{7} \int \left(\frac{x^3}{3} + x^2y_0 + xy_0^2\right) dy$

$= \frac{6}{7}\left(\frac{x^3y}{3} + \frac{x^2y^2}{2} + \frac{xy^3}{3}\right)$

FOR $0 \le x, y \le 1$

1a

F(x, y) CTD

$0,1$ $\qquad \frac{6}{7}\left(\frac{x}{3} + \frac{x^2}{2} + \frac{x^3}{3}\right)$ $\qquad 1,1$

0 $\qquad \frac{6}{7}\left(\frac{x^3y}{3} + \frac{x^2y^2}{2} + \frac{xy^3}{3}\right)$ $\qquad \frac{6}{7}\left(\frac{y}{3} + \frac{y^2}{2} + \frac{y^3}{3}\right)$

0 $\quad 0,0$ $\qquad 0$ $\qquad 1,0$

1c $\cancel{f_X}(x) = \frac{6}{7}\int (x^2 + 2xy + y^2)\,dy$

$= \frac{6}{7}\left(x^2\cancel{y} + x + \frac{1}{3}\right) \qquad 0 \le x \le 1$

1d $f_Y(y) = \frac{6}{7}\left(y^2 + y + \frac{1}{3}\right) \qquad 0 \le x \le 1$

1e $f(x)\,f(y) = \frac{36}{49}\left(x^2 + x + \frac{1}{3}\right)\left(y^2 + y + \frac{1}{3}\right)$

$\neq f(x, y)$

NOTE $f(x,y) = \frac{6}{7}(x+y)^2$

NOT INDEPENDENT

1f $E[X] = \int x f(x)\,dx$

$= \frac{6}{7}\int_0^1 \left(x^3 + x^2 + \frac{x}{3}\right) dx$

$= \frac{6}{7}\left(\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{6}\right)\Big|_0^1$

$= \frac{9}{14} = E[Y]$ $\cancel{\#}$

3

$$E[X^2] = \int x^2 f(x)\,dx$$

$$= \frac{6}{7}\int \left(x^4 + x^3 + \frac{x^2}{3}\right) dx$$

$$= \frac{6}{7}\left(\frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{9}\right)\Big|_0^1$$

$$= \frac{101}{210} = E[Y^2]$$

$$VAR[X] = E[X^2] - E[X]^2$$

$$= \frac{199}{2940}$$

$$= .068$$

FLOATING OR RATIONAL BOTH OK

$$VAR[Y] = VAR[X]$$

1 2 $E[XY] = \iint xy\, f(x,y)\, dx\,dy$

$$= \frac{17}{42}$$

$$\cancel{COV[X,Y] = \frac{17}{42} - \left(\frac{199}{2940}\right)^2 = .400}$$

4

$$COV[X,Y] = E[XY] - E[X]E[Y]$$

$$= \frac{17}{42} - \left(\frac{9}{14}\right)^2 = -\frac{5}{588}$$

$$\approx -.0085$$

$$\rho = \frac{COV[X,Y]}{\sigma_X \sigma_Y} = \frac{-.0085}{.068} = -.125$$

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2 ALL TIMES IN MINUTES AFTER 2:00

LET G = BUS ARRIVAL TIME. IT'S A RV.

$$f(G) = \begin{cases} \frac{1}{20} & 0 \le G \le 20 \\ 0 & \text{ELSE} \end{cases}$$

LET S = STUDENT ARRIVAL TIME.

LET C = COST

$$C = \begin{cases} 0 & \text{IF } S < G \\ 10 & \text{ELSE} \end{cases}$$

INTERESTING VALUES OF S ARE $0 \le S \le 20$

IF $S < 0$ $C = 0$

$S > 20$ $C = 10$

5

$$E[C] = \int_0^{20} C f(b)\, db$$

$$= \int_0^{S} C f(b)\, db + \int_S^{20} C f(b)\, db$$

$$= \int_0^{S} \frac{10}{20}\, db + \int_S^{20} 0$$

$$= \frac{S}{2}$$

$E[C] = 5$ WHEN $S = 10$

3 $f(x) = 1$

$$f(y|x) = \begin{cases} \frac{1}{x} & \text{if } y < x \\ 0 & \text{else} \end{cases}$$

$$f(y) = \int f(y|x) f(x)\, dx$$

$$= \int_y^1 \frac{1}{x}\, dx$$

$$= -\ln(y)$$

CHECK: $\int_0^1 -\lg(y)\,dy = 1$

4 $f(x|y) = \frac{1}{y}$ IF $0 \le x \le y$ ELSE 0.

THIS Q REVERSES THE NORMAL NOTATION
y IS INPUT ; x IS OUTPUT.

ML IS y THAT MAXIMIZES $f(x|y)$

THAT IS AT ~~x~~ $y = x$

5a $f_Y(y|x=0) = f_N(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$
NORMAL DIST

$f(y|x=1) = f_N(y-1)$

$f(y) = \frac{2}{3} f_N(y) + \frac{1}{3} f_N(y-1)$

$f(y \,\&\, X=0) = \frac{2}{3} f_N(y)$

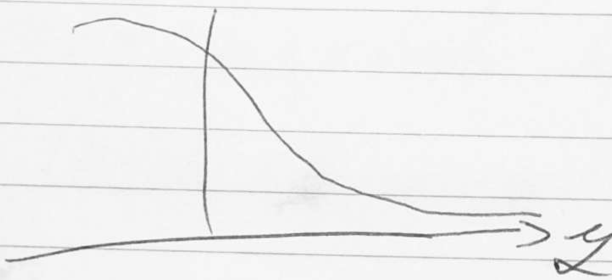
$P[X=0|y] = \cancel{P[X=0]} \frac{f(y \,\&\, X=0)}{f(y)}$

$$= \frac{\frac{2}{3} f_N(y)}{\frac{2}{3} f_N(y) + \frac{1}{3} f_N(y-1)}$$

$$= f_N(y) / \left[f_N(y) + \frac{1}{2} f_N(y-1) \right]$$

$$= \frac{e^{-\frac{y^2}{2}}}{e^{-\frac{y^2}{2}} + \frac{1}{2} e^{-\frac{(y-1)^2}{2}}}$$

5b $P[X=0 \mid y]$ LOOKS LIKE THIS:



WE WANT ~~to~~ y WHERE IT FALLS BELOW $\frac{1}{2}$

OK TO USE A COMPUTER.

THAT HAPPENS @ $y = 1.193$

$$g_{MAP}(y) = \begin{cases} 0 & \text{if } y \le 1.193 \\ 1 & \text{else} \end{cases}$$

6 X, Y_i, Z INDEPENDENT $\Longrightarrow$

$$E[X+Y_i+Z] = E[X] + E[Y_i] + E[Z]$$

$$VAR[X+Y_i+Z] = VAR[X] + VAR[Y_i] + VAR[Z]$$

$E[X+Y+Z] = 5 + \frac{1}{3} + 0 = 5\frac{1}{3} = 5.33$

$VAR[X+Y+Z] = 10 + \frac{1}{9} + \frac{1}{3} = 10\frac{4}{9} = 10.44$

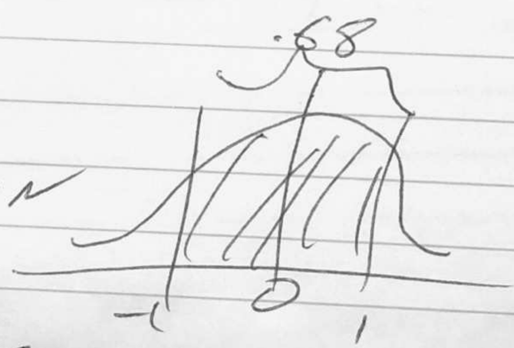
7. SAMPLE STD IS $\frac{\sigma}{\sqrt{N}} = 10 = S$

IF $P[100 - a \le \mu \le 100 + a] = .68$

THEN $100 + a = \cancel{8}\ m + s = 110$

$a = 10$

NOTE FOR GAUSSIAN



$P[\,|x| \le 1\,] = .68$

8 T-TEST

9A POISSON

B BINOMIAL

C GAUSSIAN / NORMAL - CAN'T USE FOR FIRST CASE (0.19) BECAUSE MEAN IS SO SMALL NORMAL APPROX WOULD BE BAD.

9D A χ^2 WITH 5 D.F.

B FAIR # FOR EACH FACE $= 16\frac{2}{3}$

$$\chi^2 = \sum \frac{1}{n_{pk}} (n_{ak} - n_{pk})^2$$

RADKE
NOTES
LEC 22

$$= \frac{(12 - 16\frac{2}{3})^2}{16\frac{2}{3}} + \frac{(20 - 16\frac{2}{3})^2}{16\frac{2}{3}} + \cdots$$

$$= 3.08$$

$P[\chi_5^2 \geq 3.08] = .69$ FROM A TABLE OR CALCULATOR

I.E. A DISTRIBUTION THIS FAR FROM EVEN COULD EASILY HAPPEN.
NO EVIDENCE THAT DIE IS UNFAIR.