PROB CLASS 19 R 3/24/22

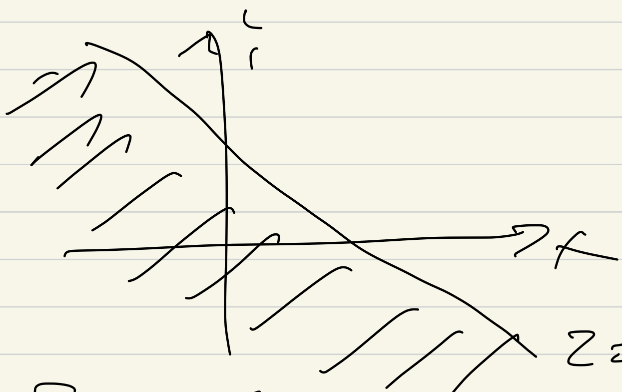
5.8 P271

FUNCTIONS OF 2 R.V.

2 RV X, Y. $f_{XY}(x,y)$

DEFINE $Z = X+Y$

WHAT ARE CDF, PDF OF Z.



$P[Z \le z] = P[X+Y \le z]$

$$F_Z(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{z-x'} f_{XY}(x', z-x)\, dy'\, dx'$$

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

$$= \int_{-\infty}^{\infty} f_{XY}(x', z-x')\, dx'$$

IF X, Y INDEPENDENT

$$f_{XY}(x', z-x') = f_X(x')\, f_Y(z-x')$$

EXAMPLE OF 2 DEPENDENT GAUSSIAN

$\mu = 0 \quad \sigma = 1$ SOME ρ

CORRELATION COEFF

$|\rho| \leq 1$

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{(x^2 - 2xy\rho + y^2)}{2(1-\rho^2)}\right]$$

SET $\rho = -\frac{1}{2}$ FOR EXAMPLE

$$f(x,y) = \frac{1}{2\pi\sqrt{\frac{3}{4}}} \exp\left[-\frac{(x^2+xy+y^2)}{3/2}\right]$$

$$z = x+y$$

$$f_Z(3) = \int f_{X,Y}(x', 3-x')\,dx$$

$$= \frac{1}{2\pi\sqrt{3/4}} \int_{x'=-\infty}^{\infty} \exp\left[\frac{-(x' + x(3-x') + (3-x')^2)}{3/2}\right] dx$$

$$= \frac{e^{-3^2/2}}{\sqrt{2\pi}}$$

GAUSSIAN

$\mu = 0 \quad \sigma = 1$

EX 5.41 p 272

SYSTEM WITH 2 WIDGETS.
1 IS SPARE.

RUN SYSTEM WITH 1ST WIDGET UNTIL IT DIES THEN START 2nd WIDGET.

EACH WIDGET IS EXPONENTIAL

$$f(x) = \lambda e^{-x} \qquad x \geq 0$$

x LIFETIME.

WIDGET 1 $\quad f_{T_1}(x) = \lambda e^{-x} \qquad x \geq 0$

WIDGET 2 $\quad f_{T_2}(3-x) = \lambda e^{-(3-x)} \qquad 3 \geq x \geq 0$

3 IS TOTAL LIFETIME
$3-x$ IS 2nd WIDGET LIFETIME

$$f(z) = \int_0^z \lambda e^{-\lambda x'} \lambda e^{-\lambda(z-x')} dx'$$

$$= \lambda^2 e^{-\lambda z} \underbrace{\int_0^z e^{-x'+x'} dx'}_{z}$$

$$= \lambda^2 z e^{-\lambda z}$$

PDF OF ~~R~~EDUNDANT SYSTEM

ERLANG-2 RV.

EX 5.43 P274

WE HAVE 2 RV X, Y

DEFINE 2 NEW ONES FUNCTIONS OF X, Y

$W = \min(X, Y)$

$Z = \max(X, Y)$

WHAT IS CDF OF W, Z ?

$$F_{WZ}(w,z) = P[W \le w \;\&\; Z \le z]$$

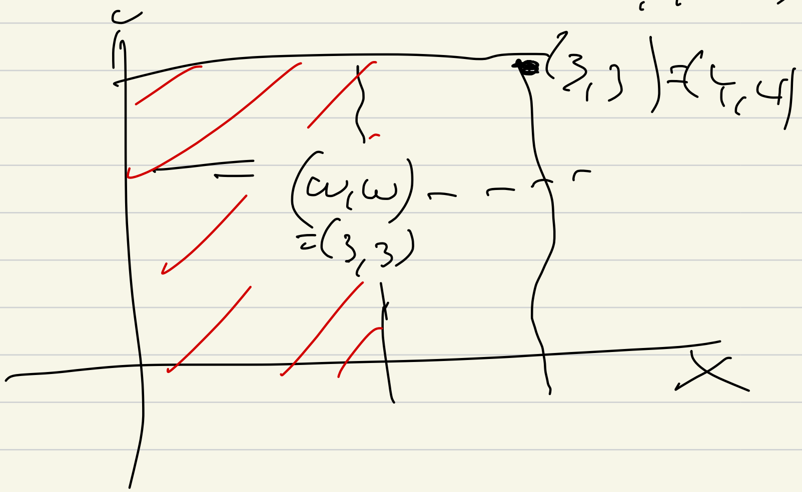
$$= P[\min(X,Y) \le w \;\&\; \max(X,Y) \le z]$$

$X = 4$

$Y = 3$

$w = 3$

$z = 4$



SEE THIS IN MORE DETAIL MON

5.8.3 LINEAR TRANSFORMATIONS.

I HAVE 2 R.V X, Y.

DEFINE
$$V = aX + bY$$
$$W = cX + dY$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = A$$

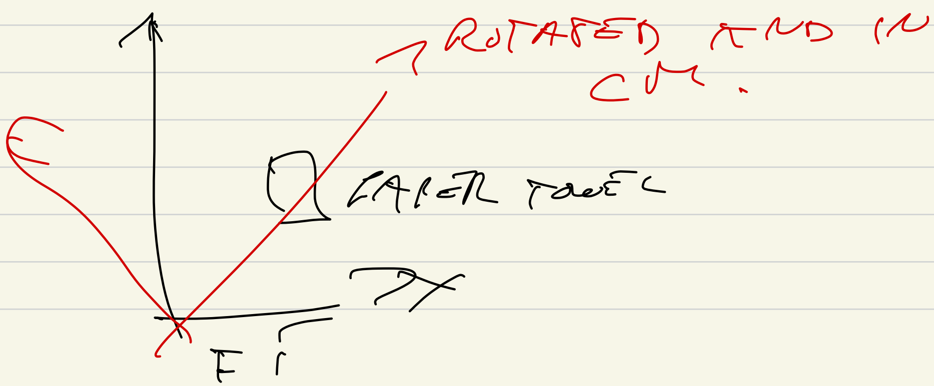
$$\underbrace{\begin{pmatrix} \cancel{uv} \\ w \end{pmatrix}}_{Z} = A \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X}$$

$$Z = AX$$

$$f_Z(z) = \frac{f_X(A^{-1} z)}{|A|}$$

X is 2D point on floor location.
measured in ft

$f_X(X)$ PDF of where chalk lands



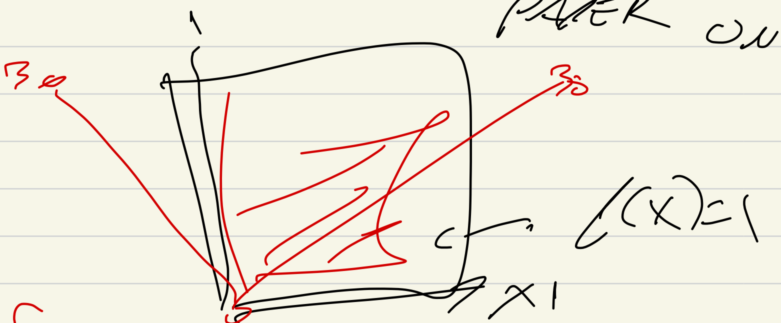
$$Z = \begin{pmatrix} 24 & 18 \\ -18 & 24 \end{pmatrix} X$$

WHAT IS $f(z)$ $\quad \hat{=} A$

$$|A| = 900$$

$$f_Z(z) = \frac{f_X(\cancel{x})}{900}$$

PAPER ON FLOOR



$$\int f_Z(z)\,dz = 1.$$

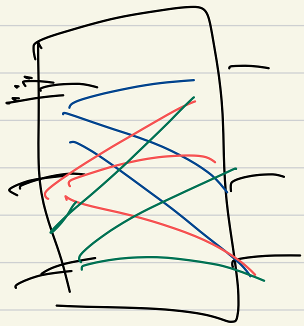
DO MORE CHAP 5 ON MONDAY

NOW DO CHAP 6.

CH 6 VECTOR R.V. P 303.

DIGITIZING SOUND OR VOLTAGE OR
LIGHT INTENSITY - - - -
VECTOR $X_1 X_2 X_3$ - - - - - -
N COMPONENTS.
VECTOR R.V. $X = \begin{bmatrix} X_1 \\ \vdots \\ X_N \end{bmatrix}$

EG 6.1



PACKET SWITCH.

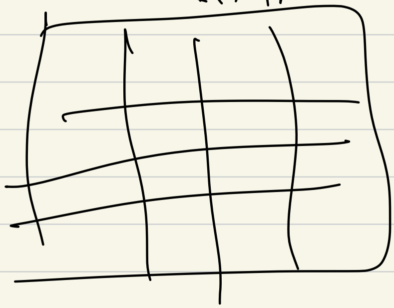
3 IN 3 OUT.

$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ # PACKETS IN @ EACH PORT. $\begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$

EACH PACKET SWITCHED TO AN OUT PORT - - - -

EX 6.2

CHIP WITH DEFECTS,



M REGIONS
M=16

n DEFECTS TOTAL.

Y_i DEFECTS ON REGION i

WANT EXPECTED NUMBER OF
REGIONS WITH 0 DEFECTS
≤ 5 DEFECTS ...

DEFINE JOINT CDF
FOR $X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_k \end{pmatrix}$

$$P\left(X_1 \leq x_1 \ \& \ X_2 \leq x_2 \ \& X_3 \leq x_3 \ldots\right)$$

EX 6.5 P 306

PACKET SWITCH WITH 3 INPUT PORTS,

ARRIVALS IS (X_1, X_2, X_3)

TOTAL # ARRIVALS = N

PROB EACH PORT, SEPARATELY HAS PACKET = $\frac{1}{2}$.

TOTAL # INPUTS N - 0, 1, 2, 3.

N IS BINOMIAL $k=3$
$p=1/2$

$$P_N(n) = \binom{3}{n}\frac{1}{8} \qquad \frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}.$$

THESE n PACKETS ARE SWITCHED TO 3 OUTPUT PORTS.

OUTPUTS @ EACH PORT IS

MULTINOMIAL -

DEFINE

$$P_{X_1,X_2,X_3}(i,j,k \mid i+j+k=n)$$

$$= \begin{cases} \dfrac{n!}{i!\,j!\,k!}\,\dfrac{1}{3^n} & i,j,k \geq 0 \\ & i+j+k=n \end{cases}$$

PROB THAT OUTPUTS GET

i, j, k PACKETS RESPECTIVELY

eg. PROB THAT IF $n=6$

1ST PORT GETS 1
2 2
3 3

$$\frac{6!}{1!\,2!\,3!}\,\frac{1}{3^6} = \frac{720}{2\cdot6\cdot3^6} = \frac{60}{3^6} = \frac{10}{729}$$

$$\underbrace{p(x \mid n)}_{\text{PDF FOR DIST OF OUT GIVEN N}}\;\underbrace{p(n)}_{\text{PDF FOR \# PACKETS = N}} = p(x)$$

DIST FOR # OUT

NOTES
$n = X_1 + X_2 + X_3$

THAT WAS AN EXAMPLE FOR DISCRETE R.V.

CONTINUOUS OBVIOUS.

YOU CAN FIND MARGINAL PDF BY INTEGRATING OUT VARIABLES.

$$f(x_1 \ldots x_{i-1}, x_{i+1}, \ldots x_n) = \int f_{x_1 x_2 \ldots x_n}\, dx_i$$

EX 6.6 P 308

$$f(x_1, x_2, x_3) = \frac{1}{2\pi\sqrt{\pi}} e^{-(x_1^2 + x_2^2 - \sqrt{2}x_1x_2 + x_3^2)}$$

CHECK $\iiint f = 1$

MARGINAL OF $x_1, x_3 = \int \frac{1}{\quad} \cdots dx_2$

$$p(x_2 \mid x_1, x_3) = \frac{p(x_1, x_2, x_3)}{p(x_1, x_3)}$$