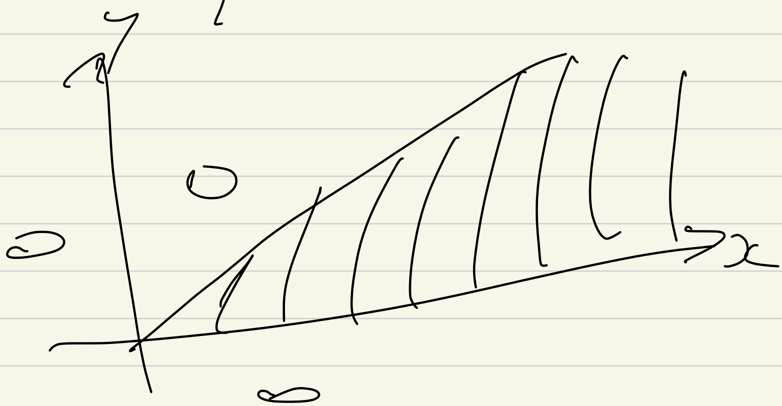
PROBABILITY C17 R 3/17/22

EX 5.16 P252

$$f(x,y) = \begin{cases} c e^{-x-y} & 0 \le y \le x < \infty \\ 0 & \end{cases}$$



C

WHAT IS C?

$$\int_0^\infty \int_0^x f(x,y)\, dy\, dx = 1$$

$$c \int_0^\infty e^{-x} \underbrace{\int_0^x e^{-y}\, dy}_{1-e^{-x}}\, dx$$

$$= c \left(\underbrace{\int_0^\infty e^{-x} - \int_0^\infty e^{-2x}}_{1 - \frac{1}{2}} \right) dx$$

$$1 = \frac{c}{2} \rightarrow c = 2$$

$$f_{XY}(x,y) = 2e^{-x-y} \qquad 0 \le y \le x < \infty$$

DO MARGINALS

$$f_X(x) = \int_0^x 2e^{-x-y}\,dy$$

$$= 2e^{-x} \underbrace{\int_0^x e^{-y}\,dy}_{1-e^{-x}}$$

$$f_X(x) = 2e^{-x}(1-e^{-x}) \qquad 0 \le x < \infty$$

$$f_Y(y) = \int_0^x f(x,y)\,dx$$

$$= 2e^{-y}\int_0^\infty e^{-x}\,dx$$

. . . .

$$= 2e^{-2y}$$

(WAVING HANDS)

5.17 $P[X+Y \le 1]$?

$f(x,y) = 2e^{-x-y}$

$0 \le y \le x \le 1$

LET $A = P[X+Y \le 1]$

INTEGRATE $f(x,y)$ OVER REGION

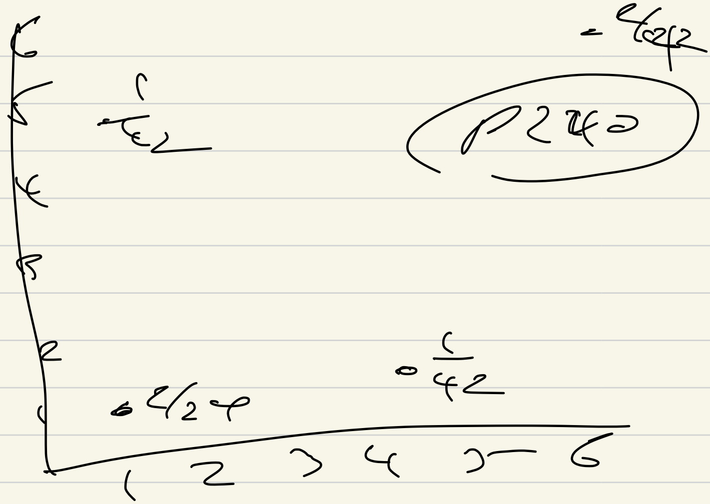
$$A = 2\int_0^1 \int_0^{\min(x,1-x)} e^{-x-y}\,dy\,dx$$

$$= 2\int_0^{1/2}\int_0^{x} e^{-x-y}\,dy\,dx + 2\int_{1/2}^{1}\int_0^{1-x} e^{-x-y}\,dy\,dx$$

EX 5.19

2 DICE

PMF



$p(k,k) = \frac{2}{42}$ $\quad$ $p(j,k) = \frac{1}{42}$

$j \neq k$

MARGINAL

$p(3) = p(1,3) + p(2,3) + \dots = \sum_{i=1}^{6} p(i,3)$

$= \frac{5}{42} + \frac{2}{42} = \frac{7}{42} = \frac{1}{6}$

EACH ONE SEPARATELY IS FAIR.

$p(i) = 1/6 \quad i = 1 \dots 6$

THE 2 DICE ARE DEPENDENT.

$p(1,2) = \frac{1}{42} \quad p(1)p(2) = \frac{1}{6}\frac{1}{6} = \frac{1}{36} \neq \frac{1}{42}$.

5.20 SPLIT A LONG MESSAGE (OR FILE) SIZE N

INTO Q BLOCKS OF SIZE M

AND 1 PARTIAL BLOCK OF

SIZE R $0 \le R < M$.

COMMON $p(N)$ GEOMETRIC.

COMPUTED $P(Q)$ ALSO GEOM

R "

COULD TEST: ARE Q, R INDEPENDENT?

IS $P[N=n] = P[Q=q \,\&\, R=r]$

$n = QM+R$.

YES.

EX 5.24 P257

SUM OF 2 R.V.

$$Z = X + Y \qquad z' = x' + y'$$

$$E[Z] = E[X+Y]$$

$$= \int_{x'=-\infty}^{\infty} \int_{y'=-\infty}^{\infty} (x'+y') f(x',y')\, dx'dy'$$

$$\boxed{= \iint x' f(x',y')\, dx dy + \iint y' f(x'y')\, dxdy}$$

$$\boxed{E[Z] = \int z' f(z')\, dz'}$$

$$\int x' \underbrace{\int f(x',y')\, dy'}_{f_X(x')} dx'$$

$$= \int x' f_X(x')\, dx' = E[X]$$

$$E[Z] = E[X] + E[Y]$$

IF $Z = X + Y$

THEN $E[Z] = E[X] + E[Y]$

REGARDLESS OF DEPENDENCE OR NOT.

$$E\left[\sum X_i\right] = \sum E[X_i]$$

FOR PRODUCT, TRUE IF INDEPENDENT

EX 5.25 p258

$Z = XY$

$$E[Z] = \int z' f(z') \, dz'$$

$$= \iint x' y' \underbrace{f(x', y')}_{= f(x') f(y') \text{ IF INDEPENDENT}} dx' \, dy'$$

$$= \iint x' y' f(x') f(y') \, dx' \, dy'$$

$$= \int x' f(x') \, dx' \int y' f(y') \, dy'$$

$$E[Z] = E[X]E[Y] \qquad Z = XY$$

IF INDEPENDENT -

SECT 5.6.2 p258

HI ORDER MOMENTS

$$E[X] = \int x f(x)\,dx$$

$$E[X^2] = \int x^2 f(x)\,dx$$

$$E[X^k] = \int x^k f(x)\,dx$$

$$VAR[X] = E[X^2] - (E[X])^2$$

JOINT MOMENTS

$$E[X^j Y^k] = \iint x^j y^k f_{X,Y}(x,y)\,dx\,dy$$

$$= \sum\sum x_i^j y^k p(x_i, y_j)$$

JOINT MOMENT $E[X^j Y^k]$

CORRELATION $E[XY]$

CENTRAL MOMENT

- SUBTRACT MEAN.

EG. $E[(X-E[X])^2]$ FOR VAR

FOR 2 VARS

COVARIANCE.

$$\mathrm{COV}(X,Y) = E((X-E[X])(Y-E[Y])).$$

$$= E[XY - XE[Y] - YE[X] + E[X]E[Y])$$

$$= E[XY] - E[X]E[Y]$$

IF INDEPENDENT THEN

$$\mathrm{COV}(X,Y) = 0.$$

2 VARS MAY BE STRONGLY RELATED BUT STILL INDEPENDENT BY OUR DEFINITION OF "INDEPENDENT"

cf 5.27 p260

Θ R.V $U[0, 2\pi]$

$X = \cos\theta$

$Y = \sin\theta$

GIVEN X, ONLY 2 CHOICES FOR Y.

$$E[XY] = E[\sin\theta\cos\theta]$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \underbrace{\sin\theta\cos\theta}_{\frac{\sin 2\theta}{2}}\, d\theta$$

$$= \frac{1}{4\pi}\int_0^{2\pi} \sin 2\theta\, d\theta = 0$$

X, Y UNCORRELATED, INDEP.

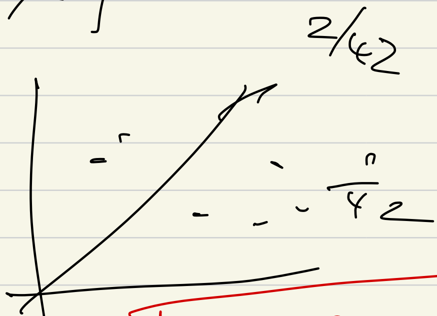
5.7.1 CONDITIONAL PROB

p261

$$P[Y|X] = \frac{P[Y \& X]}{P[X]}$$

USE 5.6

2 LINKED DICE



$P(k,k) = 2/42$

$P(j,k) = 1/42$

$j \neq k$

SEE EX 5.29 p263

X = 1st DICE Y = 2nd DICE

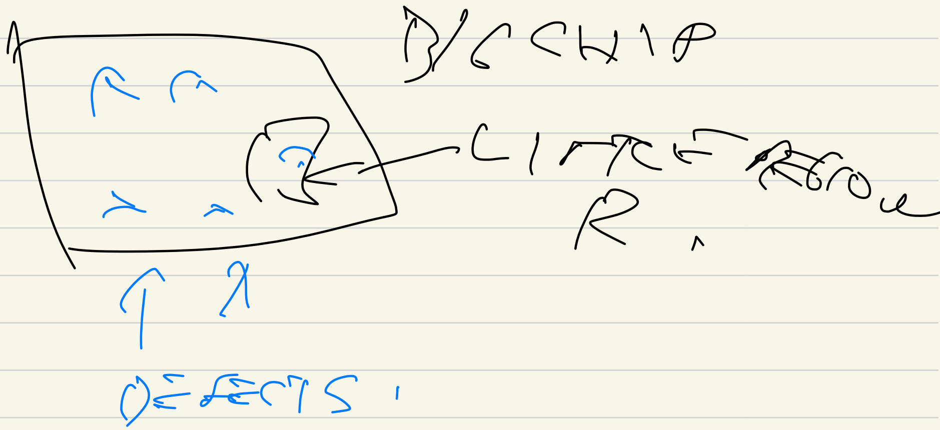
$$P[Y=1|X=1] = \frac{P[1,1]}{P[X=1]} = \frac{2/42}{1/6} = \frac{2}{7}$$

$$P[Y=2|X=1] = \frac{1/42}{1/6} = 1/7$$

$$\sum_{y=1}^{6} P[Y=y|X=1] = \frac{2}{42} + 5 \cdot \frac{1}{42} = \frac{7}{42} = \frac{1}{6}$$

EX 5.30 P263

BEAUTIFUL EX.



X: RV TOTAL # DEFECTS.

POISSON $\mu = \alpha$

p = PROB OF A SPECIFIC DEFECT BEING IN R.

LEARN ~~A~~ ABOUT DEFECTS IN R.

$$G_X(x) = \frac{e^{-\alpha} \alpha^x}{x!} \qquad x = 0, 1, 2, 3, \ldots$$

Y = R.V. ~~E~~ DEFECTS IN R.

Q: WHAT IS PMF OF Y?

$P_Y(y)$?

SEE ANSWER ON MON.