PROB CLASS 13 R 2/24/22

EXAM 1

IN CLASS

USE GS

BRING SCRATCH PAPER

MATERIAL UP TO LAST
THURS CLASS 11

BRING ONE CHEAT SHEET.

TA OFFICE HOURS.
FRI, SAT, SUN

WEBEX

CHAPTER 5
PAIRS OF R.V.

RANDOM EXPT:
PICK RPI STUDENT,
MEASURE HEIGHT H
WEIGHT W

$E[H]$ $STD[H]$
$E[W]$

CDF, PDF ETC.

FOR 2 VARS, THERE ARE
JOINT STATS,

EXPT: TOSS 2

FAIR 6-SIDED DICE.

SEE (x, y)

$P_X(x) = P_X(2) = \frac{1}{6}$

$P_Y(y) \quad P_Y(3) = \frac{1}{6}$

JOINT PROB

$P_{XY}(x, y) = \frac{1}{36}$

SIMPLER EXPT.

THROW 2 COINS.

MEASURE x, y

x = 0 TAIL FOR 1ST COIN
1 HEAD

y =

$y \backslash x$	0	1
0	$\frac{1}{6}$	$\frac{1}{3}$
1	$\frac{1}{3}$	$\frac{1}{6}$

$\boxed{\text{NOT FAIR COINS}}$

PROBABILITY MASS FN.

$p(0,1) = \frac{1}{3}$

YOU CAN SUM ROWS OR COLUMNS TO GET PMF FOR EITHER VAR SEPARATELY

	0	1	
0	$\frac{1}{6}$	$\frac{1}{3}$	$\to \frac{1}{2}$
1	$\frac{1}{3}$	$\frac{1}{6}$	$\to \frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	

$$P_X(0) = \frac{1}{2} = P_X(1) = P_Y(0) = P_Y(1)$$

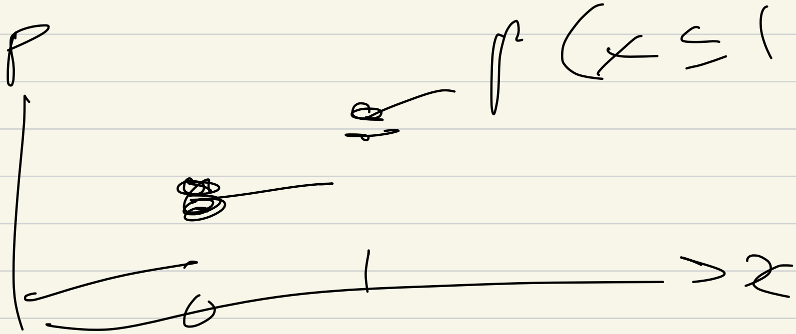
FOR 1 VAR, YOU HAVE A

CDF = CUMULATIVE DISTN FN

$$F_X(x) = P(X \le x)$$

FOR 1ST COIN $P(X \le 0) = \frac{1}{2}$

$P(X \le 1) = 1$



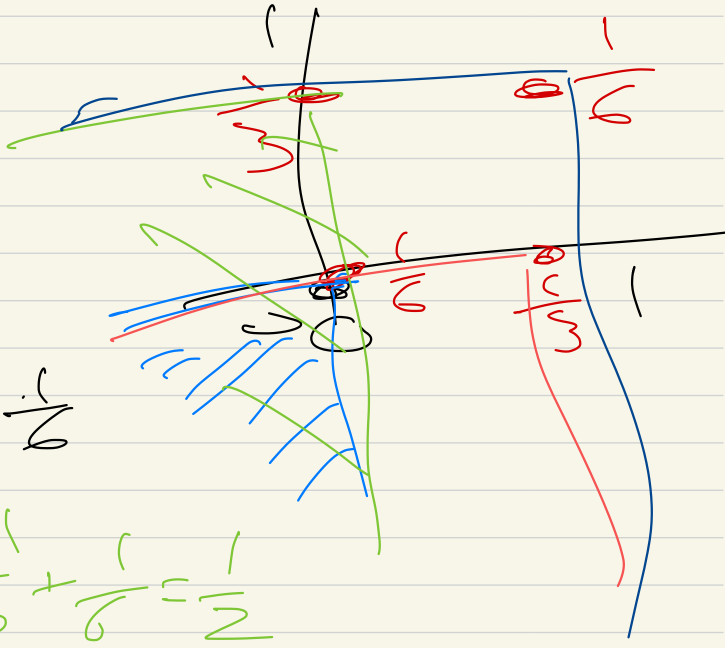
2 VARIABLE CDF

$$F_{X,Y}(x,y)$$

$$= P(X \le x \;\&\; Y \le y)$$

PMF

	0	1
0	$\frac{1}{6}$	$\frac{1}{3}$
1	$\frac{1}{3}$	$\frac{1}{6}$



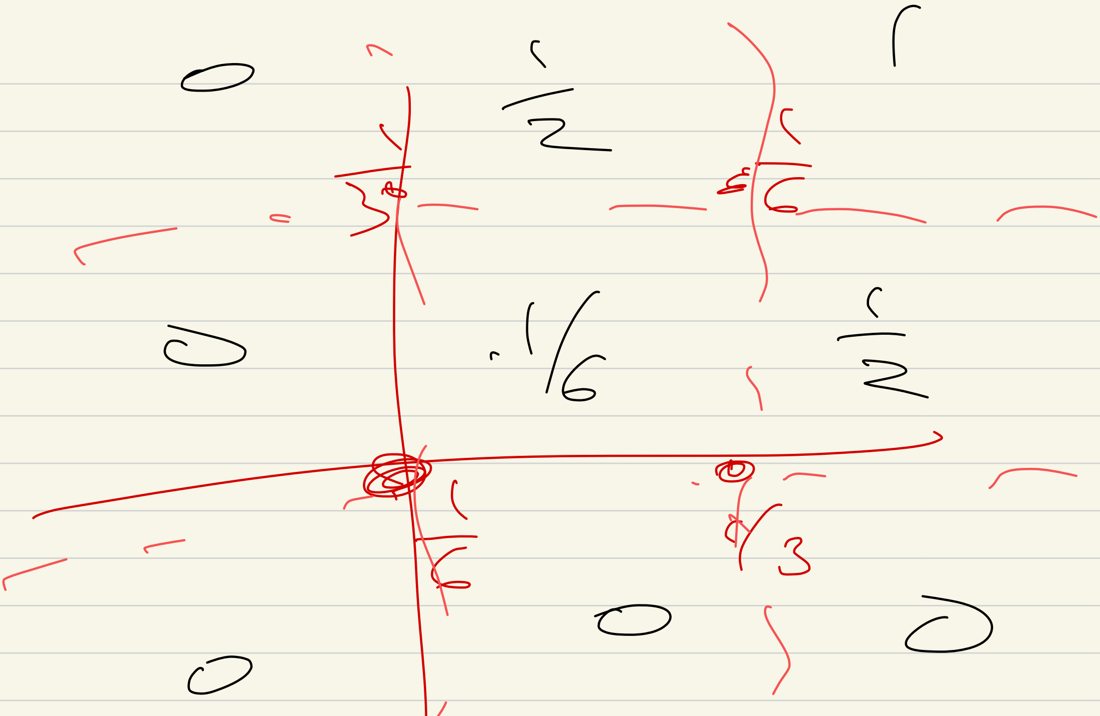
$$F_{XY}(0,0) = \frac{1}{6}$$

$$F_{XY}(0,1) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$F_{XY}(1,0) = \frac{1}{2}$$

$$F_{XY}(1,1) = 1$$

For $x<0$ OR $y<0$ $\qquad f(x,y)=0$



$F(x,y) = 0$

$x \leq 0$

$y \leq 0$

IF $0 \leq x < 1$ & $0 \leq y < 1$

$$F(x,y) = \frac{1}{6}$$

$0 \leq x < \infty$, $0 \leq y < 1$ $\quad F(x,y) = \frac{1}{2}$

BOOK HAS EXAMPLES OF 2 DICE. SEPARATELY THEY'RE FAIR, BUT JOINT PROBABILITES ARE NOT FAIR.

	0	1
0	$\frac{1}{6}$	$\frac{1}{3}$
1	$\frac{1}{6}$	$\frac{1}{3}$

$$E[X] = \frac{1}{6}\cdot 0 + \frac{1}{3}\cdot 1 = \frac{1}{3}$$

$$E[Y] = \frac{1}{3}$$

$$E[X^2] = \frac{1}{6}\cdot 0 + \frac{1}{3}\cdot 1 = \frac{1}{3}$$

$$\mathrm{VAR}[X] = E[X^2] - (E[X])^2 = \frac{2}{9}$$

COVARIANCE (P258)

$$COV(X, Y) =$$

$$E[(X - E[X])(Y - E[Y])]$$

$$COV(X, X) = VAR(X)$$

IF LARGER X AND Y GO ~~TOGETHER~~ TOGETHER

$$COV(X, Y) > 0$$

IF X, Y MOVE IN OPPOSITE DIRECTIONS

$$COV(X, Y) < 0$$

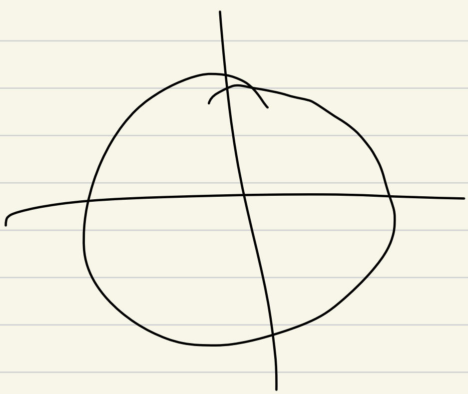
THIS CAPTURES ONLY LINEAR RELATIONSHIPS.

SAY $Y = X^2$ $\quad 0 \le X \le 1$

$\text{COV}(X, Y) = 0$ PERHAPS

BUT STRONG RELATIONSHIP.

EX 2 $\quad X^2 + Y^2 = 1$



$\text{COV} = 0$

2 coin case

cov(X, Y) =

$E[(X - E(X))(Y - E(Y))]$

	0	1
0	$\frac{1}{6}$	$\frac{1}{3}$
1	$\frac{1}{3}$	$\frac{1}{6}$

$cov = E\left((X - \frac{1}{2})(Y - \frac{1}{2})\right)$ ←

$= \frac{1}{6}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \frac{1}{3}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$

$+ \frac{1}{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + \frac{1}{6}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{1}{24} - \frac{1}{12} - \frac{1}{12} + \frac{1}{24} = -\frac{1}{12}$

X, Y INDEPENDENT IFF

$P(X=x \; \& \; Y=y) =$

$P(X=x)\,P(Y=y)$

2 COIN CASE

$P(X=0, Y=0) = \frac{1}{6}$

$P(X=0) = \frac{1}{2} = P(Y=0)$

$\frac{1}{6} \neq \frac{1}{2} \cdot \frac{1}{2}$

X, Y DEPENDENT.

$\mu_X = E(X) = \frac{1}{2}$ $\mu_Y = \frac{1}{2}$

$\sigma_X = \sqrt{VAR(X)}$

$VAR = E[X^2] - (E(X))^2$

$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$\sigma_X = \frac{1}{2} = \sigma_Y$

NEW TERM

CORRELATION COEFFICENT

$\rho_{XY} = \frac{COV(X, Y)}{\sigma_X \sigma_Y}$

$-1 \leq \rho_{XY} \leq 1$

$\rho_{xy} = 1$

PERFECT POSITIVE LINEAR RELATION

$\rho = -1$ --- NEGATIVE

e.g. $Y = 5x + 3$ $\rho = 1$

$Y = -2x + 1$ $\rho = -1$

$\rho = 0$ NO LINEAR RELATION.

ρ HAS TO BE LARGE FOR INTERESTING.

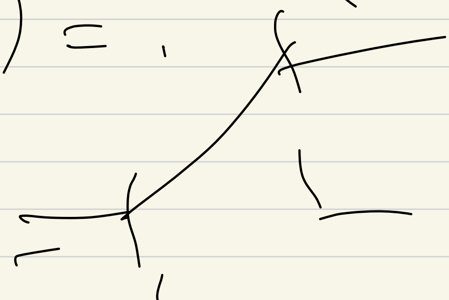
$\rho >> \frac{1}{2}$

CONTINUOUS VARIABLES

X is $U[0,1]$

$$f(x) = \begin{cases} 0 & x \le 0 \\ 1 & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

$F(x) =$



$$\begin{cases} 0 & x < 0 \\ x, & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

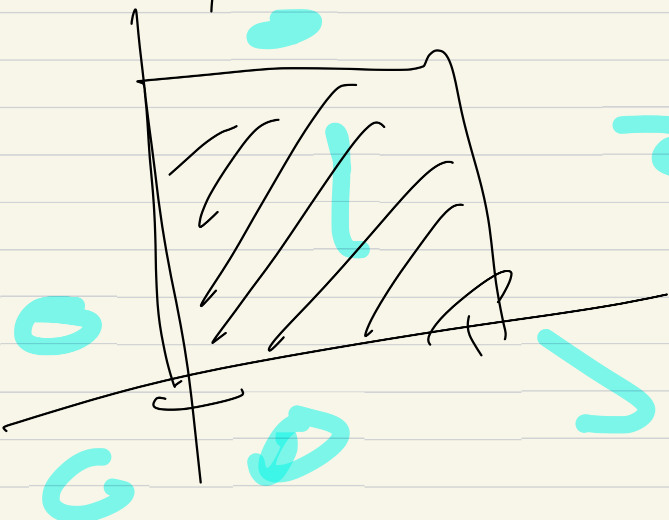
$f(y) =$ LIKE $f(x)$

X, Y INDEPENDENT

JOINT DENSITY, PDF

$f_{X,Y}(x,y) =$

$$f_{X,Y}(x,y) = \begin{cases} 1 & \text{IF } 0 \le x \le 1 \\ & \quad \wedge\ 0 \le y \le 1 \\ 0 & \text{ELSE} \end{cases}$$



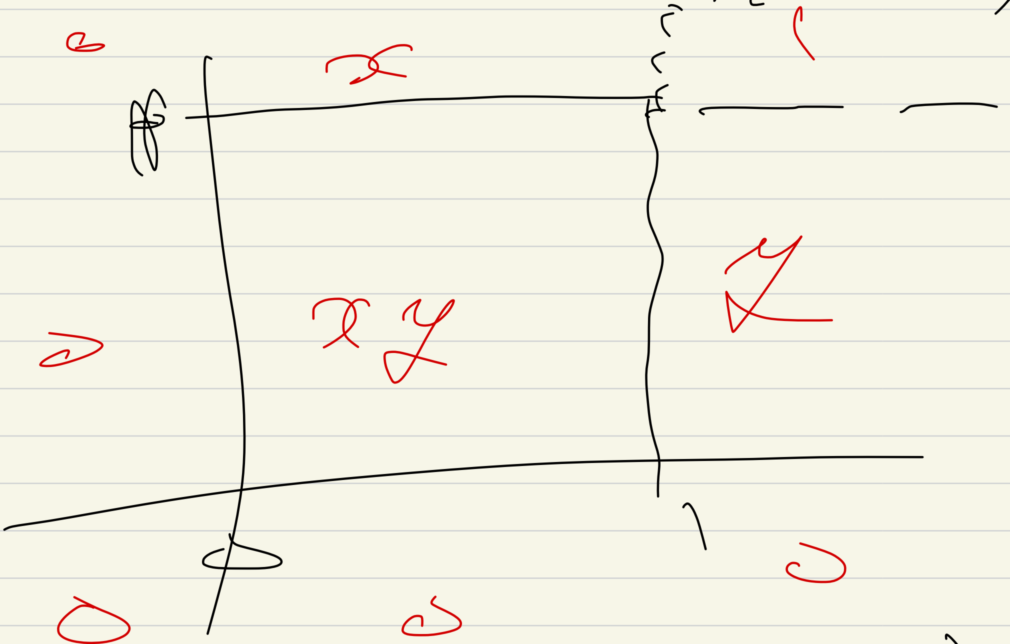
$P(x \le X \le x+dx,$
$\quad y \le Y \le y+dy)$

$\approx f(x,y)\,dx\,dy$

FOR SMALL dx, dy.

CDF:

$$F(x,y) = P(X \le x \;\&\; Y \le y)$$



$$F\left(2, \tfrac{1}{3}\right) = P\left(X \le 2 \;\&\; Y \le \tfrac{1}{3}\right)$$

$$= P\left(Y \le \tfrac{1}{3}\right)$$

$$= \frac{1}{3}$$

$$\boxed{P(X \le 2) = 1}$$

1 VARIABLE

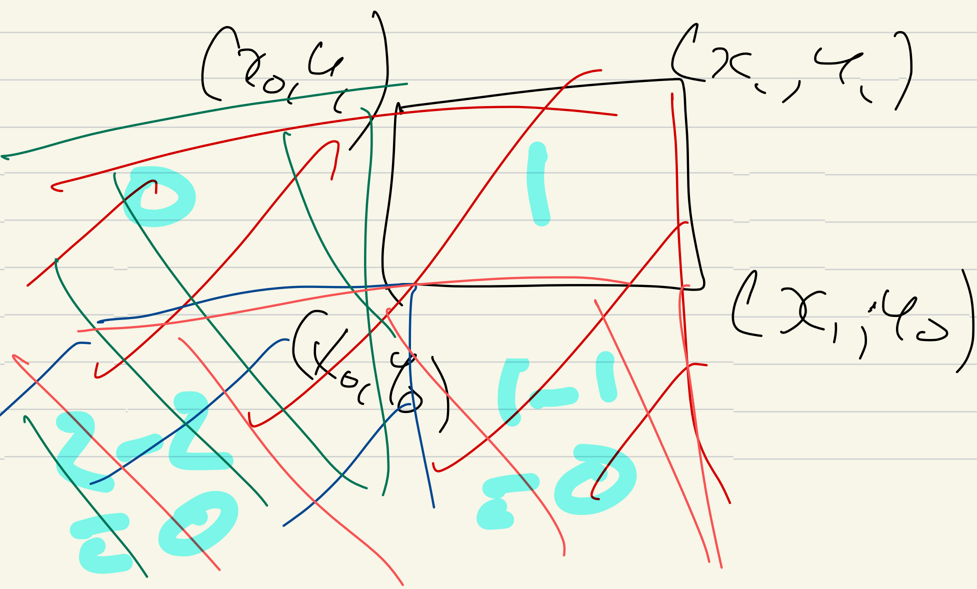
$P[l \le X \le h]$

$= F_X(h) - F_X(l)$

$P\left[\frac{1}{3} \le X \le \frac{3}{4}\right]$

$= F\left(\frac{3}{4}\right) - F\left(\frac{1}{3}\right) = \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$

$P[x_0 \le X \le x_1 \cap y_0 \le Y \le y_1]$



$$P(x_0 \le X \le x_1 \ \& \ y_0 \le Y \le y_1)$$

$$= F(x_1, y_1) + F(x_0, y_0)$$

$$- F(x_0, y_1) - F(x_1, y_0)$$

WHY USE CUMULATIVE CDF ?

IT WORKS FOR BOTH DISCRETE AND CONTINUOUS DISTNS. + MIXED DISTNS ALSO.

MIXED EXAMPLE:
GET CAR AT AIRPORT

$p = \frac{1}{2}$ (IT'S ALREADY THERE)

(IF NOT $p(t) = c\, e^{-t}$

c: SCALE

$t > 0$