PROB CLASS ~~xx~~ 12
TUES 2/22/22

MORE FROM CHAP 4 .

REMEMBER 1ST EXAM IN MON
HERE IN CLASS .
SEE BLOG .
BRING COMPUTERS TO USE
GRADESCOPE .

TA'S HAVE OFFICE HOURS .
ADD A SATURDAY OFFICE HOUR .

EX 4.22 p169

NOISY COMM CHANNEL.

V: INPUT ≥ 0

Y: OUTPUT

$$Y = \alpha V + N$$

α SOME CONSTANT $= 10^{-2}$

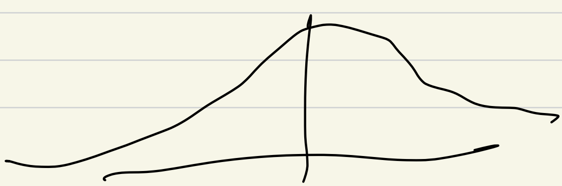
N: GAUSSIAN NOISE

$\mu = 0 \quad \sigma = 2$.

TO FIND INPUT X THAT

HAS OUTPUT Y

$$P[Y < 0] = 10^{-6}$$



OUTPUT GETS SMEARED

$$P\{Y<0\} = 10^{-6}$$

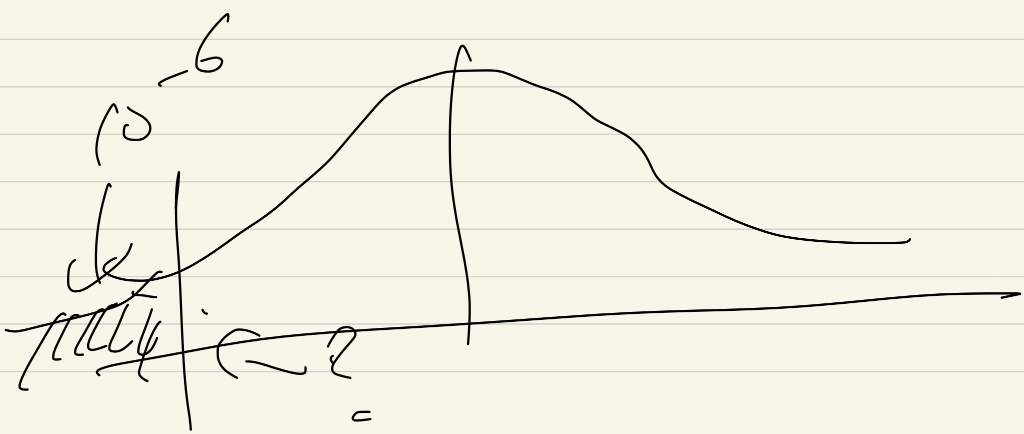
$$P[\alpha V + N < 0] = 10^{-6}$$

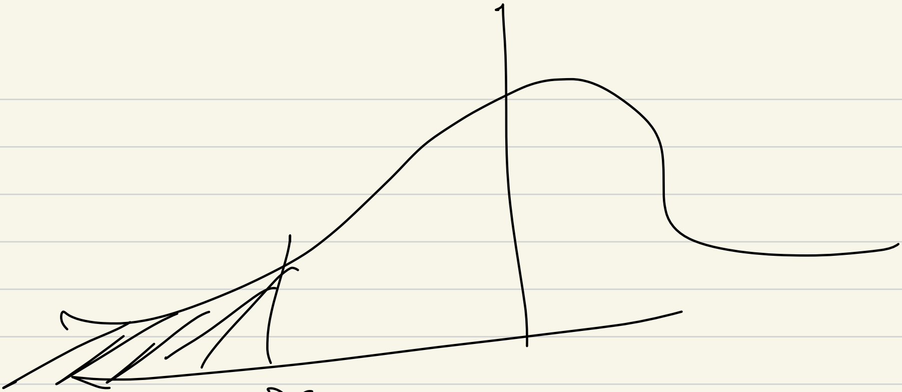
$$P[N \leq -\alpha V] = 10^{-6}$$

~~G~~ N IS GAUSSIAN $\underset{\mu}{0}, \underset{\sigma}{2}$

$N_2 = \frac{N}{2}$ GAUSSIAN $\mu = 0$, $\sigma = 1$

$$P\left[N_2 \leq -\frac{\alpha V}{2}\right] = 10^{-6}$$





x

$$\Phi(x) = Q(-x)$$

CDF RIGHT TAIL PROB.

$$Q\left(\frac{\alpha\nu}{2}\right) = 10^{-6}$$

THERE ARE TABLES FOR Q

P 169

$$\frac{\alpha\nu}{2} \approx 4.7$$

$$\nu = 4.7 \times 2 \times 100 = 950$$

THERE ARE OTHER PROBABILITY DISTRIBUTIONS: m-ERLANG
PARETO
ETC.

PARETO

PROBABILITY OF UNLIKELY TAIL EVENTS

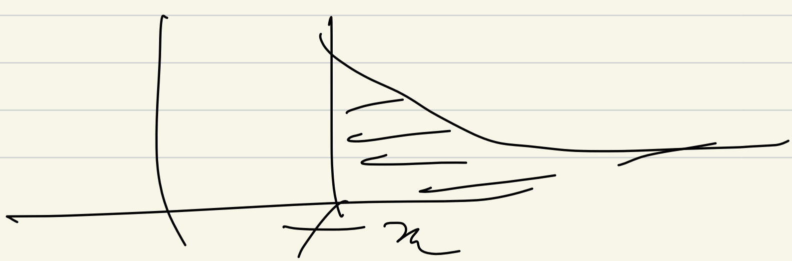
e.g. VERY WEALTHY PEOPLE.

p123

TWO PARAMETERS α

CUTOFF x_m

DEFINED FOR $x > x_m$



TAIL IS FATTER THAN FOR GAUSSIAN OR EXPONENTIAL

COMPLEMENTARY CDF

$$P[X > x] = \begin{cases} 1 & x < x_m \\ \left(\frac{x_m}{x}\right)^{\alpha} & x \geq x_m \end{cases}$$

$$CDF = 1 - \left(\frac{x_m}{x}\right)^{\alpha} \quad x \geq x_m$$

PDF DENSITY

$$\frac{dF}{dx} = \frac{d\left(1-\left(\frac{x_m}{x}\right)^{\alpha}\right)}{dx}$$

$$= \frac{-\alpha x_m^{\alpha}}{x^{\alpha+1}}$$

RELIABILITY CTD

p190

$R(t)$ = PROB DEVICE STILL WORKING AT TIME $T = t$

$= P(T > t)$

$= 1 - F(t)$

SUPPOSE SYSTEM HAS 2 PARTS, BOTH REQD.

BICYCLE WITH 2 WHEELS.

$R_1(t)$ = RELIABILITY OF WHEEL 1

$R_2(t)$ - - - - - 2

$R(t)$ = RELIAB. OF BICYCLE.

ASSUME WHEELS ARE INDEPENDENT.

BIKE IS WORKING IFF

(IFF = IF AND ONLY IF)

BOTH WHEELS WORK.

P(BIKE WORKING AT T = 4)

= P(BOTH WHEELS - - --)

= P(1ST WHEEL) × P(2ND WHEEL)

$$R(t) = R_1(t) R_2(t)$$

SAY EACH WHEEL IS

EXPONENTIAL $\lambda = 1$

$$R_1(t) = e^{-t}$$

$$R_2(t) = e^{-t}$$

$$R(t) = e^{-2t}$$

WHAT IF ONLY ONE OF TWO PARTS HAS TO WORK FOR SYSTEM TO WORK?

SYSTEM IS BROKEN IFF BOTH PARTS BROKEN -

$$P[\text{SYSTEM BROKEN}] = P[\text{PART 1 BROK}] \times P[\text{PART 2 BROK}].$$

$$1-R(t) = (1-R_1(t))(1-R_2(t))$$

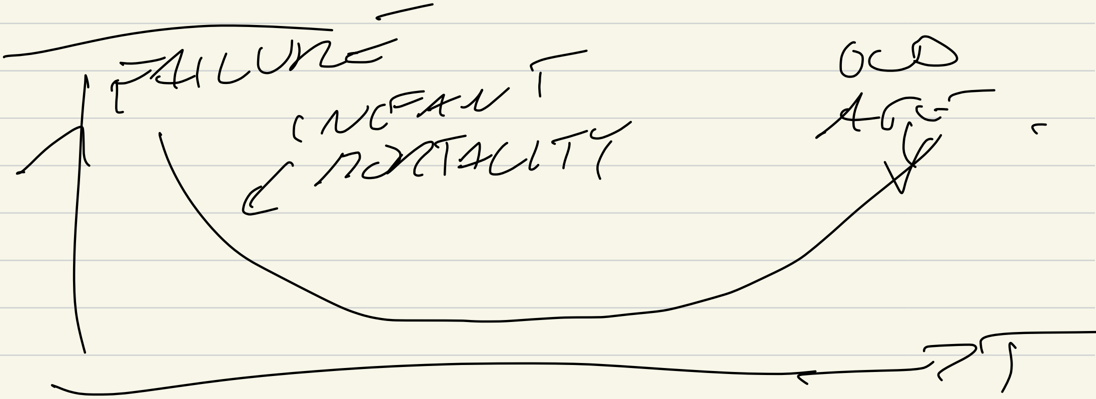
$$1 - R(t) = (1 - e^{-t})(1 - e^{-t})$$

$$R(t) = 1 - \left[1 - 2e^{-t} + e^{-2t}\right]$$

$$= 2e^{-2t} - e^{-2t}$$

MORE COMPLICATED RELIABILITY THAN EXPON.

IT'S MEMORYLESS.



25. WATER DAM.

REAL WORLD RELIABILITY

eg. COOPER TQUEBEC BRIDGE.

PHYSICAL LAWS DON'T CARE WHO YOU ARE.

P194 GENERATING RANDOM VARIABLES.

- USE A PACKAGE.

IT'S HARD.

IN PAST, YOU'D BUY A BOOK OF PSEUDO RANDOM NUMBERS.

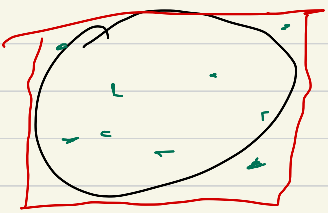
SOME TECHNIQUES:

1. START WITH A SIMPLE DISTN
SAY $U[0,1]$

GET THAT WITH MERSENNE TWISTER.

2. COMBINE + TRANSFORM.
ADD 10 $U[0,1]$, YOU HAVE A
PRETTY GOOD GAUSSIAN R.V.

3. REJECTION METHOD.
YOU WANT UNIFORM POINTS IN CIRCLE



YOU DO:
PUT SQUARE
AROUND IT.

GENERATE RANDOM POINTS IN □
REJECT THOSE OUTSIDE CIRCLE
SURVIVORS ARE UNIFORM RANDOM
POINTS IN CIRCLE.
REJECT $1-\frac{\pi}{4} < \frac{1}{4}$ OF POINTS.

ECONOMISTS MAY WANT RANDOM
POINT INSIDE HI-DIM POLYTOPE.

IGNORE STARRED SECTIONS OF BOOK.

CHAP 5 P233

PAIRS OF R.V.

POINT IN SQUARE.

TEMP + RAINFALL

2 AMPLITUDES OF SIGNAL.

MAYBE 2 RV ARE RELATED. KNOWING
ONE HELPS WITH OTHER.

WILL ADD A SATURDAY OFFICE HOUR

TEST ON MON.