PROB CLASS 10
M 2/14/22

FROM 2020 EXAM

S SMOKERS

C CANCER.

NUMBERS MADE UP

$P(C|S) = .1$

$P(S) = .2$

$P(S|C) = .9$

Q) $\boxed{P(C)}$

$P(C \,\&\, S) = P(C|S)\,P(S)$

$= P(S|C)\,P(C)$

$.022$

$P(C) = \dfrac{P(C|S)\,P(S)}{P(S|C)} = \dfrac{.1 \times .2}{.9}$

$p(c|s')$?

$p(c|s')\,p(s') = p(c\,\&\,s')$

$= p(s'|c)\,p(c)$

$$p(c|s') = \frac{p(s'|c)\,p(c)}{p(s')}$$

.1 .022 .8

$p(s|c) = .9 \rightarrow p(s'|c) = .1$

$$p(c|s') = \frac{.1 \times .022}{.8} \approx .003$$

? $\boxed{\text{GEOMETRIC}}$ PROB.

TAKING HARD CLASS AT MIT

p = PROB PASS = $\frac{1}{2}$

RETAKE IT. PASSING IS INDEPENDENT OF PREVIOUS ATTEMPT

PROB PASSING ON K^{th} TRY

$p = \frac{1}{2}$ $q = 1 - p = \frac{1}{2}$ $\quad P(K) = pq^{K-1}$

$= \frac{1}{2^K}$

$E[K] = \frac{1}{p} = 2$

$$\sum_{K=1}^{\infty} Kpq^{K-1} = p \sum_{1}^{\infty} Kq^{K-1}$$

USE DERIVATIVE

$$\sum Kq^{K-1} = \frac{d}{dq} \sum q^K = \frac{d}{dq} \frac{1}{1-q}$$

VAR[K]

2020 EXAM 1 CTD

3. 20-SIDE DIE

TOSS. SEE K ← RANDOM VAR

RANDOM EXPT. OUTCOME.

EVENTS A $1 \le K \le 10$

B $K \in \{1, 3, 5 \dots 19\}$

ODD

$C = \{2, 4, 6, 8, 10, 11, 13, 15, 17, 19\}$

ARE A, B INDEP?

$P(A)P(B) = P(A \cap B)$ $\frac{1}{4}$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ $A \cap B = \{1, 3, 5, 7, 9\}$

YES

A, C?

$A \cap C = \{2, 4, 6, 8, 10\}$

$p = \frac{1}{4}$

YES

TABLE 3.1 ON PAGE 115-116
IS IMPORTANT

DISCRETE

TABLE 4.1 PAGE 164

CONTINUOUS

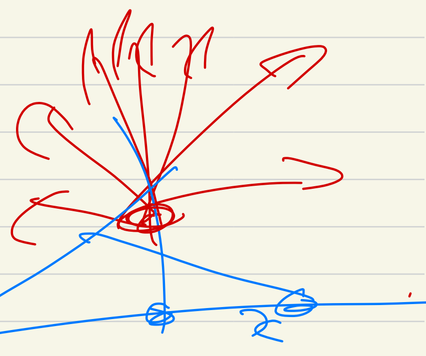
CAUCHY R.V.

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

$-\infty \le x \le \infty$

HOW IT CAN HAPPEN

SPIN A POINTER
CENTERED AT (0, 1)

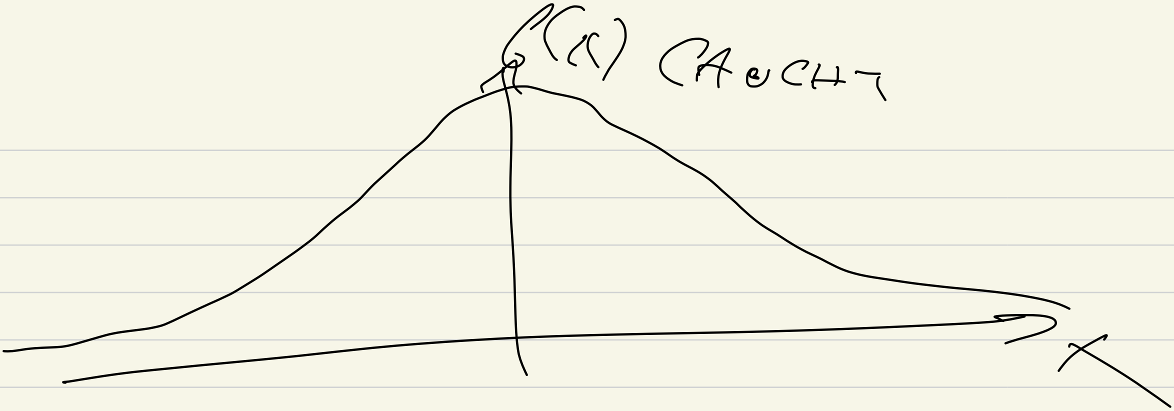


LOOK AT WHERE IT HITS. AXIS

THAT'S R.V.

$$\int f(x)\,dx = 1$$

OK



TAILS ARE THICKER THAN FOR GAUSSIAN -

$$E[x] = \int_{-\infty}^{\infty} x f(x)\,dx = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx$$

DIVERGES

E[x] DOES NOT EXIST.

IF YOU SAMPLE N OUTCOMES,

$x_1, x_2 \ldots x_N$

SAMPLE MEAN $\dfrac{\sum_{i=1}^{N} x_i}{N}$

CONVERGES TO E[x].

MEANING OF $E[X]$.

FOR MOST PROBABILITY DISTNS, YOU CAN TAKE A SAMPLE OF N R.V. + COMPUTE SAMPLE MEAN $E[\hat{S}_1]$

AS N GROWS, SAMPLE MEAN TENDS TO SETTLE DOWN.

EG TOSS COIN N TIMES

$\underbrace{.5 \pm \frac{\sqrt{N}}{2} \quad \frac{2}{3}}$ OF TIME.

NOT TRUE FOR CAUCHY.

MEAN OF LARGE SAMPLE DOES NOT SETTLE DOWN.

$E[X]$ DOES NOT EXIST

NEITHER $VAR[X]$

IN REAL WORLD MAYBE SOME ECONOMICS MODELS ALSO ARE INVALID.

SEE "LONG TERM CAPITAL MANAGEMENT"

BACK TO 2020 EXAM!

A : ZERO

B : ODD

C : ~~

$P(A) = \frac{1}{2} = P(B) = P(C)$

$A \cap B \cap C = \emptyset = \{\}$

$P(A \cap B \cap C) = 0$

$\neq P(A)P(B)P(C)$

KNOWING A TELLS YOU NOTHING ABOUT C

" " B " " " " " "

KNOWING THAT BOTH A, B HAPPENED TELLS THAT C IS IMPOSSIBLE.

$P(C|A) = \frac{1}{2}$

$\boxed{\text{BAYES}}$

DISPLAY WITH 2000^2 PIXELS.

MANUFACTURER DEFINED GOOD DISPLAY AS ≤ 10 BAD PIXELS.

P(A GIVEN PIXEL BEING BAD) = 10^{-6}.

PIXELS INDEPENDENT

APPROPRIATE PROBAB DISTN?

POISSON

PROB OF K BAD $\quad N = 4000000$

$p = 10^{-6}$ $\qquad \binom{N}{K} p^K (1-p)^{N-K}$

POISSON $\alpha = 4$ $\qquad E[K] = 4$

$VAR[K] = 4$

$\sigma = 2$

2/3 OF TIMES, WE HAVE 2 TO 6 BAD PIXELS.

$$p(K) = \frac{\alpha^K}{K!} e^{-\alpha} \qquad p(0) = e^{-4}$$

WHAT FRACTION OF DISPLAYS ARE ACCEPTABLE?

$$\sum_{k=0}^{10} p(k) = e^{-4} \sum_{k=0}^{10} \frac{4^k}{k!}$$

BACK TO CHAP 4

GAUSSIAN (NORMAL) $= f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

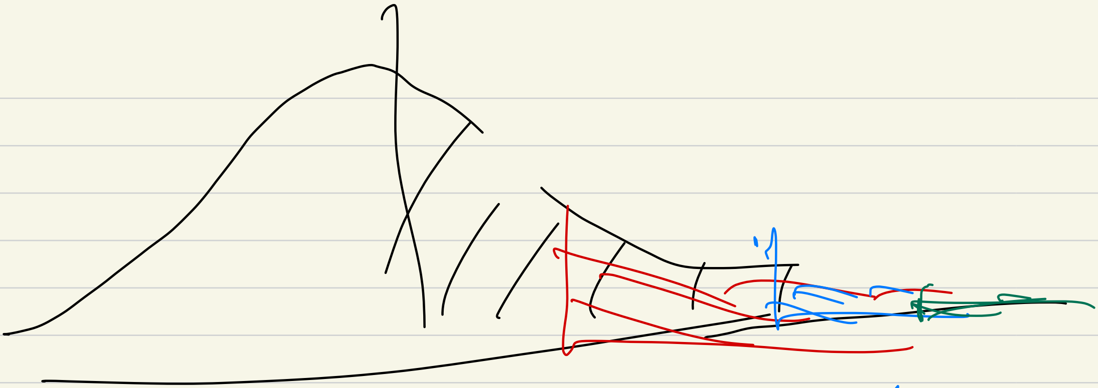
$-\infty \le x \le \infty$

DENSITY (PDF)

$$\text{CDF} = \int_{-\infty}^{x} f(x)\,dx = \Phi(x)$$

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx$$

$Q(x) = 1 - \Phi(x)$ RIGHT TAIL



$Q(0) = \frac{1}{2}$ $\quad Q(1) = \frac{1}{6}$ $\quad Q(2) = .02$

$Q(3) = .001$

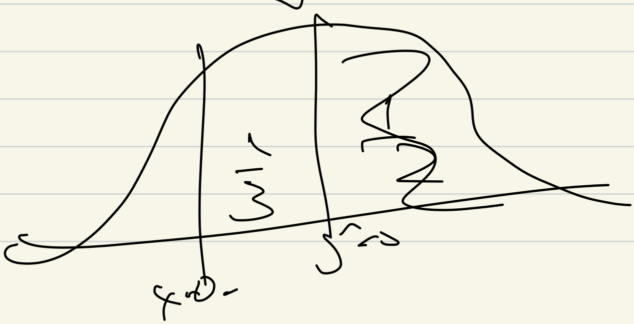
$\mu = 0, \sigma = 1$

TRANSFORM FOR OTHER μ, σ.

$\mu = 500$ $\qquad$ WANT $P[X \geq 500]$

$\sigma = 100$ $\qquad Q(0) = \frac{1}{2}$

$P[X \geq 400] = Q(-1) = 1 - Q(1) = \frac{5}{6}$



$$f(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

APPL NOISY CHANNEL

XMIT 0 OR 1 $\quad P(0) = p$

X. $\quad\quad 1 = 1-p$

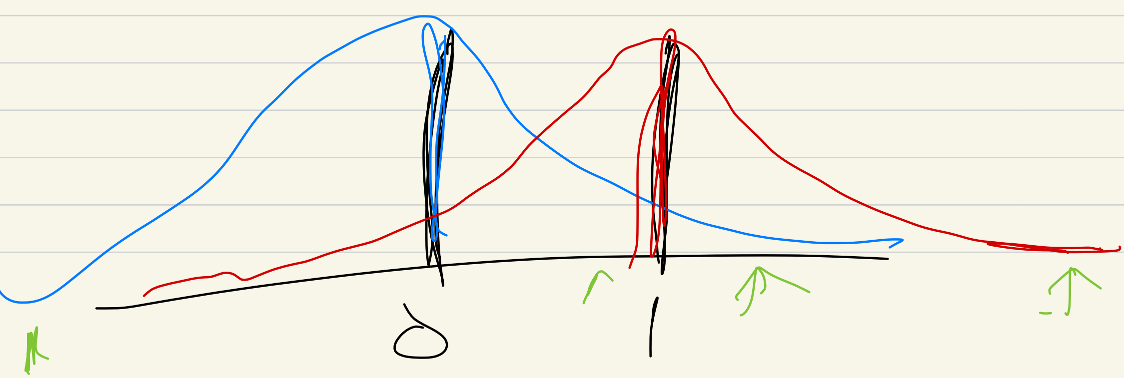
ADD NOISE N IT'S GAUSSIAN

$\mu = 0 \ \sigma = 1$

$N(0,1)$

RECD SIGNAL

$Y = X + N$



0 $\quad$ 1

WANT $f(y)$, THEN $f(x|y)$

GUESS WHAT WAS TRANSMITTED.

$$f(y) = f(y|X=0)\,\underbrace{p(X=0)}_{1-p} + f(y|X=1)\,\underbrace{p(X=1)}_{p}$$

$$f(y|X=0) = f(N) = N(0,1) \qquad X+N = 0+N$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$$

$$f(y|X=1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-1)^2}{2}}$$

$$y = X + N = 1 + N$$

$$f(y) = (1-p)\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} + p\frac{1}{\sqrt{2\pi}} e^{-\frac{(y-1)^2}{2}}$$

N IS GAUSSIAN NOISE R.V. ADDED TO TRANSMIT. ALSO $N(\mu,\sigma)$ IS GAUSSIAN R.V.