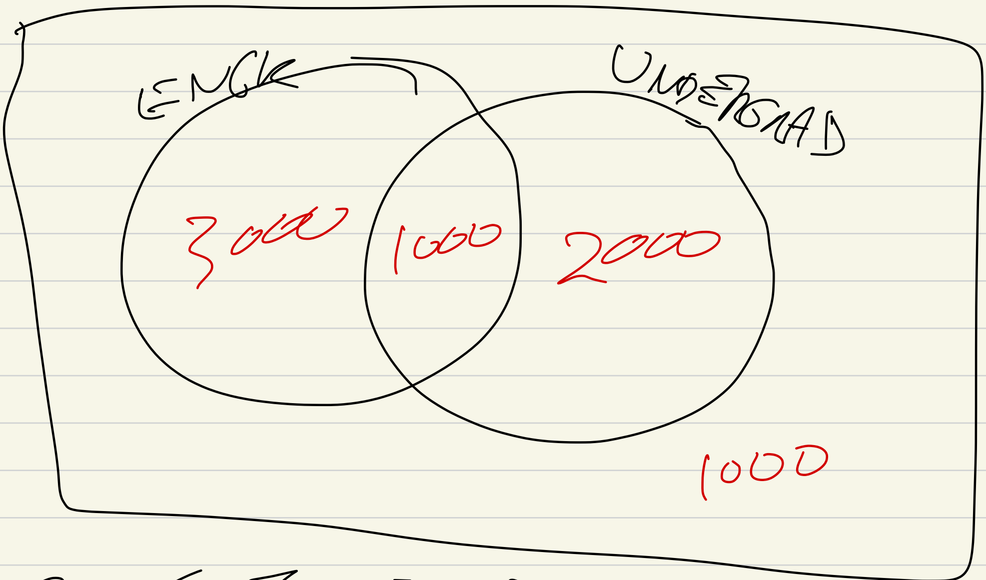
PROB 2022-01-31 p1

CONDITIONAL PROBABILITIES
NUMBERS ARE FICTITIOUS



STUDENTS AT FICTIONAL UNIVERSITY

$$N = 7$$

$$P(E) = \frac{4}{7} \qquad P(U) = \frac{3}{7}$$

$$P(E|U) = \frac{1}{3} \qquad P(U|E) = \frac{1}{4}$$

BAYES THEOREM

HOW TO COMPUTE BACKWARDS CONDITIONAL PROBABILITIES

$P(E|U)$?

$$P(E \cap U) = P(E|U)\,P(U)$$
$$= P(U|E)\,P(E)$$

$$P(U) = P(U \cap E) + P(U \cap \bar{E})$$

(TOTAL PROBABILITY)

$$P(E|U) = \frac{P(E \cap U)}{P(U)} = \frac{P(E \cap U)}{P(E \cap U) + P(\bar{E} \cap U)}$$

$$P(E \cap U) = P(U|E)\,P(E)$$
$$P(\bar{E} \cap U) = P(U|\bar{E})\,P(\bar{E})$$

$$P(E|U) = \frac{P(U|E)\overset{1/4}{}\,P(E)^{4/7}}{P(U|E)\,P(E) + P(U|\bar{E})\,P(\bar{E})}$$

$$\underset{1/4}{}\qquad \underset{4/7}{}\qquad \underset{2/3}{}\qquad \underset{3/7}{}$$

$$\frac{1/7}{3/7}$$

$$\frac{1}{3}$$

ANOTHER BAYES EXAMPLE

PEOPLE, LYCATHROPY GENE, TEST

L (gene), T (test)

$$P(L) = \frac{1}{100}$$

$$P(T|L) = \frac{3}{4} \qquad P(T|\bar{L}) = \frac{1}{10}$$

FALSE POSITIVE

$$P(L|T) \;?$$

HOW MANY TESTS ARE POSITIVE?

$$P(T) = P(TL) + P(T\bar{L})$$

$$P(TL) = P(T|L)\,P(L) = \frac{3}{4}\,\frac{1}{100} = \frac{3}{400}$$

$$P(T\bar{L}) = P(T|\bar{L})\,P(\bar{L}) = \frac{1}{10}\,\frac{99}{100} = \frac{99}{1000}$$

$$P(T) = \frac{3}{400} + \frac{99}{1000} = \frac{213}{2000}$$

$$P(L|T) = \frac{P(LT)}{P(T)} = \frac{\frac{3}{400}}{\frac{213}{2000}} = \frac{15}{213} \approx 7\%$$

PRIOR PROBABILITY = 1%

POSTERIOR PROBABILITY = 7%

A POSITIVE TEST RAISES PROBABILITY

BUT NOT ALWAYS EVEN TO 50%

BECAUSE – PRIOR UNLIKELY,

TEST IS ONLY APPROXIMATE.

BAYES

TOSS 2 COINS

YOU WANT # HEADS TO BE 1,

EVENT T : TOTAL # HEADS 1.

EVENT A : 1ST COIN IS H.

B 2nd " " H.

$$P(A) = \frac{1}{2}$$

$$P(AT) = \frac{1}{4}$$

$$P(T|A) = \frac{P(AT)}{P(A)} = \frac{1}{2}$$

B \ A	0	1
0	0	1
1	1	2

T

WANT $P(A|T)$?

$$P(A|T)\,P(T) = P(AT)$$

$$P(A|T) = \frac{P(AT)}{P(T)} = \frac{\overset{\frac{1}{2}}{P(T|A)}\,\overset{\frac{1}{2}}{P(A)}}{\underset{\frac{1}{2}}{P(T|A)}\,\underset{\frac{1}{2}}{P(A)} + \underset{\frac{1}{2}}{P(T|\bar{A})}\,\underset{\frac{1}{2}}{P(\bar{A})}}$$

$$= \frac{1}{2}$$

CHAP 3 DISCRETE

FUNCTIONS OF R.V

RANDOM VARIABLE

X R.V

CAPITAL: IT'S THE R.V

x : A VALUE

LOWER

$E[X]$: EXPECTED VALUE

AKA MEAN

AVERAGE

$$E[X] = \sum x\, p(x)$$

TOSS COIN $X = 1$ IF HEAD

0 IF TAIL

$$E[X] = 1\, p(1) + 0\, p(0)$$

FAIR : $\frac{1}{2}$

EXPECTED VALUE OF FUNCTION OF R.V.

TOSS FAIR DIE. PAYOFF IS

	PAYOFF
1	0
2	0
3	0
4	1
5	2
6	4

$$E[f] = \frac{1}{6}\cdot 0 + \frac{1}{6} 0 + \frac{1}{6}\cdot 0$$

$$+ \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \frac{1}{6}\cdot 4$$

$$= \frac{7}{6}$$

OPTION ON TESLA STOCK.
STRIKE 1000

X : STOCK PRICE ON CLOSE DATE

PAYOFF : $X \leq 1000 \rightarrow 0$

$X > 1000 \quad X - 1000$

$P(1100) = \frac{1}{4}$ $\qquad P(\leq 1000) = \frac{5}{8}$

$P(1200) = \frac{1}{8}$ $\qquad S = \{900, 1000, 1100, 1200\}$

$$E[X] = \frac{5}{8} \cdot 0 + \frac{1}{4} \cdot 100 + \frac{1}{8} \cdot 200$$

$$= 50$$

A FAIR VALUE OF THAT OPTION IS \$50.

$\boxed{\text{EXPECTED VALUE OF FUNCTION OF R.V.}}$

ANOTHER PROPERTY: VARIANCE
HOW SPREAD OUT THE VALUES ARE.

$$VAR[X] = E[(X - E[X])^2]$$

TOSS COIN $\quad P(0) = \frac{1}{2}$

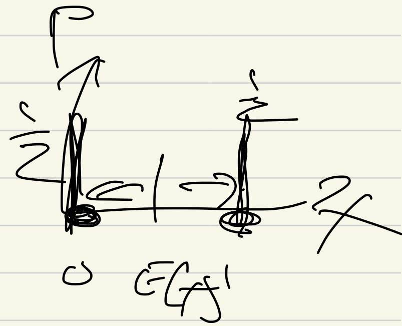
$P(1) = \frac{1}{2}$

$$E[X] = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$VAR[f] = E[(x - \underbrace{E[f]}_{\frac{1}{2}})^2]$$

$p = \frac{1}{2}$ $\quad (0 - \frac{1}{2})^2 = \frac{1}{4}$

$p = \frac{1}{2}$ $\quad (1 - \frac{1}{2})^2 = \frac{1}{4}$



$$VAR[f] = \frac{1}{4}$$

$$STD[f] = \sqrt{VAR[f]} = \frac{1}{2}$$

$$VAR[f] = E[(x - E[f])^2]$$

$$= E[x^2 - 2 \cdot x \cdot E[f] + E[f]^2]$$

PROPERTIES OF $E[x]$:

PROPERTIES OF $E[X]$

$$E[cX] = cE[X]$$

$$E[cX] = \sum c x p(x)$$

$$= c \sum x p(x)$$

$$= cE[X]$$

$$E[X+Y] = E[X] + E[Y]$$

REGARDLESS OF ANY CORRELATION OF X, Y.

ALWAYS TRUE.

(2

$$VAR[X] = E[(X - E[X])^2]$$

$$= E[X^2 - 2XE[X] + E[X]^2]$$

$$= E[X^2] - 2E[X]E[X] + E[X]^2$$

$$= E[X^2] - (E[X])^2$$

$$\boxed{VAR[X] = E[X^2] - (E[X])^2}$$

EX COIN TOSS

$P(0) = \frac{1}{2}$ $\qquad E[X] = \frac{1}{2}$

$P(1) = \frac{1}{2}$ $\qquad E[X^2] = \frac{1}{2}$

$$VAR[X] = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

BINOMIAL

FAIR
TOSS a COIN
4 TIMES

$X = \cancel{\#}$ HEADS

$\boxed{\text{ORDER DOES NOT MATTER}}$

$$P(k) = \binom{4}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{4-k} = \frac{\binom{4}{k}}{16}$$

$P(0) = \frac{1}{16}$ $\quad P(2) = \frac{6}{16}$ $\quad P(4) = \frac{1}{16}$

$P(1) = \frac{4}{16}$ $\quad P(3) = \frac{4}{16}$

$$E[X] = 0 \cdot \frac{1}{16} + 1 \cdot \frac{4}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{4}{16} + 4 \cdot \frac{1}{16}$$

$$= \frac{32}{16} = 2.$$

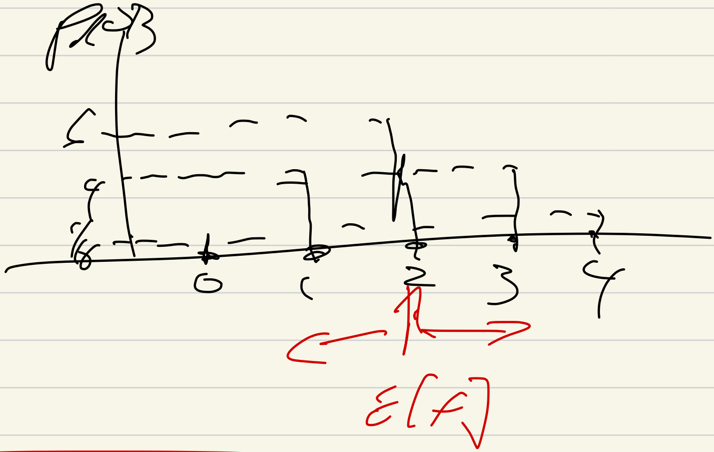
$$E[X^2] = 0 \cdot \frac{1}{16} + 1 \cdot \frac{4}{16} + 4 \cdot \frac{6}{16} + 9 \cdot \frac{4}{16} + 16 \cdot \frac{1}{16}$$

$$= \frac{80}{16} = 5 \;;$$

$$VAR[X] = E(X^2) - (E(X))^2 = 5 - 4 = 1$$

PLOT PROBABILITY MASS FUNCTION

P.M.F =



N COINS NOT JUST 4, FAIR

$$p(k) = \binom{N}{k} 2^{-N}$$

$$E[X] = \left[\sum_{k=0}^{N} k \binom{N}{k}\right]\left[2^{-N}\right]$$

$$\sum_{k=0}^{N} k \frac{N!}{k!(N-k)!} = N \sum \frac{(N-1)!}{(k-1)!(N-k)!}$$

$(k-1)!$

$$\binom{N}{k} p^k q^{N-k} \qquad p = q = \frac{1}{2}$$

$$E[X] = \frac{N}{2}$$

$$VAR[X] = \sum_{K=0}^{N} K^2 \binom{N}{K} 2^{-N}$$

BAYES - RUNS CONDITIONAL PROBS BACKWARDS

FUNCTIONS OF RV

$E[X]$ VAR$[X]$

READING: CHAP 3

EXAM: FEB 28