PROB THURS 1/27/22

DIFFERENT TYPES OF RANDOM EXPERIMENTS

(1) BERNOULLI eg COIN TOSS

$S = \{0, 1\}$

MAYBE UNFAIR $p_0 = p$

$p_1 = 1-p = q$

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(2) BINOMIAL

TOSS UNFAIR COIN N TIME

OUTCOME: # HEADS

ORDER DOESN'T MATTER

$S = \{0, 1, \ldots N\}$

$$P(K \text{ HEADS}) = \binom{N}{K} p^K q^{N-K}$$

EG. $p = .7 \quad q = .3$

$N = 4 \quad K = 2$

$$\binom{4}{2} .7^2 .3^2$$

$$= 6 \times .49 \times .09$$

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(3) MULTINOMIAL

TOSS SOMETHING N TIMES

THE "THING" HAS MORE THAN 2 OUTCOMES

E.G. TOSS COIN THAT
COULD LAND

HEAD .5
TAIL .4
ON EDGE .1

TOSS 3 TIMES.
WANT PROB OF 1 H
1 T
1 E

ORDER DOESN'T
MATTER.

IF ORDER DOES MATTER

$P(HTE) = .5 \times .4 \times .1 = .02$

ORDER DOESN'T MATTER

PROBS: P_H, P_T, P_E

NUMBER N_H N_T N_E

$$P_H + P_T + P_E = 1$$

$$N_H + N_T + N_E = N$$

$$P = \boxed{\binom{N}{N_H \, N_T \, N_E}} P_H^{N_H} P_T^{N_T} P_E^{N_E}$$

$$\frac{N!}{N_H! \, N_T! \, N_E!}$$

6

$p_H = .5 \quad p_T = .4 \quad p_E = .1$

$N_H = N_T = N_E = 1 \qquad N = 3$

$$p = \binom{3}{1\ 1\ 1} .5^1 .4^1 .1^1 = .02$$

$$\frac{3!}{1!1!1!}$$

$\boxed{.12}$

SAY $N = 6$

PROB(3H, 2T, 1E)

$$\binom{6}{3\ 2\ 1} .5^3 .4^2 .1$$

$$\frac{6!}{3!2!1!} = 60 \times .125 \times .16 \times .1$$

(FROM BOOK)

IN YEAR THERE ARE 12 RANDOM PLANE CRASHES

WHAT'S PROB THAT EACH MONTH HAS 1?

$$\binom{12}{\underbrace{1\,1\,1\,1\,1\,1}_{12}} \underbrace{\frac{1}{12}\,\frac{1}{12}\cdots\frac{1}{12}}_{12}$$

$$= \frac{12!}{12^{12}}$$

(4) GEOMETRIC

TOSS UNFAIR COIN UNTIL SEE HEAD

OUTCOME: TOSS # THAT GOT 1ST HEAD.

$S = \{1, 2, 3 \ldots\}$

COUNTABLY ∞.

p = PROB THAT A TOSS IS HEAD.

LET $q = 1 - p$

FAIR $p = \frac{1}{2}$

CALL OUTCOME N

$p(N=3) = q \cdot q \cdot p = \frac{1}{8}$

$p_N = 2^{-N}$

NOW UNFAIR

$$P(N) = q^{N-1}\,p$$

(5) UNIFORM DISCRETE

$$S = \{0, 1, 2, 3, \ldots, N-1\}$$

$$P(k) = \frac{1}{N}$$

(6) POISSON

\# COSMIC RAYS TO HIT ME DURING LECTURE.

\# ATOMS IN THIS BLOCK OF URANIUM THAT WILL DECAY DURING NEXT MINUTE.

LET α = EXPECTED # OF ATOMS THAT WILL DECAY IN NEXT SECOND

k = ACTUAL # DECAYS.

$S = \{0, 1, 2, 3, 4, \ldots\}$

$$P(k) = \frac{\alpha^k k!}{e^\alpha}$$

(?) MAYBE WRONG.

INDEPENDENCE OF EVENTS

A, B : 2 EVENTS.

IF A HAPPENS, DOES THAT TELL YOU SOMETHING ABOUT B ?

TOSS DIE $S = \{1, 2, 3, 4, 5, 6\}$

A : EVEN $\{2, 4, 6\}$ $P(A) = \frac{1}{2}$

B : PRIME $\boxed{\{2, 3, 5\}}$ $P(B) = \frac{1}{2}$

AB $\{2\}$ P $1/6$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1/6}{1/2} = \frac{1}{3}$$

COMPARE $P(AB)$ TO $P(A)P(B)$

IF $P(AB) = P(A)P(B)$

A, B $\boxed{\text{INDEPENDENT}}$

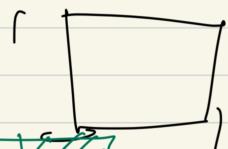
IF THERE ARE SEVERAL EVENTS
$E_1, E_2, E_3, \ldots$

YOU CAN INDEPENDENCE FOR PAIRS
AND FOR LARGER GROUPS

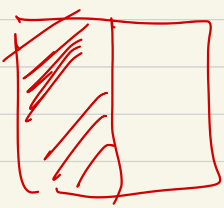
E_1, E_2, E_3 ARE INDEPENDENT
IF $P(E_1 E_2 E_3) = P(E_1) P(E_2) P(E_3)$

NOT SAME AS PAIRWISE INDEP.

PICK A POINT IN SQUARE

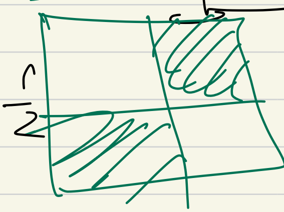


E_1 $\frac{1}{2}$



E_2 $\frac{1}{2}$

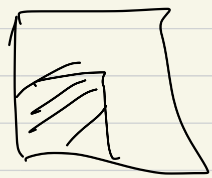
E_3 $\frac{1}{2}$



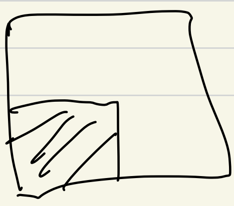
$E_1 E_2$ $\frac{1}{4}$

$E_2 E_3$ $\frac{1}{4}$

$E_1 E_3$ $\frac{1}{4}$



$E_1 E_2 E_3$ $\frac{1}{4}$



$\frac{1}{4} \neq \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

NOT INDEP.

KNOWING THAT E_1 IS TRUE TELLS YOU NOTHING ABOUT WHETHER E_3 IS TRUE

$P(E_3) = \frac{1}{2}$ $\qquad$ $P(E_3|E_1) = \frac{1}{2}$

" " " " E_2 " " " " " " " SAME

$\underline{P(E_3) = \frac{1}{2}}$ $\qquad$ $\underline{P(E_3|E_2) = \frac{1}{2}}$

BUT KNOWING THAT BOTH E_1 AND E_2 ARE TRUE TELLS YOU THAT E_3 IS TRUE. SURPRISE.

$P(E_3|E_1,E_2) = 1$

CHAPTER 3

EXPECTED VALUE

TOSS A FAIR COIN. GET PAID \$1 FOR H, \$0 FOR TAIL

WHAT'S FAIR PRICE TO PAY?

$$\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 0 = \frac{1}{2}$$

X IS A RANDOM VAR.
OUTCOME OF RANDOM EXPT

x_i PARTICULAR VALUES

$$E[X] = \sum x_i p(x_i)$$

(1) BERNOULLI $\quad 1\cdot p + 0\cdot(1-p) = p$

$p(H) = p \quad p(T) = 1-p$

② BINOMIAL NTOSES

$$P(k) = \binom{N}{k} p^k q^{N-k}$$

$$E[k] = \sum_{k=0}^{N} k \binom{N}{k} p^k q^{N-k}$$

$$= \sum_{k=0}^{N} k P(k)$$

$$= \sum \frac{k\, N!}{k!\,(N-k)!} p^k q^{N-k}$$

$$= N \sum \frac{(N-1)!}{(k-1)!\,(N-k)!} p^k q^{N-k}$$

$$= Np \sum_k \binom{N-1}{k-1} p^{k-1} q^{N-k}$$

LET $k_2 = k-1$

$$= Np \boxed{\sum_{k_2} \binom{N-1}{k_2} p^{k_2} q^{(N-1)-k_2}} = 1$$

THERE ARE SOME OFF-BY-1 ERRORS
BUT IDEA IS OK.

$$E[K] = Np$$

USES

$$\sum_{k=0}^{N} \binom{N}{K} p^{K} q^{N-K} = 1$$

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(3) GEOMETRIC

TOSS UNTIL GET HEAD

N = ~~# OF~~ TOSS THAT WINS

$$p(N) = q^{N-1} p$$

$$E(N) = \sum_{N=0}^{\infty} N q^{N-1} p$$

SAY $p = \frac{1}{2}$

$$E(N) = \frac{1}{2} + 2 \cdot \frac{1}{4} + \frac{3}{8} + \frac{4}{16} \cdots$$

④ UNIFORM

$S = \{0, 1, 2, \ldots N-1\}$

CALL OUTPUT K

$$E(K) = \frac{0}{N} + \frac{1}{N} + \frac{2}{N} + \cdots \frac{N-1}{N})$$

$$= \sum_{K=0}^{N-1} \frac{K}{N} = \frac{1}{N} \sum_{K=0}^{N-1} K$$

$$\frac{N(N-1)}{2}$$

$$= \frac{N-1}{2}$$

$N = 4$ $\qquad \frac{1}{4}(0+1+2+3) = \frac{6}{4} = \frac{3}{2}$