PROBABILITY M 1/24/22 -

CHAP 2

RANDOM EXPERIMENT
OUTCOMES - SAMPLE SPACE
EVENTS - SET OF OUTCOMES
INTERVAL ON LINE.
ASSIGN PROBABILITY
1. EACH OUTCOME HAS EQUAL PROBABILITY
TOSS COIN
2 USE PHYSICS

ASSIGNING PROBABILITY NOT ALWAYS EASY.

EXPT: TOSS COIN 3 TIMES
OUTCOME: NUMBER OF HEADS
S = {0, 1, 2, 3}
ARE THEY EQUAL PROBABILITIES?

PROBABILITY   M 11/24/22

ASSIGNING PROBABILITIES TO OUTCOMES OR EVENTS

EXPT: TOSS COIN 3 TIMES

OUTCOMES: $S = \{0, 1, 2, 3\}$

USE COMMON SENSE,
PHYSICS,
ETC.

HERE: THEY'RE NOT EQUAL.

FOR SAME EXPT,
ANOTHER SET OF OUTCOMES

$S_2 = \{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\}$

#0H : TTT 1

#1H : THT, TTH, HTT : 3

#2H : 3

#3H 1

P(# HEAD) : 0	1/8
1	3/8
2	3/8
3	1/8

THESE 2 WAYS OF GETTING PROBABILITY CAN'T BOTH BE RIGHT BECAUSE THEY ARE DIFFERENT.

RULES FOR PROBABILITIES

OF BOOLEAN COMBOS OF EVENTS.

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$P\left(\bigcup_{i=1}^{3} A_i\right) = \sum_{i=1}^{3} P(A_i) - \sum_{i=1}^{2}\sum_{\substack{j=2 \\ i<j}}^{3} P(A_i \cap A_j) + P(A_1 \cup A_i \cup A_j)$$

H: HEARTS

Q: QUEEN

F: FACE JQK

$P(H) = 1/4$

$P(Q) = 1/13$

$P(F) = 3/13$

$P(H \cap Q) = 1/52$

$P(H \cap F) = 3/52$ $\quad Q \subset F$

$P(F \cap Q) = 1/13$

$P(H \cap F \cap Q) = 1/52$

$P(H \cup F \cup Q) = \frac{1}{4} + \frac{1}{13} + \frac{3}{13}$

$- \frac{1}{52} - \frac{3}{52} - 1/13$

$+ 1/52$

NONUNIFORM DISTRIBUTION
CHIP FAILURE
RANDOM VARIABLE T
WHEN CHIP DIES.
CAPITAL LETTER: RANDOM VAR
LOWER CASE t: SOME
VALUE.
RANDOM EXPT: RUN CHIP UNTIL
DIES
OUTCOME: TIME IT DIES

NEED PROBABILITY ASSIGNMENT.
HOW? - TEST A LOT OF CHIPS
- ASK PHYSICIST
- ?

ONE RULE: AGE OF CHIP DOESN'T MATTER.

IF CHIP IS ALIVE NOW,
$P[\text{DYING IN NEXT } \Delta t]$
DOES NOT DEPEND ON AGE.

MAYBE IT DIES WHEN COSMIC RAY ~~H~~ HITS IT. DOESN'T DEPEND ON AGE.

$$\underset{\text{THIS CHIP'S LIFETIME}}{P[T > t]} = e^{-\underset{\text{SOME CONSTANT}}{\alpha} t}$$

THIS CHIP'S LIFETIME.

SOME CONSTANT

EXPONENTIAL PROBABILITY DISTRIBUTION

LET $\alpha = 1$

$P[T>t] = e^{-t}$

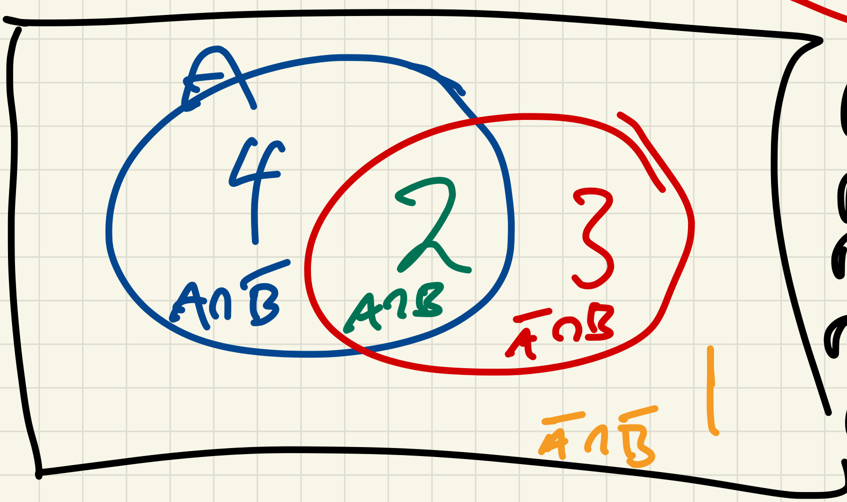
$P[T>0] = e^{-0} = 1$

$P[t_1<T<t_2] = e^{-t_1} - e^{-t_2}$

$P[t<T<t+dt \mid T>t]$

$=$

CONDITIONAL PROB $\longrightarrow$ $\boxed{P[A|B] = \dfrac{P[A\cap B]}{P[B]}}$



$P(A) = 6/10$

$P(B) = 5/10$

$P(A\cap B) = 2/10$

$P(\bar{A}\cap\bar{B}) = 1/10$

$P(A|B) = 2/5$

IMPORTANT

$P[T > t] = e^{-t}$

$P[t < T < t + \Delta t \mid T > t]$?

$$= \frac{P[t < T < t+\Delta t \;\&\; \cancel{T > t}]}{P[T > t]}$$

$$= \frac{e^{-t} - e^{-(t+\Delta t)}}{e^{-t}} = 1 - e^{t-t-\Delta t}$$

$$= 1 - e^{-\Delta t}$$

THIS DOES NOT DEPEND ON t.

$\boxed{\text{MEMORYLESS}}$

$\Delta t = 0 \quad P = 0$ ✓

$\Delta t = \infty \quad P = 1$ ✓

$\boxed{\text{COMMONSENSE CHECKS}}$

COUNTING EXPERIMENTS

APPROVAL VOTING

USED BY IEEE

BALLOT HAS N CANDIDATES
VOTE FOR AS MANY AS YOU WANT.
MOST VOTES WINS.

NOT RANKED

SAY N=5 CANDIDATES ABCDE
YOU VOTE FOR 3
HOW MANY BALLOTS

ABC
ABD
ABE
⋮

$$\frac{5 \cdot 4 \cdot 3}{6} = \frac{5!}{2!\,3!}$$

$$\frac{N!}{K!\,(N-K)!}$$

SELECTION W/O REPLACEMENT
W/O ORDER.

COMBINATION

$$\binom{N}{K}$$

RANKED CHOICE VOTING

SELECT AS MANY AS YOU WANT

RANK THEM.

WAYS TO SELECT 3

5 · 4 · 3

$N=5$

$K=3$

$$\frac{N!}{(N-K)!}$$

ORDER DOES MATTER HERE.

HOW TO GENERATE RANDOM NUMBERS?

DON'T TRY TO COOK UP SOMETHING YOURSELF. USE A GOOD BUILT-IN FUNCTION

e.g. MERSENNE TWISTER

FOR SELECTION IF $N >> K$ REPLACEMENT OR NOT DOESN'T MATTER. THERE ARE NICE APPROXIMATIONS.

1000000 BALLS.
1000 RED
999000 BLUE
YOU DRAW 5 BALLS.
P[EXACTLY 1 RED BALL]
$P(\text{RED}) = \frac{1}{1000} = p$
(GETTING INTO BINOMIAL PROBS)
5 WAYS TO DRAW 1 R 4 B
RBBBB $p(1-p)^4$
⋮
$5p(1-p)^4$
$\binom{5}{1}$

DREW $N=5$
SEE $k=1$ RED
PROB?

$\binom{N}{k} p^k (1-p)^{N-k}$

$\boxed{\text{BINOMIAL PROB}}$

IF N VERY BIG, USE NORMAL APPROX
$N \gg k$ POISSON

TOSSING ONE COIN

BERNOULLI PROB

BINOMIAL: RED & BLACK BALLS

MULTINOMIAL: ≥ 3 COLORS.

URN HAS 1M RED
2M BLACK
3M GREEN

WAYS TO DRAW

1 BALL : 3 WAYS $P(R) = 1/6$ P_1
$B = 2/6$ P_2
$G = 3/6$ P_3

2 BALLS: RR :
RB :
RG :

ORDER DOESN'T MATTER. MESSY.

$P(R \& B) : \frac{1}{6} \cdot \frac{2}{6} \cdot 2$

MORE ON THURS. THINK.

REVIEW BAYES RULE

EG. TRANSMISSION

SEND 0 OR 1

REC 0 OR 1,

A: SEND 0 $\quad \bar{A}$ SEND 1

B: REC 0 $\quad \bar{B}$ REC 1

$P(A) = .9 \quad P(\bar{A}) = .1$

$P(B|A) = .9 \quad P(\bar{B}|A) = .1$

$P(\bar{B}|\bar{A}) = 1 \quad P(B|\bar{A}) = 0$

IF YOU REC 0, WHAT DO YOU KNOW?

WANT $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \frac{.81}{.81} = 1 \qquad P(A \cap B) = P(B|A)P(A) = .9 \cdot .9 = .81$$

NEED $P(B)$

$P(B) = P(B \cap A) + P(B \cap \bar{A})$

$P(\bar{A} \cap B) = \underbrace{P(B|\bar{A})}_{0} P(\bar{A}) = 0$

$P(B) = .81 + 0 = .81$

IF I RECEIVE A "0"
I KNOW A 0 WAS TRANSMITTED

$P(A|B) = 1$.

$$P(\bar{A}|\bar{B}) = \frac{P(\bar{A}\bar{B})}{P(\bar{B})} \quad \frac{.1}{.19}$$

$$P(\bar{A}\bar{B}) = P(\bar{B}\bar{A}) = P(\bar{B}|\bar{A})\,P(\bar{A})$$

$= .1 \qquad 1 \qquad .1$

$$P(\bar{A}|\bar{B}) = \frac{10}{19} \approx \frac{1}{2}$$

IF I RECEIVE A 1, IT'S A LITTLE BETTER THAN 50/50 THAT '1' WAS TRANSMITTED.

$$P(A) = P(A\bar{B}) + P(AB)$$