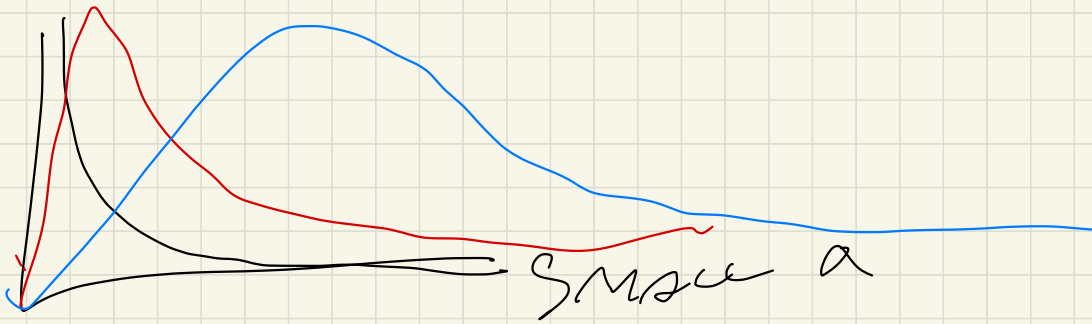
PROBABILITY CLASS 26

M 5/3/21

05/31/21 P1 PROB

POISSON



POISSON IS APPROPRIATE WHEN THERE ARE A LARGE NUMBER OF POSSIBLE EVENTS BUT EACH IS VERY UNLIKELY, SO THE EXPECTED NUMBER IS SMALL.

EXPONENTIAL IS MEMORYLESS.

INDEPENDENT → VARIANCES ADD

ALWAYS → EXPECTATIONS ADD.

6. $E[4] = E(2) = .5$

$E[X+Y] = 1$

7. FOR MAX, MIN, CDF IS APPROPRIATE.

$Z = MAX(X,Y)$ X, Y INDEP.

$F_Z(3) = P[Z \le 3]$

$= P[MAX(X,Y) \le 3]$

$= P[X \le 3 \text{ & } Y \le 3]$

$= P[X \le 3]\, P[Y \le 3]$ INDEP

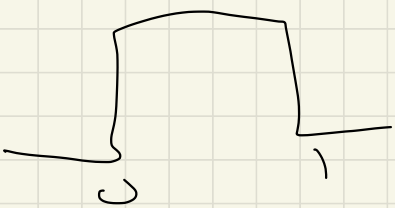
$= F_X(3)\, F_Y(3)$

IF X_1, X_2 IID

$$F_Z(z) = F_X(z)^2$$

IF X_1, X_2 ARE $U[0,1]$

$f_X(x) =$



$F_X(x) =$

$$\begin{cases} 0, & x \le 0 \\ x, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

$$F_Z(z) = z^2 \qquad 0 \le z \le 1$$

$$0 \le z \le 1$$

$$f_Z(z) = 2z$$

$$E[Z] = \int_z f(z)\,dz = 2\int_0^1 z^2\,dz$$

$$= \frac{2}{3}$$

MEAN OF MAX OF N $U[0,1]$

$$= \frac{N}{N+1}$$

---

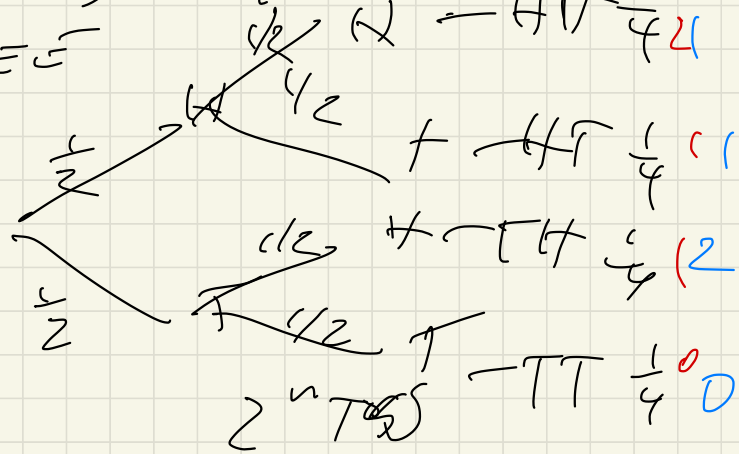
4 OUTCOMES, EQUAL PROB.

HH, HT, TH, TT

$E[\#H] = 1$ $P[X=1] = 1/2$

WHY ARE THOSE 4 OUTCOMES EQUAL PROBABILITY?

DRAW A TREE

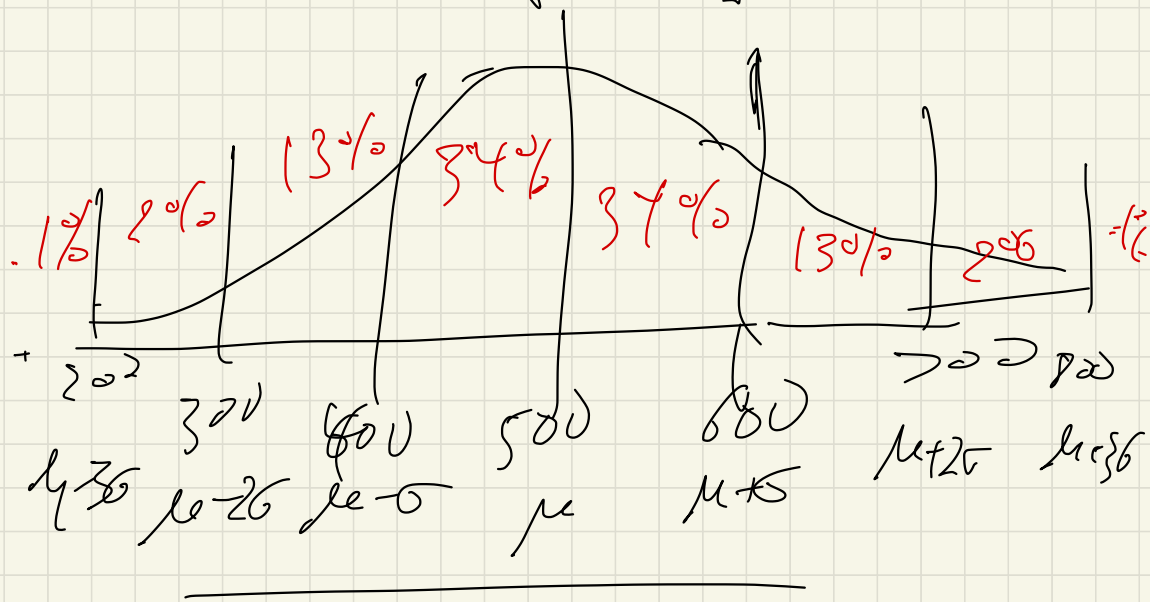


$P[Y=1] = \frac{1}{2}$

$P[X=1 \mid Y=1] = \frac{1}{4}$

$P[Y=1 \mid X=1] = \frac{1}{2}$ $P[X=1 \mid Y=1] = \frac{1}{2}$

$$P[Y_i=1 \mid X=1] = \frac{P[Y_i=1 \text{ & } X=1]}{P[X=1]}$$



14. POP $\sigma = 100$

TAKE SAMPLE OF 4 STUDENTS. σ OF SAMPLE

MEAN $= \frac{100}{\sqrt{4}} = 50$

MEAN OF SAMPLE IS AN ESTIMATOR OF MEAN OF POPULATION. BUT IT BOUNCES AROUND. DIFFERENT SAMPLES HAVE DIFFERENT MEANS. LARGER SAMPLES BOUNCE LESS.

$400 \leq$ SAMPLE MEAN ≤ 600 ? THATS 2 σ EACH WAY

$P = .96$

WE HAVE A POPULATION OF SAT SCORES 200–800.

LOOK AT ONE SCORE X_1

$E[X_1] = 500$ $\quad STD(X_1) = 100$ $\quad VAR[X_1] = 10000$

$= E[X^2] - E[X]^2$

DRAW 4 RANDOM SCORES $X_1 - X_4$ AND

CALC $Y = \frac{X_1+X_2+X_3+X_4}{4}$

Y IS ALSO A RANDOM VAR.

$E[Y] = 500$ $\quad STD[Y] = \frac{STD(X)}{\sqrt{N}} = \frac{100}{2} = 50$

IF WE AVERAGED 100 SCORES

$Z = \frac{\sum_{i=1}^{100} X_i}{100}$ $\quad E[Z] = 500$

$STD[Z] = \frac{STD(X)}{\sqrt{N}} = \frac{100}{10} = 10$

Q14 Y: MEAN OF 4 SCORES

$\mu \; E[Y] = 500$ $\quad STD[Y] = 50 = \sigma$

$400 : \mu - 2\sigma$

$600 : \mu + 2\sigma$

$P[400 \le Y \le 600] = F[2] - F[-2] = .98 - .02$

$= .96$