PROBABILITY

CLASS 6

R 2(11/21)

$N = 7000$

$N_{ENG} = 4000$

$N_U = 5000$

$P[ENG] = \frac{4}{7}$

$P(UG) = \frac{5}{7}$

$P[ENG \& UG] = \frac{3}{7}$

$P[ENG | UG]$ = IF WE KNOW THAT STUDENT S IS AN UNDERGRAD, WHAT'S PROB S IS AN ENG?

LOOKING: $\frac{3}{5}$

VERT BAR : "GIVEN THAT"

$$P[ENG \cap US] = P[ENG \mid US]\, P[US]$$

$$P[ENG \mid US] = \frac{P[ENG \cap US]}{P[US]} = \frac{3/7}{5/7} = \frac{3}{5}$$

$$P[US \mid ENG] = \frac{3/7}{4/7} = \frac{3}{4}$$

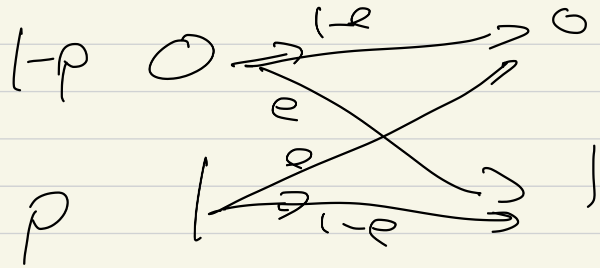
$$P[US \mid \overline{ENG}] = \frac{2/7}{2/5} = \frac{5}{7} \; ?$$

$$P[\overline{ENG}] = \frac{3}{7}$$

$$P[US \cap \overline{ENG}] = \frac{2}{5}$$

EVERYONE THINK.

BINARY COMMUN



T0 : TRANSMIT 0

T1 : TRANSMIT 1

R0 RECEIVE 0

R1 RECEIVE 1

$P[T1] = p \qquad P[T0] = 1-p$

$P[R0 | T0] = 1-e$

$P[R1 | T0] = e$

$P[T0 \cap R0] = (1-p)(1-e)$

$P[T0 \cap R1] = (1-p)e$

$$P[T1 \cap R0] = pe$$

$$P[T1 \cap R1] = p(1-e)$$

$$P[R1|T1] = \frac{P[T1 \cap R1]}{P[T1]}$$

$$= \frac{p(1-e)}{p} = 1-e$$

WE'RE ACTUALLY INTERESTED IN $P[T0|R0]$, $P[T1|R1]$

$$P[T0|R0] = \frac{P[T0 \cap R0]}{P[R0]}$$

$$P[R0] = P[T0 \cap R0] + P[T1 \cap R0]$$

$$= (1-p)(1-e) + pe$$

$$= 1 - p - e + 2p$$

$$P[TO|RO] = \frac{(1-e)(1-p)}{(1-p)(1-e)+pe}$$

E.G. $p = \frac{1}{2}$

$e = .1$

$$P[TO|RO] = \frac{.9 \times .5}{.45 + .05} = .9$$

EG. $p = .1$

$e = .1$

$$P[TO|RO] = \frac{.9 \times .9}{.81 + .02} = \frac{.81}{.83}$$

$\approx .98$

$$P[T1|R1] = \frac{P[T1 \cap R1]}{P[R1]}$$

$$= \frac{p(1-e)}{(1-p)e + p(1-e)}$$

$$P[R1] = P[T0 \cap R1] + P[T1 \cap R1]$$
$$(1-p)e + p(1-e) = .09 + .09 = .18$$

e.g. $p = .1$, $e = .1$ $\qquad P[T1|R1] = \frac{.09}{.18} = .5$

10% OF TRANSMITTED BITS ARE "1"

IF WE RECEIVE "1", THEN 50% CHANCE THAT "1" WAS TRANSMITTED.

THIS IS SURPRISING.

TOTAL PROBABILITY THEOREM

$N_{GRAD} = 1320$ $N_{UGRAD} = 4100$

$N_{ENG} = 2500$ $N_{SCIE} = 1400$ $N = 5420$

$N_{MGT} = 800$ $N_{ARC} = 600$ $N_{HASS} = 120$

$P[ENG] = \frac{2500}{5420}$ $P[ENG \cap G] = \frac{500}{5420}$

$$P[ENG] = P[ENG \cap UG] + P[ENG \cap G]$$

$$\frac{2000}{5420} + \frac{500}{5420}$$

$$= \frac{2500}{5420}$$

$$P[ENG \cap UG] = P[ENG \mid UG]\, P[UG]$$

CHIP QUALITY CONTROL

$$P[\text{ALIVE @ } t] = P[\text{ALIVE @ } T \mid \text{GOOD}]\, P[\text{GOOD}] + P[\text{ALIVE @ } T \mid \text{BAD}]\, P[\text{BAD}]$$

$$= P[\text{ALIVE @ } T \cap \text{GOOD}] + P[\text{ALIVE @ } T \cap \text{BAD}]$$

$$= e^{-at}(1-p) + e^{-1000at}\, p$$

REVERSE IT.

$P[\text{GOOD} \mid \text{ALIVE @ } t]$?

BAYES RULE

W = OUTCOME THAT YOU'RE A WEREWOLF

$$p(W) = 10^{-6}$$

THERE'S A TEST T FOR THIS. IT'S PRETTY GOOD $P(T \mid W) = .999$

$$P(\bar{T} \mid \bar{W}) = .999$$

T : TEST WAS +VE

$\bar{T}$: ... -VE.

WANT $P(W \mid T)$?

$P(W) = 10^{-6}$

$P(T|W) = .999 \qquad P(T \wedge W) = .999 \times 10^{-6}$

$P(F|\bar{W}) = .999 \qquad P(F \wedge \bar{W}) = .999 \times .999999$

$\approx .999$

$P(T \wedge \bar{W}) = .001$

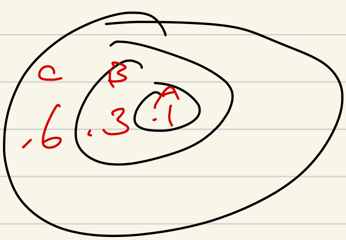
$P(T) = .001 + .000000999 \approx .001$

$$P(W|T) = \frac{P(W \cap T)}{P(T)} = \frac{.000000999}{.001} \approx .000999$$

$\approx .001$

EVEN IF TEST IS POSITIVE, PROB YOU'RE A WEREWOLF IS ONLY .001

DARTBOARD MULTINOMIAL PROB



TOSS 5 DARTS

GET 2A
2B
C

PROB?

$$\binom{5}{2\,2\,1} .1^2 .3^2 .6^1$$

$$\leftarrow \frac{5!}{2!2!1!}$$