SAMPLING $\begin{Bmatrix} w \\ w/o \end{Bmatrix}$ REPLACEMENT

$\begin{Bmatrix} w \\ w/o \end{Bmatrix}$ ORDERING.

Q = QUANTUM
C = CLASSICAL

$N = 2$ COINS IN URN
DRAW $k = 2$ COINS.

w/o REPL. w ORD.	QD, DQ	2	$N(N-1)\cdots\frac{N!}{(N-k)!}$
w/o REPL w/o ORDER	{Q, D}	1	$\binom{N}{k} = \frac{N!}{k!(N-k)!}$
w REPL w ORDER	QQ, QD, DQ, DD	4	N^k
w REPL w/o ORDER:	{QQ, QD, DD}	3	

8 3 BALLS FROM 5 W REPL.

1st: 5 CHOICES

2 5

3 5

125 CHOICES.

#CASES WHERE DIFF $5 \times 4 \times 3 = 60$

PROB ALL DIFF $= \frac{60}{125}$

PERMUTATIONS: #WAYS TO ORDER N OBJECTS

FACTORIAL.

$3! = 6$ ABC CAB
ACB CBA.
BAC
BCA

FOR EACH CRASH, WHICH MONTH?
HOW MANY WAYS?

$12!$ (IF CRASHES ALL IN DIFFERENT MONTHS

12^{12} (IF DON'T CARE

$P(12 \text{ CRASHES ALL IN DIFF MONTHS}) = \frac{12!}{12^{12}}$

MULTINOMIAL EXAMPLE

BINOMIAL N=4 COINS

#WAYS TO DIVIDE INTO 2 SETS OF 2.

4 WAYS TO PICK 1ST

3

2

X=6

÷2 EACH SET PICKED 2 WAYS

$$\frac{N(N-1)\cdots(N-K+1)}{K!} = \frac{N!}{(N-K)!K!} = \binom{N}{K}$$

DIVIDE INTO SET OF 3 AND OF 1

$$\binom{4}{3} = \frac{4!}{3!1!} = 4$$

MULTI

DIVIDE 4 COINS INTO 3 SETS: SIZE 2, 1, 1

$$\binom{4}{2\,1\,1} = \frac{4!}{2!\,1!\,1!} = 12$$

4 COINS: QDNP

{QD}, {N}, {P}

QP	N	D
	D	N
PD	Q	N
	N	Q
ND	P	Q
	Q	P
QN	P	D
	P	Q
NP	Q	D
	D	P

TOTAL 12

5

9 BALLS

WANT TO DIVIDE INTO 3 SETS

SIZES 4, 3, 2

WAYS?

$\frac{9 \cdot 8 \cdot 7 \cdot 6}{24} = \frac{9!}{4!5!}$ WAYS TO PICK 1ST SET.

5 BALLS LEFT

$\frac{5 \cdot 4 \cdot 3}{6} = \frac{5!}{3!2!}$ WAYS TO PICK 2ND.

1 WAY TO PICK 3rd.

$\frac{9!}{4!5!} \frac{5!}{3!2!} = \frac{9!}{4!3!2!} = \binom{9}{4\ 3\ 2}$

$\frac{N!}{K_1! K_2! K_3! \cdots}$

BINOMIAL PROBS

FAIR COIN $P(\text{HEAD}) = \frac{1}{2}$

TOSS $N=2$ $P(HH) = \frac{1}{4}$

$P(HT, TD) = \frac{1}{2}$

$P(TT) = \frac{1}{4}$

#WAYS TO GET EXACTLY K HEADS $= \binom{N}{K}$

TOTAL # OUTCOMES $= 2^N$

$$P(\text{EXACTLY } K \text{ HEADS}) = \frac{\binom{N}{K}}{2^N}$$

$N=4$ $\binom{N}{K} = 1, 4, 6, 4, 1$

$K=0 \quad 1 \quad 2 \quad 3 \quad 4$

6 WAYS TO GET 2H.

$P(2H) = \frac{6}{16} = .38$ $\quad P(\underset{K=0}{0H}) = \frac{1}{16}$

UNFAIR COIN

$P = \frac{2}{3}$ OF HEAD.

TOSS N TIMES

SEE K HEADS

$P(N=2, K=0) = \frac{1}{3} \cdot \frac{1}{3} = (1-p)^N$

$P(N, K) =$

P(K HEADS IN A ROW, THEN N-K TAILS (IN A ROW)

$= p^K (1-p)^{N-K}$

$P(\text{K HEADS, ANY ORDER}) = \binom{N}{K} p^K (1-p)^{N-K}$

PROB OF TAIL IN 1 TOSS

$N=4$
$K=2$
$p=2/3$

$\binom{4}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2$

$= \frac{6 \cdot 4}{81} \approx .3$

$N=4, K=4 \quad \left(\frac{2}{3}\right)^4 \approx .2$

NON CAN EXAMPLE

BUILD WIDGETS $P = .01$ BAD

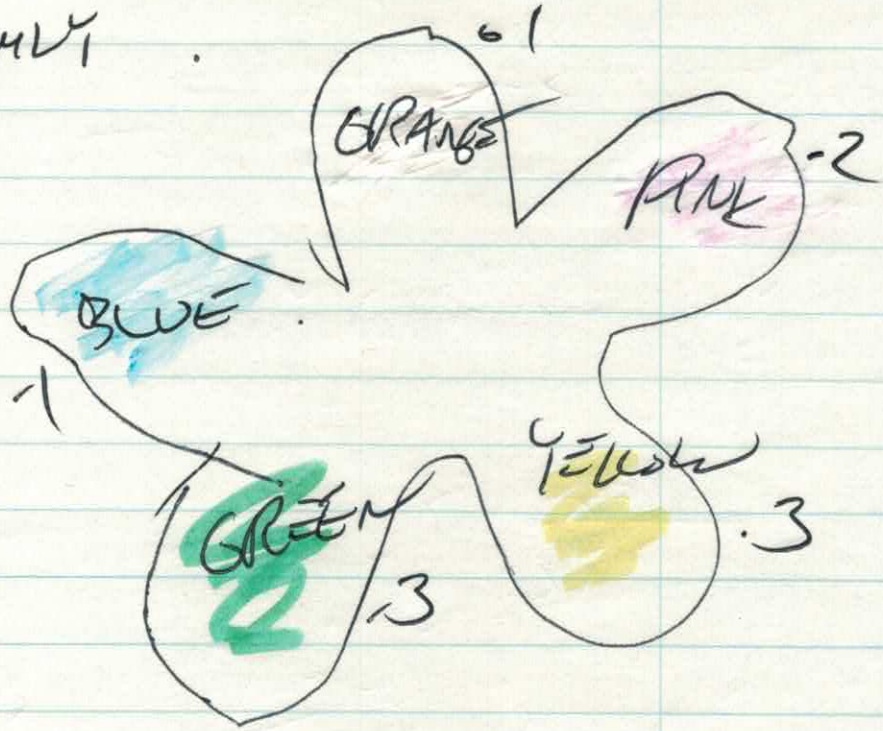
P(2 OF 10 WIDS) BAD.

$$\binom{10}{2} (.01)^2 (.99)^8$$

MULTINOMIAL

PROB OF SELECTING A COLOR

PICK A COLOR RANDOMLY.



PICK ~~3~~ COLORS

PICK 10 COLORS.

$P[\text{ALL BLUE}] = (.1)^{10}$

$P[2\text{BLUE}, 2\text{GRN}, 2\text{YEL}, 2\text{PINK}, 2\text{ORG}]$

$$= \binom{10}{2\,2\,2\,2\,2} \cdot .1^2 \cdot .3^2 \cdot .3^2 \cdot .2^2 \cdot .1^2$$

$$\frac{10!}{2!\,2!\,2!\,2!\,2!}$$

PLAYING FUNNY CARD GAME.

$N = 52$ CARDS.

15 H
20 C
10 S
7 D

$P(4) = \frac{15}{52}$

$P(2H, 1C, 2D)$

$$\binom{5}{2\,1\,2} \left(\frac{15}{52}\right)^2 \left(\frac{20}{52}\right)\left(\frac{7}{52}\right)^2$$

$\binom{5}{2\,1\,2}$ "5 CHOOSE 2, 1, 2"

$= \frac{5!}{2!1!2!}$ #WAYS TO DIVIDE A SET OF 5 THINGS INTO A SET OF 2
AND " 1
AND " 2

FOR SET OF 2: 5 WAYS TO PICK 1ST THING
+4 - - - - - 2ND...

$\div 2$ BECAUSE EACH SET CAN BE PICKED 2 WAYS.

5 THINGS: A, B, C, D, E

~~10 WAYS TO PICK A SET OF~~

10 SETS OF 2 FROM THAT

AB AC AD AE BC BD BE CD CE DE

4 + 3 + 2 + 1 = 10

5 WAYS TO PICK 1ST

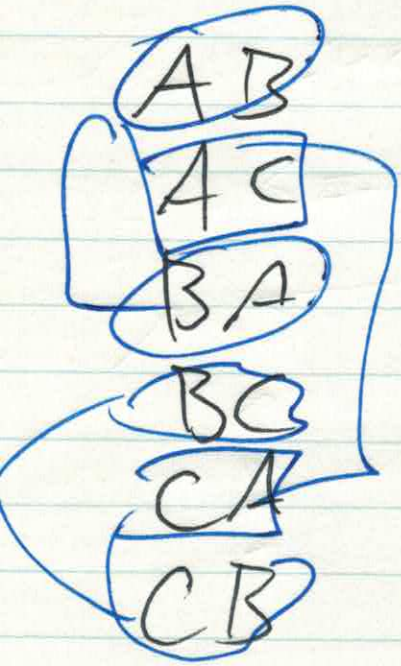
$\times 4$ $\cdots$ 2^{n-1}

20

$\div 2$ WAYS TO PICK ANY SET

LETS.

PICK 2 ~~COINS~~ FROM 3



EACH SET WAS PICKED TWICE

SO DIVIDE BY 2

3 SETS OF YOU CAN PICK FROM 3

$\frac{N!}{(N-K)!K!} = \binom{N}{K}$

$\frac{N!}{(N-K)!}$

$N=3, K=2$

TOSS 6-SIDED DIE

1. FAIR $P(1)=P(2)\ldots=\frac{1}{6}$

TOSS 10 TIMES

$P(1\text{-}1, 2\text{-}2, \underbrace{3\text{-}3}_{\text{3 CAME UP 3 TIMES}}, 4\text{-}4, 0\text{-}5, 0\text{-}6)$

$$\binom{10}{1\,2\,3\,4\,0\,0}\left(\frac{1}{6}\right)^{10}$$

$$\frac{10!}{1!\,2!\,3!\,4!\,0!\,0!}$$

2 UNFAIR $P(1)=50\%=.5$

$P(2)=P(3)\ldots P(6)=.1$

$$\binom{10}{1\,2\,3\,4\,0\,0}(.5)(.1)^3(.1)^3(.1)^4(.1)^0(.1)^0$$