DISCRETE PROB 2/1/17

FAIR 6 SIDED DIE.

$S = \{1, 2, 3, 4, 5, 6\}$

A = EVEN $\{2, 4, 6\}$

B: PRIME $\{2, 3, 5\}$

C: SQUARE $\{1, 4\}$

NEED PROBS OF INDIVIDUAL OUTCOMES.

HOW? IS ANOTHER QUESTION

PHYSICS? COMMON SENSE?

$P[1] = P(2) = \ldots = \frac{1}{6}$

$P[A] = \frac{1}{2} = P[B]$

$P[C] = 1/3$

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$P[A \cap B] = 1/6$

$A \cap B = \{2\}$

$A \cup B = \{2, 3, 4, 5, 6\}$

$P[A \cup B] = 5/6$

$P[A \cup B] = P[A] + P[B] - P[A \cap B]$

$\frac{5}{6} = \frac{1}{2} + \frac{1}{2} - 1/6$

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3. WHAT COMBOS

$P[A \cup B \cup C] = P[A] + P[B] + P[C]$

$\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{3}$

$- P[A \cap B] - P[B \cap C] - P[A \cap C]$

$\frac{1}{6} \quad 0 \quad \frac{1}{6}$

$+ P[A \cap B \cap C]$

0

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{3} - \frac{1}{6} - 0 - \frac{1}{6} + 0 = \frac{7}{6}$ ERROR!

$= 1$

$A \cap C$?

A = EVEN: $\{2, 4, 6\}$

C: SQUARE $\{1, 4\}$

$A \cap C$: $\{4\}$

$P[A \cap C] = \frac{1}{6}$

~~$A \cap B \cap C$~~

ASSUME: INDEPENDENCE

DEFINE $D = \{4\}$

$D \subset C$

$P(D) \leq P[C]$

$P(D) = 1/6$ $P[C] = 1/3$

HOW TO GET LOW LEVEL PROBS?

EXPT: TOSS A COIN TWICE.

YOU GET TO DEFINE OUTCOMES.

OUTCOMES ~~#HEADS~~.

$S = \{0, 1, 2\}$

WHAT ARE PROBS??

HARD

$P[0] = P[1] = P[2] = \frac{1}{3}$

COMMENTS?

UNLIKELY.

NOT POSSIBLE IF THE TWO TOSSES ARE INDEPENDENT.

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REASONABLE IS

$P[0] = \frac{1}{4}$ $P[1] = \frac{1}{2}$ $P[2] = \frac{1}{4}$

BECAUSE LOOKING AT DETAILED OUTCOMES

$S = \{\underset{\frac{1}{4}}{TT}, \underset{\frac{1}{4}}{TH}, \underset{\frac{1}{4}}{HT}, \underset{\frac{1}{4}}{HH}\}$

FOR EACH TOSS $\{H, T\}$

$\frac{1}{2}$ $\frac{1}{2}$

THIS ONE IS EASY.

MONTY HALL PROB CAUSES ARGUMENTS

$$\sum \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$A = \sum_{k=1}^{\infty} \frac{1}{2^k}$$

$$\sum \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = \frac{1}{2} + \sum \frac{1}{4} + \frac{1}{8} + \cdots$$

$$= \frac{1}{2} + \frac{1}{2} \sum \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$A = \frac{1}{2} + \frac{1}{2} A \Rightarrow A = 1$$

2, UNIFORM $[0,1]$

$P[a < x < b] = \frac{1}{b-a}$

COMBO TYPE.

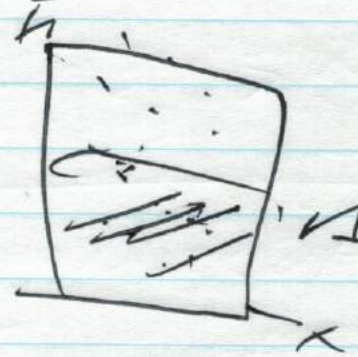
WAITING FOR CAR AT AIRPORT

T = TIME WAITING.

$P[0] = \frac{1}{2}$

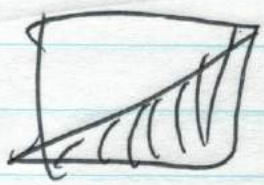
$P[T] \equiv P[t > T] = c2^{-T}$

RANDOM VARIABLE — PARAM



A: $T = \frac{1}{2}$

$P(A) = \frac{1}{2}$



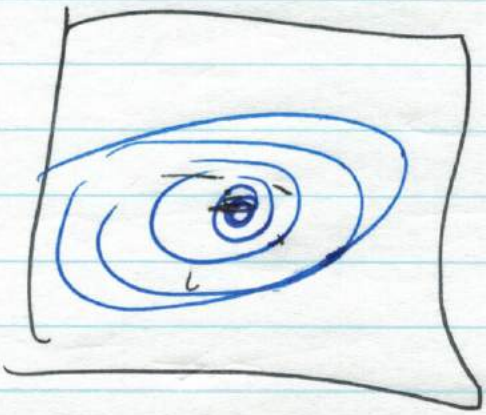
B: $x > \frac{1}{2}$

$P[B] = 1/2$

$C = A \cap B$  $P[C] = \frac{3}{8}$

$D = A \cup B$  $P[C] = \frac{5}{8}$

ASSUMES POINTS CAME UNIFORMLY. IS IT TRUE?



ASSUME YOU'RE TRYING TO HIT TARGET

POINTS NOT UNIFORM.

COMBINATORICS

Q. TOSS COIN N TIMES WANT PROB OF K HEADS.

$N=2$ $P: \frac{1}{4}, \frac{1}{2}, \frac{1}{4}$

$0, 1, 2$

GENERALIZE

$N=3$ $P = \frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$

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PROB OF 1 HEAD IN 3 TOSSES IS # WAYS YOU CAN GET 1 HEAD, 2 TAILS

$\frac{3}{8}$

HTT
THT
TTH

$\} 3$

$P = \frac{3}{8}$

TOTAL # WAYS.

N TOSS. K HEAD $P = \binom{N}{K}$

N CHOOSE K

$= \frac{N!}{K!(N-K)!}$

$\binom{10}{5} = \frac{10!}{5!5!} = \frac{1}{4}$

11 COINS. 5 QUARTERS
2 DIMES
2 NICKELS
2 PENNIES

URN

P(DRAW QUARTER) = $\frac{5}{11}$

P(DIME) = $\frac{2}{11}$

P(2 QUARTERS)

= P(DRAW QUARTER) · P(DRAW ANOTHER)

$\frac{1}{11} \cdot \frac{4}{10}$

FOR 2ND DRAW, GIVEN 1ST COIN WAS QUARTER, WE HAVE NOW 10 COINS, 4 QUARTERS.

"DRAWING W/O REPLACEMENT"