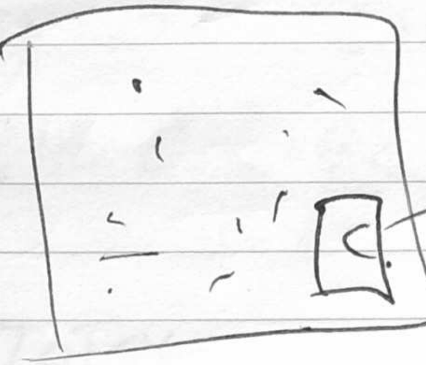
PROB CLASS 25 THURS 4/23/20 P1

EX 5.30 P 263



PROB OF A DEFECT LANDING HERE IS p.

DEFECTS ON CHIP IS POISSON α

$$P(k) = e^{-\alpha} \frac{\alpha^k}{k!}$$

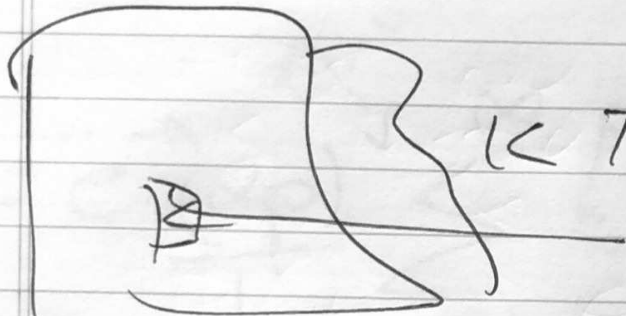
CHECK $$\sum_{k=0}^{\infty} p(k) = e^{-\alpha} \underbrace{\sum \frac{\alpha^k}{k!}}_{e^{\alpha}} = 1$$

GIVEN A DEFECT THAT LANDED ON CHIP SOMEWHERE, PROB IT LANDED IN LITTLE BOX $= p$

WE WANT PROB DST FOR # DEFECTS IN LITTLE BOX.

LET Y = # DEFECTS IN LITTLE BOX

CONDITIONAL : BINOMIAL



$$P_Y(J|K) = \begin{cases} 0 & \text{IF } J > K \\ \binom{K}{J} p^J (1-p)^{K-J} & \end{cases}$$

$$P_X(K) = e^{-\alpha} \frac{\alpha^K}{K!}$$

$J \le K$

$$P(Y = J \;\&\; X = K) = \binom{K}{J} p^J (1-p)^{K-J} e^{-\alpha} \frac{\alpha^K}{K!}$$

$$P_Y(Y = J) = \sum_{K=J}^{\infty}$$

$$= \sum_{K=J}^{\infty} \binom{K}{J} p^J (1-p)^{K-J} e^{-\alpha} \frac{\alpha^K}{K!}$$

?

$$\frac{K!}{J!\,(K-J)!}$$

$$= \frac{e^{-\alpha} p^{j}}{j!} \sum_{k=j}^{\infty} \frac{\cancel{k!}}{(k-j)!} (1-p)^{k-j} \frac{\alpha^{k}}{\cancel{k!}}$$

$\cancel{\alpha}^{j}$ α^{-j}

$$= \frac{e^{-\alpha} (\alpha p)^{j}}{j!} \sum_{k=j}^{\infty} \frac{[(1-p)\alpha]^{k-j}}{(k-j)!}$$

IF K GOES FROM J TO ∞ THEN
K−J . . . 0 . . .

$$P_Y(j) = \cancel{\alpha}\, \frac{e^{-\alpha} (\alpha p)^{j}}{j!} \underbrace{\sum_{m=0}^{\infty} \frac{[(1-p)\alpha]^{m}}{m!}}_{e^{(1-p)\alpha}}$$

$m = k-j$

$\cancel{P_Y(j) = \sum e^{-p\alpha}}$

$$p(j) = \frac{e^{-p\alpha} (p\alpha)^{j}}{j!}$$

THAT'S A POISSON WITH PARAM $p\alpha$

C4

EX 5.33 P267

RV1 # CUSTOMERS IN TIME T

IS POISSON βT

RV2 TIME TO SERVE ~~THE~~ ONE CUSTOMER

$f_T(t)$ IS EXPONENTIAL α

RV3 # NEW CUSTOMERS ARRIVING
DURING 1ST CUSTOMER

N - LOWERS QUALITY OF SERVICE.

$$P[N=k] = \int_0^\infty P[N=k \mid T=t] f_T(t)\, dt$$

$$= \int_0^\infty e^{-\beta t} \frac{(\beta t)^k}{k!} \alpha e^{-\alpha t}\, dt$$

$$= \frac{\alpha \beta^k}{k!} \int_0^\infty e^{-(\alpha+\beta)t} t^k\, dt$$

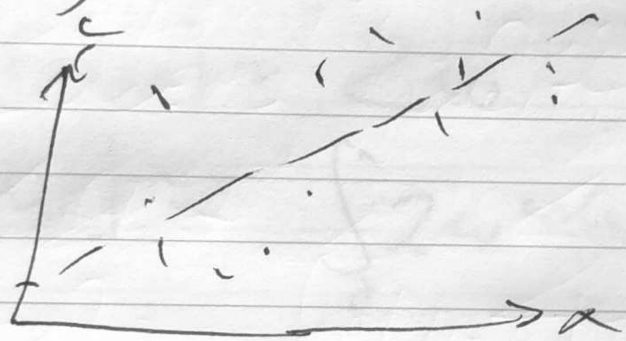
ETC.

RQ1 $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int e^{-\frac{x^2}{2}} = \sqrt{2\pi}$$

2



$COV[X,Y] = E[(X - E[X])(Y - E[Y])]$

3 $X, Y \quad N[0,1]$

$VAR[X] = VAR[Y] = 1$

IF INDEP, $VAR[X+Y] = 2$

IF $\rho = 1$, $VAR[X+Y] = 4$, $Y = X$

IF $\rho = -1$ $VAR[X+Y] = 0$ IF $Y = -X$

7 $W = \max[X, Y]$

$P[W \le w] = P[X \le w \ \& \ Y \le w]$

(IF INDEP) $= P[X \le w]\, P[Y \le w]$

$\qquad\qquad\quad w \qquad\qquad w$

$= w^2$

$CDF[W] = w^2$

$PDF(w) = 2w$

$E[W] = \int_0^1 2w^2\,dw = \frac{2}{3} w^3 \Big|_0^1 = \frac{2}{3}$

8

		X = 0	1	2		
Y	0	$\frac{1}{4}$	0	0	~~1/4~~	$\frac{1}{4}$
	1	0	$\frac{1}{2}$	~~1/4~~	~~1/2~~	$\frac{1}{2}$
	2	0	$\frac{1}{4}$	0	~~0~~	$\frac{1}{4}$
		$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$		

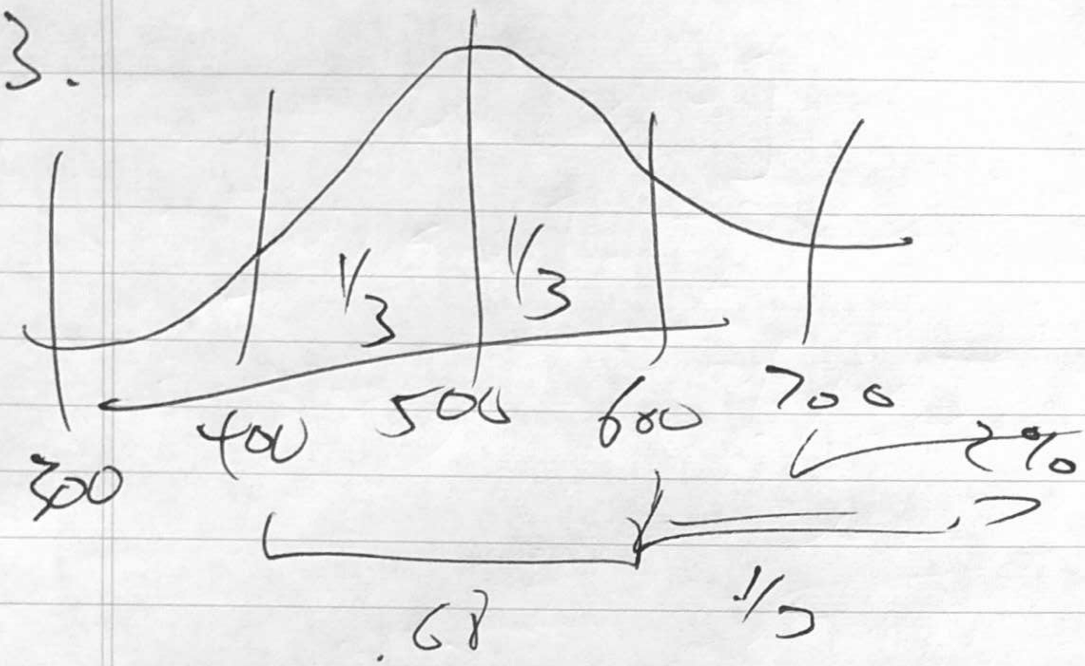
TT

TH

HT

HH

13.



14. SAMPLE MEAN $= \mu$

SAMPLE σ $= \frac{\sigma}{\sqrt{N}} = 50$

$400 \to 600$

$\mu \pm 2\sigma_s = \cancel{.98}\ .96$