

PROB C23 4/16/20 p1

X : RV - μ NOT KNOWN

SAMPLES: X_i

X_i : IID.

SAMPLE MEAN: $M_N = \frac{\sum X_i}{N}$

M_N ESTIMATES μ

$M_N = RV$.

M_N HAS MEAN, VAR

WHAT ARE THEY?

$$M_N = \frac{\sum X_i}{N}$$

X_i : μ, σ

$\sum X_i$: $N\mu, \sqrt{N}\sigma$

M_N : $\mu, \frac{\sigma}{\sqrt{N}}$

WHAT'S PROB THAT M_N IS
WITHIN ϵ OF μ ?

TRY CHEBYSHEV INEQUALITY.

$$P[|X - \mu| \geq \epsilon] \leq \frac{\sigma^2}{\epsilon^2}$$

$$P[|M_N - \mu| \geq \epsilon] \leq \frac{\sigma^2}{\epsilon^2}$$

$$= \frac{\sigma^2}{N\epsilon^2}$$

EX 7.9 $X_i = \mu + N_i$

WE WANT $N \ni P[|M_N - \mu| \leq \epsilon]$

$$N_i: \sigma = \mu\sqrt{N}$$

≈ 99

$$M_N: \sigma = \frac{1}{\sqrt{N}} \mu\sqrt{N}$$

$$P[|M_N - \mu| \geq 1] \leq \frac{1}{N}$$

$$\frac{1 - \sigma^2}{N \epsilon^2} = 1 - \alpha = .99 \quad \left\{ \begin{array}{l} 3 \end{array} \right.$$

$$N = 100$$

EX 7.10 HOW MANY TIMES TO TOSS UNFAIR COIN SO WE PROBABLY KNOW ITS BIAS PRETTY CLOSELY?

"PRETTY CLOSELY" = WITHIN 1%

"PROB" = 95% OF TIME.

N TOSSES ~~IS~~ RV = # HEADS SEEN

$$\mu = Np$$

p - UNKNOWN PROB

$$\text{VAR} : Np(1-p) \leq \frac{N}{4}$$

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EX 7.11

 $X_n = 1^{\text{th}}$ CUSTOMER ORDER COST

$$E[X_n] = \$8 \quad \text{STD}[X_n] = 2$$

$$S_{100} = \sum_{i=1}^{100} X_i$$

$$M[S] = 800$$

$$\text{STD}[S] = \sqrt{100} = 20$$

WANT $P[S > 840]$

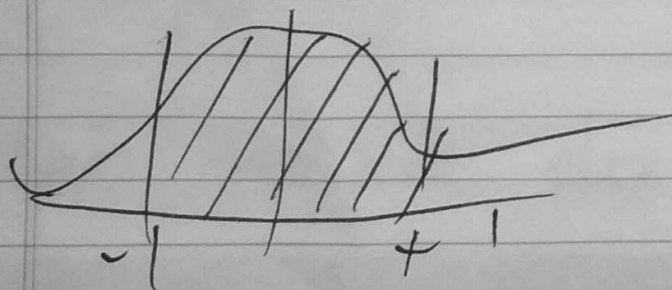
S : GAUSSIAN $\mu = 800$
 $\sigma = 20$

$$P[S > 840] = Z(2) \approx 0.02$$

\uparrow
 $\mu + 2\sigma$

$$P[780 \leq S \leq 820] \approx \frac{2}{3}$$

$\mu - \sigma$ $\mu + \sigma$



EX 7.12

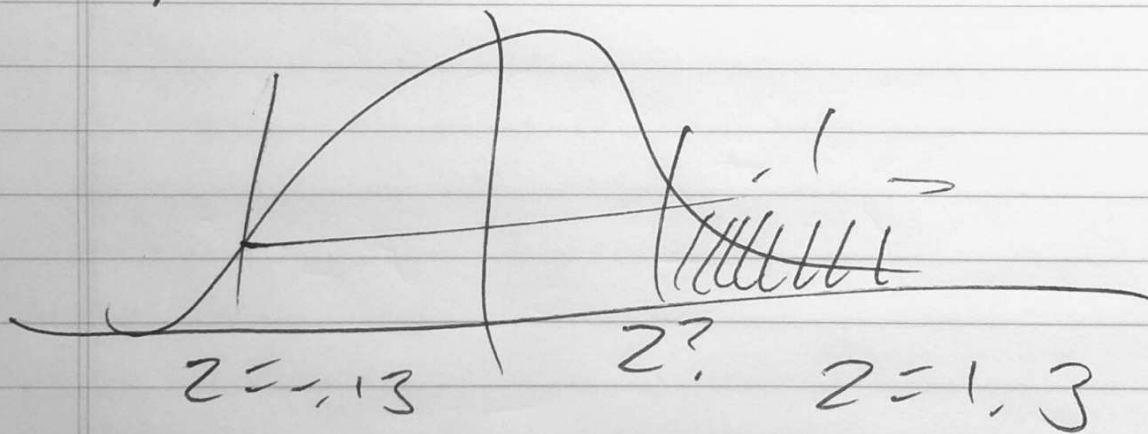
N UNKNOWN

$$S_N : \mu = 8N \quad \sigma = 2\sqrt{N}$$

1000 :

CONVERT 1000 TO Z SCORE

$$P \left[Z \leq \frac{1000 - 8N}{2\sqrt{N}} \right] = .9$$



$$-1.3 = \frac{1000 - 8N}{2\sqrt{N}} \quad N?$$

$$N = 129$$

ETAM LAST CLASS 4/27