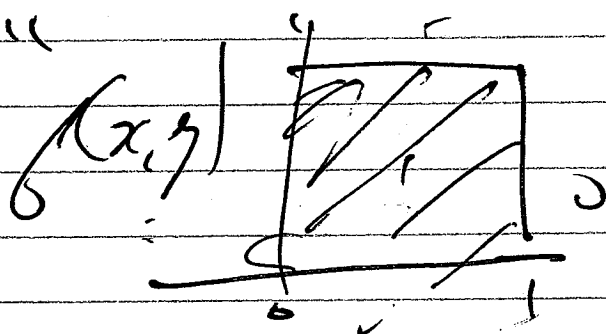


PROB dis 3/5/20 p1
JOINT CDF

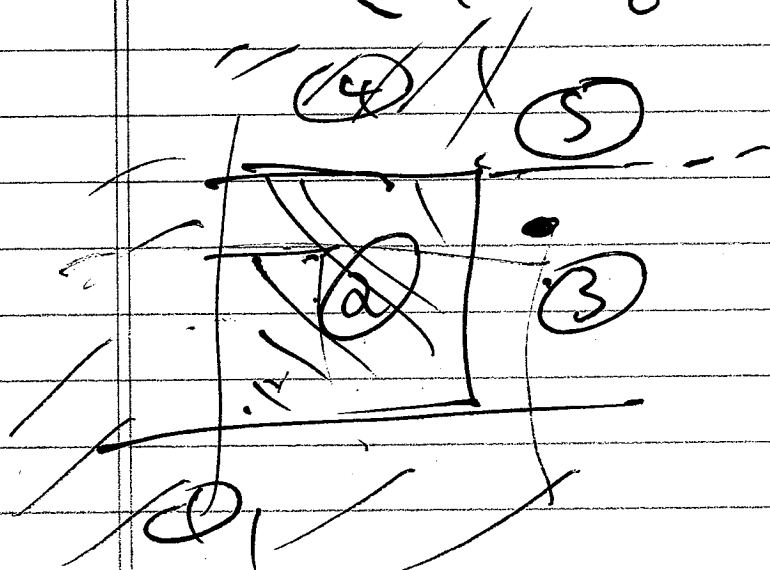
EX5.



$$f(x,y) = \begin{cases} xy & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F_{X,Y}(x,y) = P[X \leq x \text{ and } Y \leq y]$$

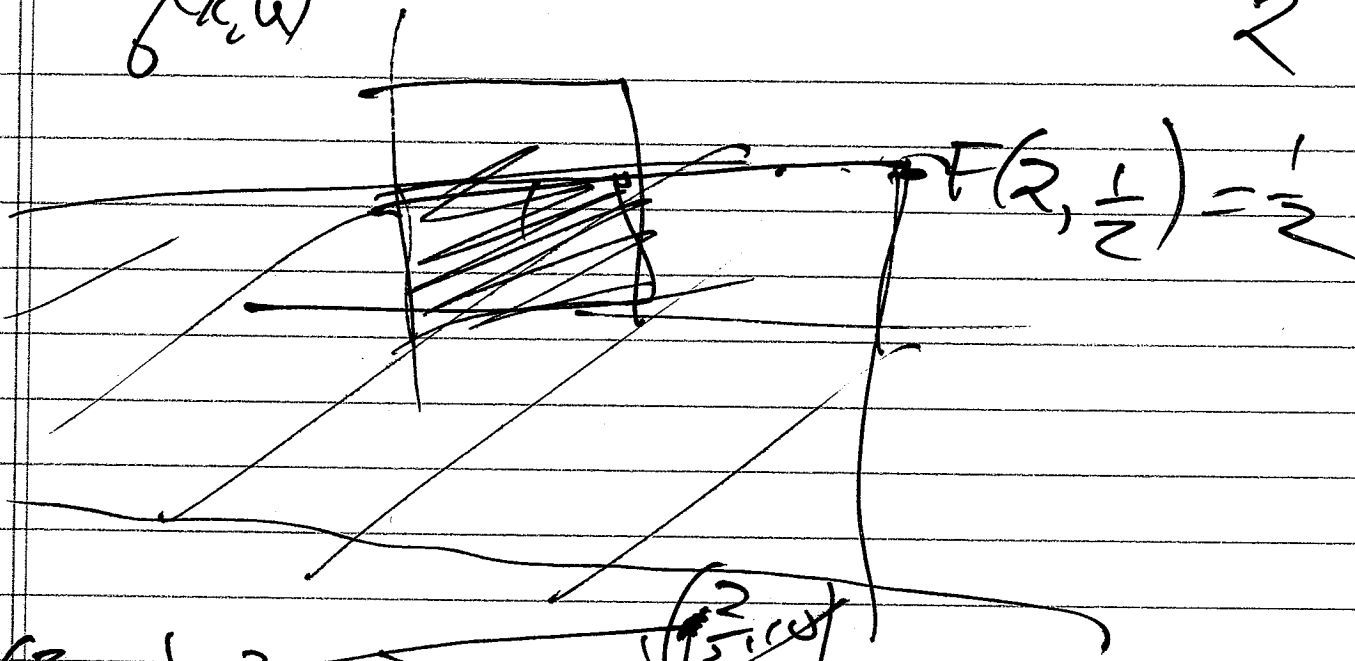
$$F(x,y) = \begin{cases} 0 & \text{if } x < 0 \text{ or } y < 0 & \textcircled{1} \\ xy & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 & \textcircled{2} \\ y & \text{if } x > 1 \text{ and } 0 \leq y \leq 1 & \textcircled{3} \\ x & \text{if } 0 \leq x \leq 1 \text{ and } y > 1 & \textcircled{4} \\ 1 & \text{if } x > 1 \text{ and } y > 1 & \textcircled{5} \end{cases}$$



$f(x, y)$

0

2

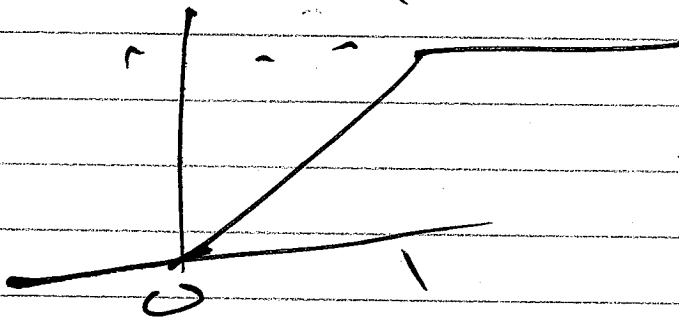


$$F\left(\frac{2}{3}, 0\right) = \frac{2}{3}$$

$$F\left(\frac{2}{3}, \frac{1}{2}\right)$$

$-\infty$

$$F(x, \infty) = \begin{cases} 0 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$



EX 5.12

$$F_{X,Y}(x,y) = \begin{cases} (1-e^{-\alpha x})(1-e^{-\beta y}) \\ 0 \end{cases}$$

if $x \geq 0, y \geq 0$

$$F_X(x) = F_{X,Y}(x, \infty) = 1 - e^{-\alpha x}$$

$$F_Y(y) = F_{X,Y}(\infty, y) = 1 - e^{-\beta y}$$

TESTS $F_X(\infty) = 1$ ✓

$F_Y(\infty) = 1$ ✓

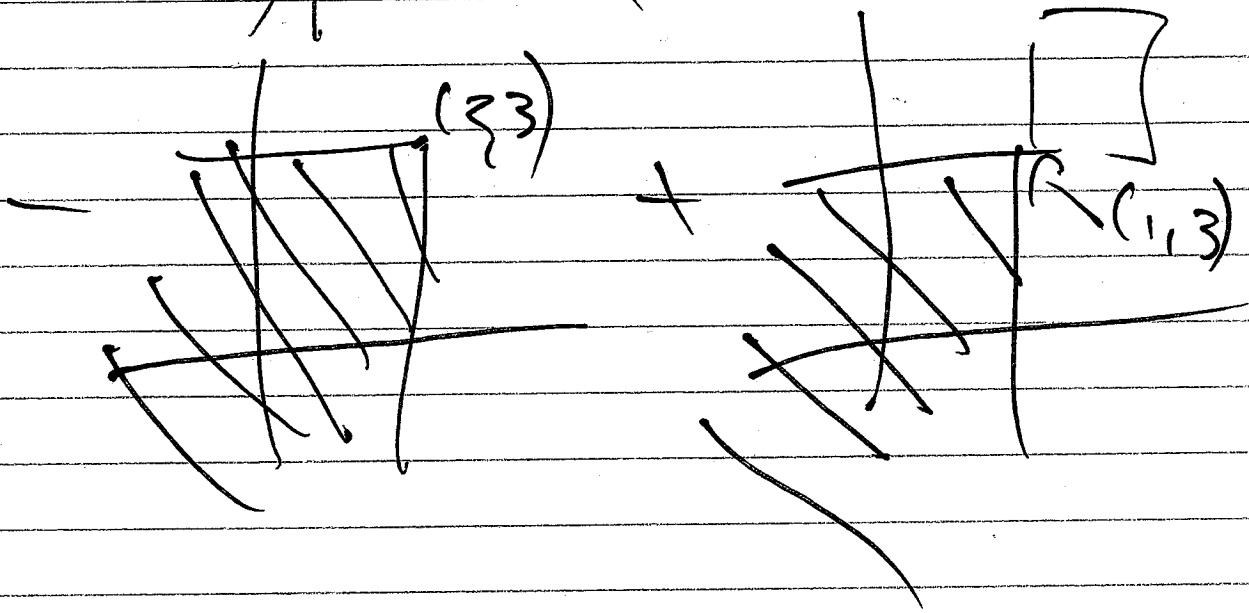
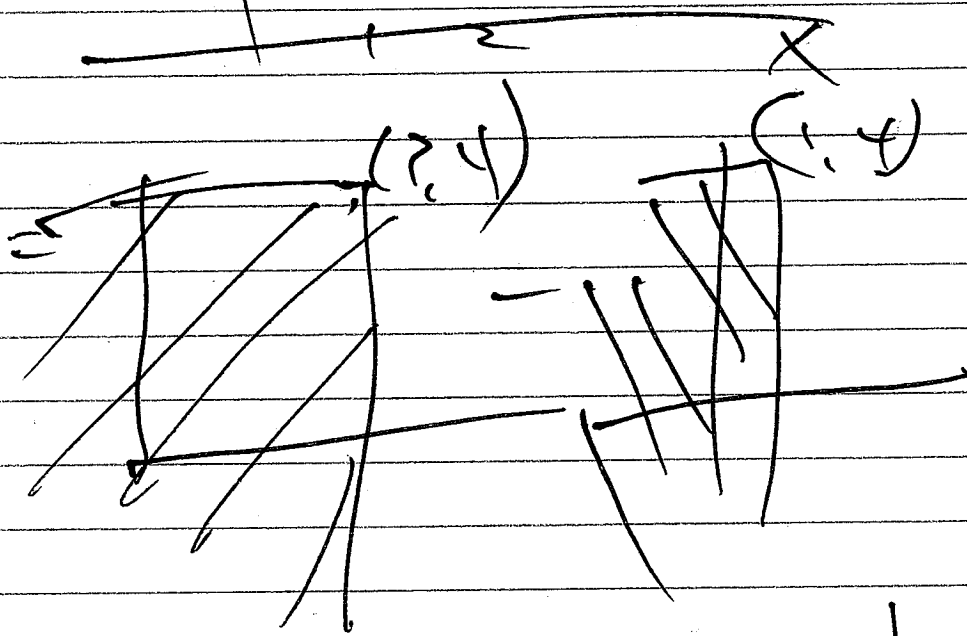
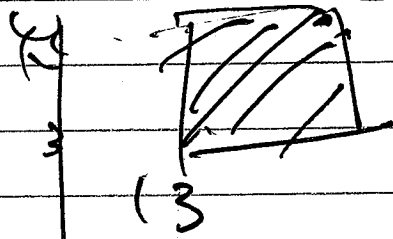
$F_{X,Y}(x)$ INCREASING —

EX 5.13

p. 247

$$P[X \leq 1 \wedge Y \leq 1] = F_{X,Y}(1,1) = (1-e^{-\alpha})(1-e^{-\beta})$$

$$p \left[\begin{matrix} 1 \leq x \leq 2 & \Delta & 3 \leq y \leq 4 \\ & & \text{ef} \end{matrix} \right] \quad 4$$



$$F_{X,Y}(x,y) = (1 - e^{-\alpha x})(1 - e^{-\beta y})$$

LE (5) 14, 22, 47

X TRANS

$$P(X = -1) = P(X = 1) = \frac{1}{2}$$

N NOISE $\in U[-2, 2]$

$$f_N(v) = \begin{cases} \frac{1}{4} & \text{if } -2 \leq v \leq 2 \\ 0 & \text{else} \end{cases}$$

Y RCV

$$Y = X + N$$

$$P[X = 1 | Y \leq y] \quad ?$$

$$P[X = 1 | Y \leq y] = P[X = 1 | Y = y] P[Y \leq y]$$

$$Y = X + N$$

$$P[X = 1 | Y \leq y] = P[X = 1 | X + N = y] P[X + N \leq y]$$

~~$P[X=1 | Y \leq 4]$~~

~~$P[X=1 | X+N \leq 4]$~~

$P[X=1 \& Y \leq 4]$

$P[X=1 \& 1+N \leq 4]$

$= P[X=1 \& N \leq 4-1]$

$= \frac{1}{2} P[N \leq 4-1]$

$= \begin{cases} 0 & \text{if } 4-1 < -2 \\ \frac{1}{4} & \text{if } -2 \leq 4-1 \leq 2 \\ 0 & \text{if } 4-1 > 2 \end{cases}$

$= \begin{cases} 0 & \text{if } 4 \leq -1 \\ \frac{1}{4} & \text{if } -1 \leq 4 \leq 3 \\ 0 & \text{if } 4 > 3 \end{cases}$

$$P[X=1 | Y=3] ?$$

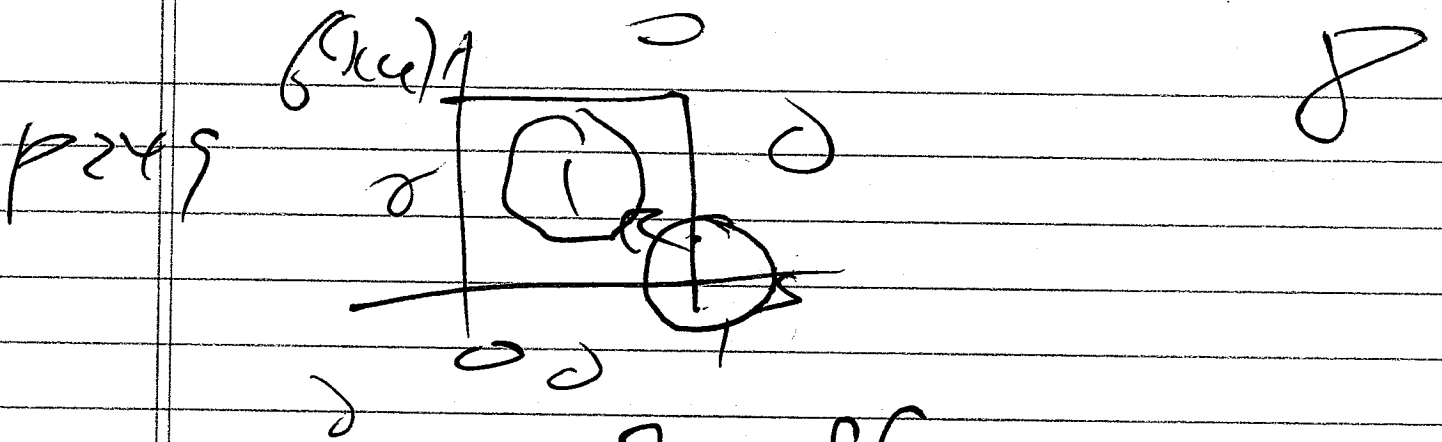
$$= P[X=1 \cap Y=3]$$

$$P[Y=3]$$

$$P[Y=3] = P[Y=3 \cap X=1] P[X=1]$$

$$+ P[Y=3 \cap X=0] P[X=0]$$

⋮



$$P[(x, y) \in R] = \iint_R f(x, y) dx dy$$

8. R CIRCLE CENTER $(\frac{1}{2}, \frac{1}{2})$

RADIUS $\frac{1}{4}$

$$P[(x, y) \in R] = \iint_R 1 dx dy$$

$$= \text{AREA OF CIRCLE} = \frac{\pi}{16}$$

9. S CIRCLE CENTER $(1, 0)$

RADIUS $\frac{1}{4}$

$$P[(x, y) \in S] = \iint_S f(x, y) dx dy$$

$$= \frac{\pi}{64}$$

$$F(x, y) = \begin{cases} xy \\ x \\ 1 \\ 0 \end{cases}$$

9

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y} = \begin{cases} 1 & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

EX 5.15 P 251

$$P[X+Y=1]?$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy$$

$$x, y = -\infty$$

$$= \frac{1}{2}$$

