C8 2/10/20-1

BERNOULLI

$S_X = \{0,1\}$

$P(1) = p \quad P(0) = 1-p = q$

$E(X) = \sum 1 \cdot p + 0 \cdot (1-p) = p$

$VAR(X) = \underbrace{E(X^2)}_{p} - \underbrace{(E(X))^2}_{p^2}$

$= p - p^2 = pq$

BINOMIAL

$P(k) = \binom{n}{k} p^k q^{n-k}$

$E[X] = np$

$E[X^2] = \sum_{k=0}^{n} k^2 \binom{n}{k} p^k q^{n-k}$

$= \sum \frac{k^2 n!}{k!(n-k)!} p^k q^{n-k}$

C8 - 2

$$\sum_{k=1}^{n} \frac{k \, n!}{(k-1)!\,(n-k)!} p^k q^{n-k}$$

LET $j+1 = k$, $j = k-1$

$$\sum_{j+1=1}^{n} - - - - \qquad n-(k+1)$$

$$\sum_{j=0}^{n-1} (j+1) \frac{n!}{j!\,(n-j-1)!} p^{j+1} q^{n-j-1}$$

$$= np \sum_{j=0}^{n-1} (j+1) \frac{(n-1)!}{j!\,(n-j-1)!} p^j q^{n-j-1}$$

$$= np \left[\sum_{j=0}^{n-1} j \binom{n-1}{j} p^j q^{n-j} + \sum_{j=0}^{n-1} \binom{n-1}{j} p^j q^{n-j-1} \right.$$

$$= np \left[\quad p(n-1) \quad + \quad 1 \quad \right]$$

$$E[X^2] = np(np - p + 1)$$

$$E[X] = np$$

$$VAR[X] = E[X^2] - E[X]^2$$

$$= np(np - p + 1) - (np)^2$$

$$= npq$$

$$STD[X] = \sqrt{npq}$$

e.g. FAIR COIN 100 TOSSES

$n = 100 \quad p = \frac{1}{2} = q$

$STD[T] = 5$

$E[X] = 50$

2/3 OF TIME HEADS IN 45 → 55

AS p GETS FARTHER FROM $\frac{1}{2}$

$VAR[X]$ SHRINKS.

POISSON

$$P(K) = \frac{\alpha^K}{K!} e^{-\alpha}$$

e.g. $\alpha = 1$ $\qquad \frac{1}{eK!}$

$$p(0) = \frac{1}{e}$$

$$\cancel{E[x] =} \sum p(k) = \sum \frac{\alpha^k}{k!} e^{-\alpha}$$

$$= e^{-\alpha} \underbrace{\sum_{k=0}^{\infty} \frac{\alpha^k}{k!}}_{e^{\alpha}}$$

$$= 1$$

$$E[x] = \sum_{k=0}^{\infty} k \frac{\alpha^k}{k!} e^{-\alpha} = \alpha \sum_{k=1}^{\infty} k \frac{\alpha^{k-1}}{k!} e^{-\alpha}$$

$$= \alpha \sum_{k=1}^{\infty} \frac{\alpha^{k-1}}{(k-1)!} e^{-\alpha}$$

C8-5

$v = k-1$

$$E[X] = \alpha e^{-\alpha} \underbrace{\sum_{v=0}^{\infty} \frac{\alpha^{v}}{v!}}_{e^{\alpha}}$$

$$E[X] = \alpha$$

SHOW POISSON APPROX THE BINOMIAL WHEN n BIG BUT np SMALL

$$p(k) = \binom{n}{k} p^{k} q^{n-k} \qquad np = \alpha$$

$$\underbrace{n(n-1)(n-2)\cdots(n-k+1)}_{\approx n^{k}} / k!$$

$$\approx n^{k} p^{k} (1-p)^{n-k}$$

$$= \frac{\alpha^{k}}{k!} (1-p)^{n-k}$$

USE $(1-x)^{\cancel{xa}\ a/x} \to e^{-a}$ AS $x \to 0$

CP-6

$$(1-p)^{n-k} \approx e^{-p(n-k)}$$

$$= e^{-\alpha + \cancel{pk}}$$

$$p(k) \approx \frac{\alpha^k}{k!} e^{-\alpha}$$

CH 4

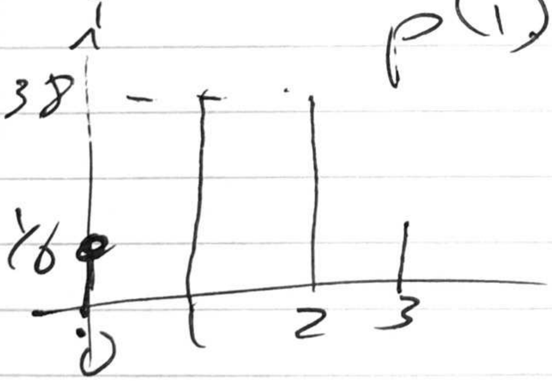
EX 4.1 p142

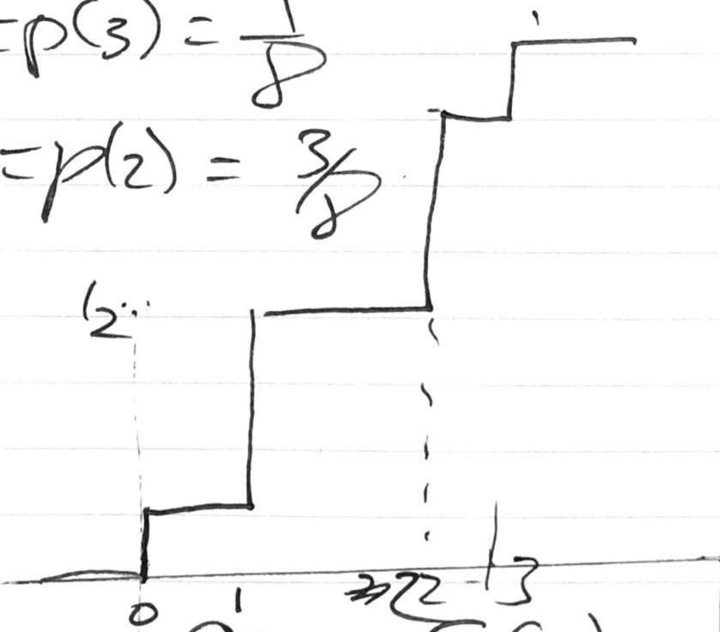
3 COIN TOSSES

$p(0) = p(3) = \frac{1}{8}$

$p(1) = p(2) = 3/8$

PMF





CDF $F(x)$

$P(X \leq 2) = \frac{7}{8}$

$P(X \leq -\frac{1}{2}) = 0$

$P(X \leq 123.4) = 1$

C8-7

NEW: CONTINUOUS R.V.

UNIFORM $U[0,1]$

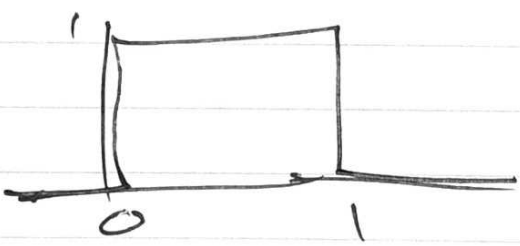
$$f(x) = \begin{cases} 0 & x \le 0 \\ 1 & 0 < x < 1 \\ 0 & x \ge 1 \end{cases}$$

$$P(a \le x \le b) = b - a$$

$$0 \le a \le b \le 1$$

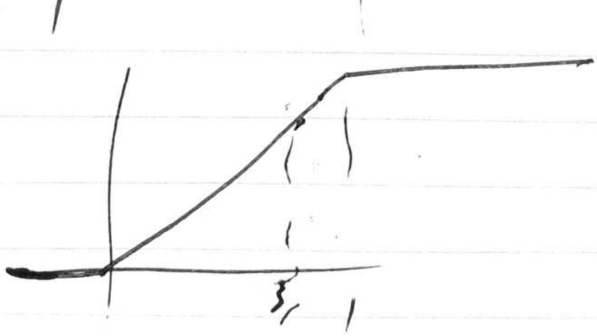
pdf

$f(x) =$



$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

CDF $F(x) =$



$$= \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$

C8-8

$p(x \le 3/4) = 3/4$

$p(x \le 5) = 1$

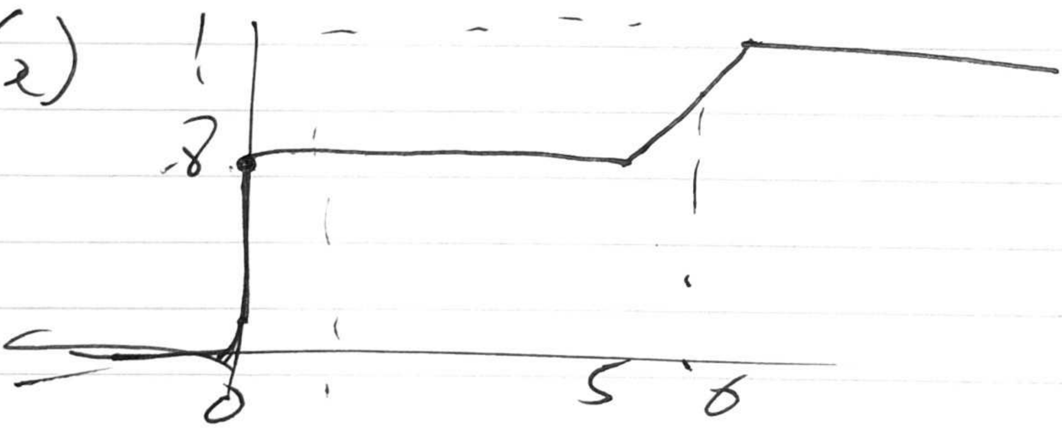
$p(x \le -2) = 0$

TAXI 80% - NO WAIT

20% UNIFORM IN 5→6 MIN.

MIX OF DISCRETE + CONTIN

CDF $F(x)$



$P(X \le 5\frac{1}{2}) = .9$

$P(X \le 10) = 1$

$P(X \le 2) = .8$

$\cancel{P(X}$