C7 2/6/20 - 1

MULTINOMIAL

$n=3$ $K_1=2$ $p_1=.5$ HEAD
$K_2=0$ $p_2=.4$ TAIL
$K_3=1$ $p_3=.1$ EDGE

$\binom{3}{2\,0\,1} \cdot .5^2 \cdot .4^0 \cdot .1^1$

$\frac{3!}{2!1!1!} \cdot .125 \times .1 \ldots$

$\binom{n}{K_1 K_2 \cdots K} p_1^{K_1} p_2^{K_2} \cdots$

$\frac{n!}{K_1! K_2! \cdots}$

GEOMETRIC

$p = P[\text{EDGE}] = .1 \quad q = 1-p = .9$

$P[1^{\text{st}} \text{ EDGE on } K^{\text{th}} \text{ toss}] = q^{k-1} p$

$q^2 \cdot .1 = .081$

E[BINOMIAL]

$$p(k) = \binom{n}{k} p^k q^{n-k}$$

$$\frac{n!}{k!(n-k)!}$$

$$E[p] = \sum_{k=0}^{n} k\, p(k)$$

$$= \sum_{k=0}^{n} \frac{k\, n!}{k!(n-k)!} p^k q^{n-k}$$

$k=0$ / $k=1$

$$\frac{k!}{k} = (k-1)!$$

$$= pn \sum_{k=1}^{n} \frac{1}{(k-1)!} \frac{(n-1)!}{(n-k)!} p^{k-1} q^{n-k}$$

$$= np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} q^{n-k}$$

$$= np \sum_{k=0}^{n-1} \binom{n-1}{k} p^{k} q^{n-k-1}$$

$$= 1$$

$$= np$$

$\sum p(k) = 1$?

$(p+q)^n = p^n + np^{n-1}q + \cdots$

$= \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k}$

$p+q=1 \rightarrow 1^n = 1$

$\sum p(k) = 1$

FUNCTION OF R.V.

1Ω RESISTOR $P = V^2$

9.6 $S_V = \{1, 2, 3\}$ $P_1 = P_2 = P_3 = \frac{1}{3}$

$S_P = \{1, 4, 9\}$ $P_1 = P_4 = P_9 = \frac{1}{3}$

$E[P] = \frac{1}{3}\cdot 1 + \frac{1}{3}\cdot 4 + \frac{1}{3}\cdot 9 = \frac{14}{3}$

POWER

$E[V] = \frac{1}{3} + \frac{2}{3} + \frac{3}{3} = 2$

$E[P] \neq E[V]^2$

$$VAR[x] = E[(x - m_x)^2]$$

$S_x = \{1, 2, 3\}$ $\quad p = 1/3$

$$E[x] = \frac{1}{3}\cdot 1 + \frac{1}{3}\cdot 2 + \frac{1}{3}\cdot 3 = 2$$

$$VAR[x] = E[(x-2)^2]$$

$x - 2$: -1, 0, 1 $\quad (x-2)^2$: 1, 0, 1 $\quad p = 1/3$

$$VAR[x] = \frac{1}{3}\cdot 1 + \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot 1 = \frac{2}{3}$$

$$STD[x] = \sqrt{VAR} = \sqrt{\frac{2}{3}} \approx 2.8$$

x : 1, 2, 3 $\quad x^2$: 1, 4, 9

$$E[x^2] = \frac{1}{3} + \frac{4}{3} + \frac{9}{3} = \frac{14}{3}$$

$E[x] = 2 \quad E[x]^2 = 4$

$$VAR[x] = E[x^2] - E[x]^2$$

$$= \frac{14}{3} - 4 = \frac{2}{3}$$

$X = \{1, 2, 3\}$ $p = 1/3$ $E[X] = 2$

$Y = X + 1$

$VAR[X] = E[X^2] - E[X]^2$

$= \frac{14}{3} - 4 = \frac{2}{3}$

$Y = \{2, 3, 4\}$ $p = 1/3$

$E[Y] = \frac{2+3+4}{3} = 3 = E[X] + 1$

$E[Y^2] = \frac{4+9+16}{3} = \frac{29}{3}$

$VAR[Y] = \frac{29}{3} - 9 = \frac{2}{3}$

$Z = 2X$

$Z: \{2, 4, 6\}$

$E[Z] = \frac{2+4+6}{3} = 4$

$E[Z^2] = \frac{4+16+36}{3} = \frac{56}{3}$

$VAR[Z] = \frac{56}{3} - 16 = \frac{56-48}{3} = \frac{8}{3} = 4\,VAR[X]$

$Y = aX + b$

$E[Y] = aE[X] + b$

$VAR[Y] = a^2 VAR[X]$

BINOMIAL

X: # HEADS IN N TOSSES

$P[X=k] = \binom{n}{k} p^k q^{n-k}$

$E[X] = np$

$VAR[X] = npq$

$\sigma_X = STD[X] = \sqrt{npq} = \frac{\sqrt{n}}{2}$ IF $p = 1/2$

APPROX $P[\mu_X - \sigma_X < X < \mu_X + \sigma_X] = 2/3$

$n = 100$ $\mu_X = 50$ $\sigma_X = 5$

$P[45 \le X \le 55] = 2/3$

$n = 10000$ $\mu = 5000$ $\sigma = 50$

$P[4950 < X < 5050] = 2/3$

C7 - 7

TOSS COIN 100 TIMES

#HEADS IS WITHIN 5% OF MEAN

2/3 OF TIME.

TOSS COIN 10,000 TIMES

WITHIN .05% OF MEAN

2/3 OF TIME.

B/C N → TIGHTER CLUSTER.