4.38

a) $F_Y(x) = F_Y(x|B_0)P[B_0] + F_Y(x|B_1)P[B_1]$

$= P[Y \le x \mid X=-1](1-p) + P[Y \le x \mid X=1]p$

$= P[X+N \le x \mid X=-1](1-p) + P[X+N \le x \mid X=1]p$

$= P[N \le x+1](1-p) + P[N \le x-1]p$

$= F_N(x+1)(1-p) + F_N(x-1)p$

$f_Y(x) = \frac{d}{dx} F_Y(x)$

$= (1-p) f_N(x+1) + p f_N(x-1)$

$f_Y(x|B_0) = f_N(x+1) = \frac{\alpha}{2} e^{-\alpha|x+1|}$

$f_Y(x|B_1) = f_N(x-1) = \frac{\alpha}{2} e^{-\alpha|x-1|}$

$f_Y(x) = \frac{1}{2}\left[\frac{\alpha}{2} e^{-\alpha|x+1|} + \frac{\alpha}{2} e^{-\alpha|x-1|}\right] = \frac{1}{4}\alpha\left[e^{-\alpha|x+1|} + e^{-\alpha|x-1|}\right]$

b) $P[Y<0 \mid B_1] = P[X+N<0 \mid X=1] = P[N<-1]$

$= \frac{\alpha}{2} e^{-\alpha|-1|} = \frac{\alpha}{2} e^{-\alpha}$

$P[Y \ge 0 \mid B_0] = P[X+N \ge 0 \mid X=-1] = P[N \ge 1]$

$= \frac{\alpha}{2} e^{-\alpha}$

c) $P_E = P[Y<0 \mid B_1]P[B_1] + P[Y \ge 0 \mid B_0]P[B_0]$

$= 0.5 \frac{\alpha}{2} e^{-\alpha} + 0.5 \quad \frac{\alpha}{2} e^{-\alpha} \quad = \frac{\alpha}{2} e^{-\alpha}$

(4.67)

a) $F_Y(X+N \le y \mid X=+1) = F_N(y-1)$

$F_Y(X+N \le y \mid X=-1) = F_N(y+1)$

$$f_Y(y \mid X=+1) = f_N(y-1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-1)^2/2\sigma^2}$$

$$f_Y(y \mid X=-1) = f_N(y+1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y+1)^2/2\sigma^2}$$

b) $f_Y(y \mid X=-1)\, P[X=-1] > f_Y(y \mid X=+1)\, P[X=+1]$ decide "0"

$$\frac{e^{-(y+1)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}(3p_1) > \frac{e^{-(y-1)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\, p_1$$

$$3e^{-(y+1)^2/2\sigma^2}\, e^{(y-1)^2/2\sigma^2} > 1$$

$$3e^{-y^2-2y-1+y^2-2y+1/2\sigma^2} > 1$$

$$3e^{-4y/2\sigma^2} > 1$$

$$\frac{-4y}{2\sigma^2} > \ln\left(\frac{1}{3}\right)$$

$y < -\frac{\sigma^2}{2}\ln\left(\frac{1}{3}\right)$ decide "0" $\quad T = -\frac{\sigma^2}{2}\ln\left(\frac{1}{3}\right)$

c) $P[X+N<T \mid X=+1] = P[N<T-1] = \Phi\left(\frac{T-1}{\sigma^2}\right)$

$P[X+N \geq T \mid X=-1] = P[N \geq T+1] = \left(1-\Phi\left(\frac{T+1}{\sigma^2}\right)\right)$

d) $P[X+N<T \mid X=+1]\, P[X=+1] + P[X+N \geq T \mid X=-1]\, P[X=-1] =$

$$= p_1\, \Phi\left(\frac{T-1}{\sigma^2}\right) + \left(1-\Phi\left(\frac{T+1}{\sigma^2}\right)\right) 3p_1$$

4.68

$$P[X_1 > x] = Q\left(\frac{x-20}{5}\right)$$

$$P[X_2 > x] = Q\left(\frac{x-22}{1}\right)$$

$$P[X_1 > 20] = Q\left(\frac{20-20}{5}\right) = Q(0) = \frac{1}{2}$$

$$P[X_2 > 20] = Q\left(\frac{20-22}{1}\right) = Q(-2) = 1 - Q(2) = 0.9722$$

} pick #2

$$P[X_1 > 24] = Q\left(\frac{24-20}{5}\right) = Q(0.8) = 0.212$$

$$P[X_2 > 24] = Q\left(\frac{24-22}{1}\right) = Q(2) = 0.023$$

} pick #1

4.69

$$P[X > 10] = 1 - P[X \leq 10]$$

$$= \sum_{k=0}^{7-1} \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

$$= \sum_{k=0}^{6} \frac{10^k}{k!} e^{-10}$$

$$= 0.1301$$

4.85 X : Gaussian, $Y = aX + b$, a linear combination of X.
Y is also Gaussian

$$E[Y] = aE[X] + b = am + b = m'$$

$$Var[Y] = a^2 Var[X] = a^2\sigma^2 = \sigma'^2$$

$$a = \sigma'/\sigma, \ \ b = m' - am = m' - m\sigma'/\sigma$$

4.90

$$X = \pm\sqrt{\frac{P}{R}} \qquad \frac{dx}{dp} = \pm\frac{1}{2}\frac{P^{\frac{1}{2}-1}}{\sqrt{R}} = \pm\frac{1}{2\sqrt{RP}}$$

$$f_P(p) = [f_X(x) + f_X(-x)]\left|\frac{dx}{dP}\right|$$

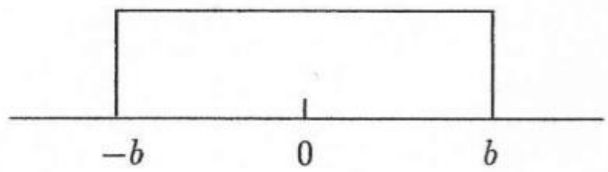
$$= \left[f_X\left(\sqrt{\frac{P}{R}}\right) + f_X\left(-\sqrt{\frac{P}{R}}\right)\right]\frac{\cdot 1}{2\sqrt{RP}}$$

$$= \left[\frac{1}{2\sqrt{2\pi}}\left(e^{-\left(\sqrt{\frac{P}{R}}-1\right)^2/2(2)} + e^{-\left(-\sqrt{\frac{P}{R}}-1\right)^2/2(2)}\right)\right]\frac{1}{2\sqrt{RP}}$$

$$= \frac{1}{2\sqrt{2\pi}}\frac{1}{2\sqrt{RP}}\left(e^{-(\sqrt{P}-\sqrt{R})^2/2(2R)} + e^{-(\sqrt{P}-(-\sqrt{R}))^2/2(2R)}\right)$$

4.99

81 **a)** For a uniform random variable in $[-b, b]$ we have



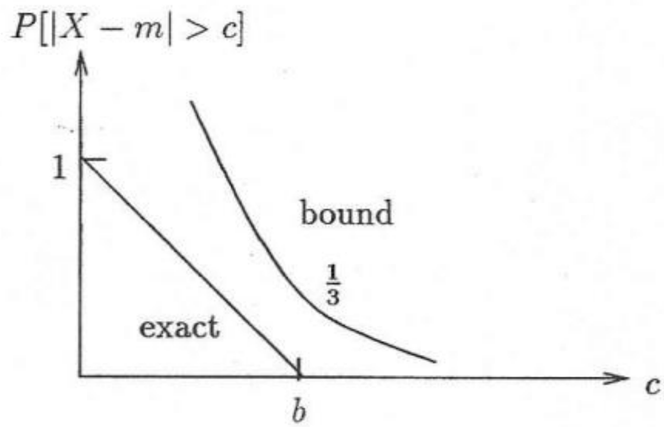
$$\mathcal{E}[X] = 0$$

Exact:

$$P[|X-m|>c] = \begin{cases} 1-\frac{c}{b} & 0 \le c \le b \\ 0 & c > b \end{cases}$$

Chebyshev Bound gives

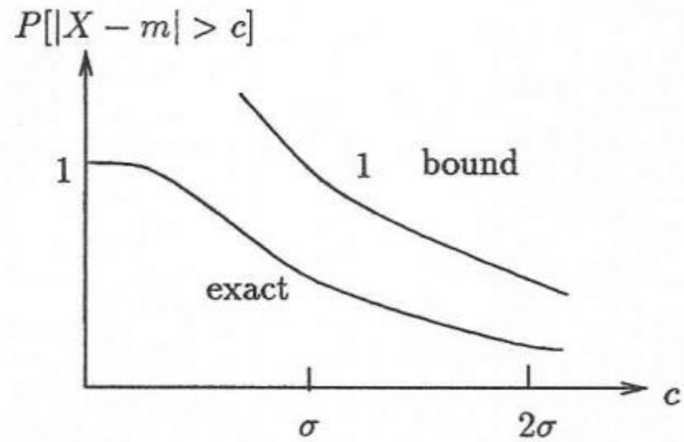
$$P[|X-m|>c] \le \frac{\sigma_X^2}{c^2} = \frac{b^2}{3c^2}$$



c) For the Gaussian random variable $\mathcal{E}[X] = 0$ and $VAR[X] = \sigma^2$

Exact: $P[|X - m| > c] = 2Q\left(\frac{c}{\sigma}\right)$

Bound: $P[|X - m| > c] \leq \frac{\sigma^2}{c^2}$



4.126 $R(t) = P[T > t]$

$$= \sum_{n=0}^{m-1} e^{-\lambda t} \frac{(\lambda t)^n}{n!} \qquad \text{Erlang.}$$

b) $r(t) = \dfrac{-f_T(t)}{R(t)}$

$$= \frac{-\lambda^m t^{m-1} e^{-\lambda t}}{(m-1)!} \cdot \frac{1}{\sum_{n=0}^{m-1} e^{-\lambda t} \frac{(\lambda t)^n}{n!}}$$

$$= \frac{-\lambda (\lambda t)^{m-1} / (m-1)!}{\sum_{n=0}^{m-1} (\lambda t)^n / n!}$$