3.90 We want to find n so that the nth arrival is after more than 2 minutes 10% of the time:

$$P[N(2) \leq n] = 0.90 = \sum_{k=0}^{n} \frac{2^k}{k!} e^{-2}$$

By trial and error we find $n = 5$.

3.91

58 a)

$$\begin{aligned}
& P[\text{signal present} | X = k] \\
= & \frac{P[\text{signal present}, X = k]}{P[X = k | \text{signal present}] P[\text{present}] + P[X = k | \text{signal absent}] P[\text{absent}]} \\
= & \frac{\frac{\lambda_1^k}{k!} e^{-\lambda_1} p}{\frac{\lambda_1^k}{k!} e^{-\lambda_1} p + \frac{\lambda_0^k}{k!} e^{-\lambda_0} (1-p)} \\
= & \frac{\lambda_1^k e^{-\lambda_1} p}{\lambda_1^k e^{-\lambda_1} p + \lambda_0^k e^{-\lambda_0} (1-p)}
\end{aligned}$$

Similarly,

$$P[\text{signal absent} | X = k] = \frac{\lambda_0^k e^{-\lambda_0} (1-p)}{\lambda_1^k e^{-\lambda_1} p + \lambda_0^k e^{-\lambda_0} (1-p)}$$

b) Decide signal present if P[signal present|X=k] $>$ P[signal absent|X=k], i.e.,

$$\lambda_1^k e^{-\lambda_1} p > \lambda_0^k e^{-\lambda_0}(1-p)$$

$$\left(\frac{\lambda_1}{\lambda_0}\right)^k > \frac{1-p}{p} e^{\lambda_1-\lambda_0} \qquad (\lambda_1 > \lambda_0)$$

$$k > \frac{ln\frac{1-p}{p} + \lambda_1 - \lambda_0}{ln\lambda_1 - ln\lambda_0}$$

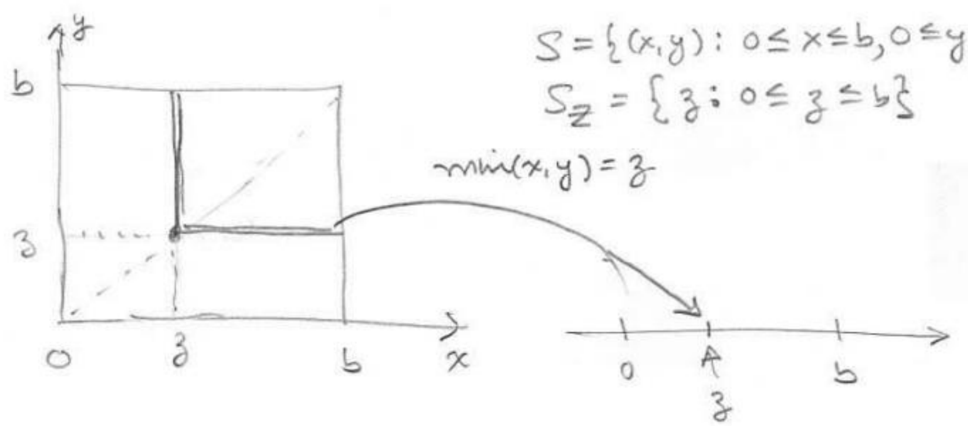
The threshold T is

$$T = \frac{ln\frac{1-p}{p} + \lambda_1 - \lambda_0}{ln\lambda_1 - ln\lambda_0}$$

c)

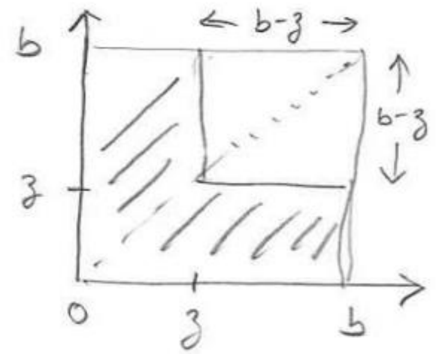
$$\begin{aligned} P_e &= P[X < T|\text{signal present}]P[\text{present}] + P[X > T|\text{signal absent}]P[\text{absent}] \\ &= p\sum_{k=o}^{\lfloor T \rfloor} \frac{e^{-\lambda_1}\lambda_1^k}{k!} + (1-p)\sum_{k=\lceil T \rceil}^{\infty} \frac{e^{-\lambda_0}\lambda_0^k}{k!} \end{aligned}$$

4.7

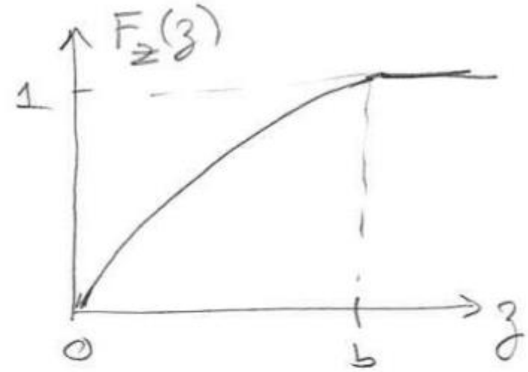


$S = \{(x,y) : 0 \le x \le b, 0 \le y \le b\}$

$S_Z = \{z : 0 \le z \le b\}$



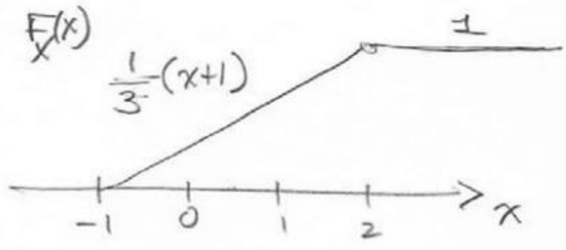
$$P[Z \le z] = 1 - \left(\frac{b-z}{b}\right)^2$$
$$= 1 - \left(1 - 2\frac{z}{b} + \frac{z^2}{b^2}\right)$$
$$= \frac{2z}{b} - \frac{z^2}{b^2}$$
$$= \frac{z}{b}\left(2 - \frac{z}{b}\right)$$



$$P[Z > 0] = 1$$
$$P[Z > b] = 0$$
$$P\left[Z \le \frac{b}{2}\right] = F_Z\left(\frac{b}{2}\right)$$
$$= \frac{1}{2}\left(2 - \frac{1}{2}\right) = \frac{3}{4}$$

$$P\left[Z > \frac{b}{4}\right] = 1 - F_Z\left(\frac{b}{4}\right)$$
$$= 1 - \frac{1}{4}\left(2 - \frac{1}{4}\right)$$
$$= \frac{9}{16}$$

4.11



$$P[X<0] = F_X(0) = \frac{1}{3}$$

$$P\left[\left|X-\tfrac{1}{2}\right|<1\right] = P\left[-1<X-\tfrac{1}{2}<1\right] = P\left[-\tfrac{1}{2}<X<\tfrac{3}{2}\right]$$

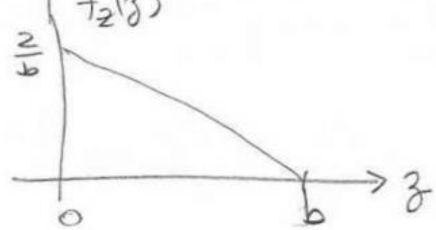
$$= \frac{1}{3}\left(\frac{3}{2}+1\right) - \frac{1}{3}\left(-\frac{1}{2}+1\right)$$

$$= \frac{1}{3}\left(\frac{3}{2}+1+\frac{1}{2}-1\right) = \frac{2}{3}$$

$$P\left[X>-\tfrac{1}{2}\right] = 1-P\left[X\le -\tfrac{1}{2}\right] = 1-\frac{1}{3}\left(-\frac{1}{2}+1\right) = \frac{5}{6}$$

4.20 $F_Z(z) = 2\frac{z}{b} - \frac{z^2}{b^2}$ from prob. 4.7

a) $f_Z(z) = \frac{d}{dz}F_Z(z) = \frac{2}{b} - \frac{2z}{b^2} = \frac{2}{b}\left(1 - \frac{z}{b}\right)$



b) $P\left[Z > \frac{b}{3}\right] = \int_{b/3}^{b} \frac{2}{b}\left(1 - \frac{z}{b}\right)dz = \frac{2}{b}\left[z \Big|_{b/3}^{b} - \frac{1}{b}\frac{z^2}{2}\Big|_{b/3}^{b}\right]$

$= \frac{2}{b}\left(b - \frac{b}{3}\right) - \frac{2}{2b^2}\left[b^2 - \frac{b^2}{9}\right]$

$= \frac{4}{3} - \frac{8}{9} = \frac{4}{9}$