1. (6 pts) A die is tossed twice and the number of dots facing up in each toss is counted and noted in the order of occurrence.

(a) Find the sample space.

(b) Find the set A corresponding to the event \number of dots in rst toss is not less than number of dots in second toss."

(c) Find the set B corresponding to the event \number of dots in rst toss is 6."

(d) Does A imply B or does B imply A?

(e) Find A $\cap B^c$ and describe this event in words.

(f) Let C correspond to the event \number of dots in dice differs by 2." Find A ∩ C.

Answer :

(a) S = {(x, y|1 $\le$x$\le$6,1$\le$y$\le$6,x,y ∈ ℤ}={(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}.

(b) A = {(x; y)|1$\le$y$\le$x$\le$6, x,y ∈ ℤ}={(1,1), (2,1), (2,2), (3,1), (3,2), (3,3) ,(4,1), (4,2), (4,3), (4,4), (5,1), (5,2), (5,3), (5,4), (5,5), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}.

(c) B ={(6,x)|1$\le$x$\le$6,x ∈ ℤ}={(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}.

(d) B implies A. Because B ⊆ A and A ⊈ B.

(e) A$\cap B^c$ ={ (1,1), (2,1), (2,2), (3,1), (3,2), (3,3) ,(4,1), (4,2), (4,3), (4,4), (5,1), (5,2), (5,3), (5,4), (5,5)}.

The event is that the number of dots in the first toss is not less than the number of dots in the second toss. The first toss is not 6.

(f) A∩C= {(3,1), (4,2), (5,3), (6,4)}.

2. (6 pts) A binary communication system transmits a signal X that is either a +2 voltage signal or a -2 voltage signal. A malicious channel reduces the magnitude of the received signal by the number of heads it counts in two tosses of a coin. Let Y be the resulting signal.

(a) Find the sample space.

(b) Find the set of outcomes corresponding to the event "transmitted signal was denitely +2."

(c) Describe in words the event corresponding to the outcome Y = 0.

Answer:

Version 1

(a)$S = \{+2, +1, 0, -1, -2, -3, -4\}$

(b) $S = \{+2, +1, 0\}$

(c) The transmitted signal is +2 and both of coins are heads.

Version 2

(a)$S = \{+2, +1, 0, -1, -2\}$

(b) $S = \{+2, +1\}$

(c) The transmitted signal is +2 or -2, then the number of heads is 2, reducing the magnitude to 0.

3. (6 pts) Three friends (Al, Bob, and Chris) put their names in a hat and each draws a name from the hat. (Assume Al picks rst, then Bob, then Chris.)
(a) Find the sample space.
(b) Find the sets A, B, and C that correspond to the events “Al draws his name," “Bob draws his name," and “Chris draws his name."
(c) Find the set corresponding to the event, “no one draws his own name."
(d) Find the set corresponding to the event, “everyone draws his own name."
(e) Find the set corresponding to the event, “one or more draws his own name."

Answer :
(a) S_0 = {(Al, Bob, Chris), (Al, Chris, Bob), (Bob, Al, Chris), (Bob, Chris, Al), (Chris, Al, Bob), (Chris, Bob, Al)}
b) Set A={(Al, Bob, Chris), (Al, Chris, Bob)}
Set B={(Al, Bob, Chris), (Chris, Bob, Al)}
Set C={(Bob, Ali, Chris), (Al, Bob, Chris)}
(c) S_1 = {(Bob, Chris, Al), (Chris, Al, Bob)}
(d) S_2 = {(Al, Bob, Chris)}
(e) S_3 = {(Al, Bob, Chris), (Al, Chris, Bob), (Bob, Al, Chris), (Chris, Bob, Al)}

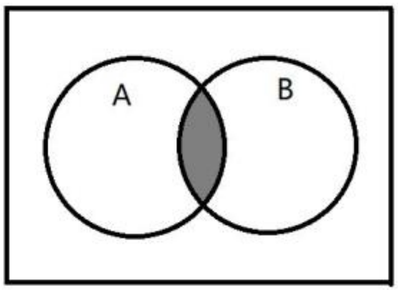
4. (6 pts) A die is tossed and the number of dots facing up is noted.
(a) Find the probability of the elementary events under the assumption that all faces of the die are equally likely to be facing up after a toss.
(b) Find the probability of the events: A = {more than 3 dots};B = {odd number of dots}.
(c) Find the probability of A∪B,A∩B, A^c.

Answer :
(a) P= $\frac{1}{6}$
(b) P[A] = $\frac{1}{2}$, P[B] = $\frac{1}{2}$.
(c) P[A∪B] = $\frac{5}{6}$
P[A∩B] = $\frac{1}{6}$
P[A^c] = $\frac{1}{2}$

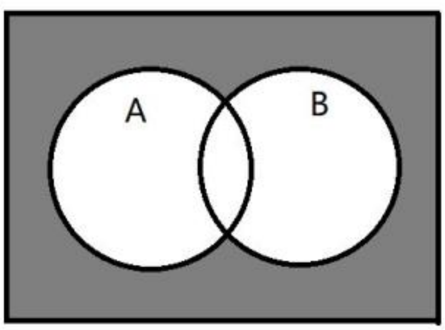
5. (6 pts) Let the events A and B have P[A] = x, P[B] = y and P[A ∪ B] = z. Use Venn diagrams to find P[A∩B], P[$A^c \cap B^c$], P[$A^c \cup B^c$], P[A ∩ B^c],P[A^c ∪ B].

<u>Answer :</u>

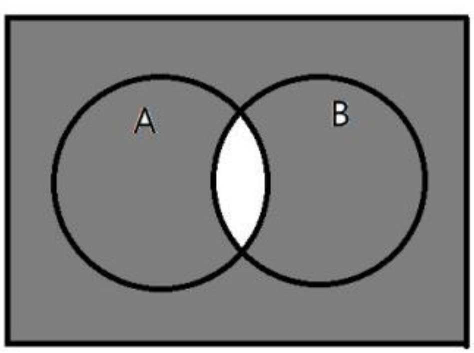
(a) P[A∩B]= x + y - z



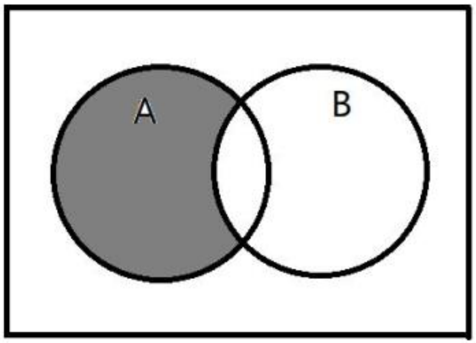
(b) P[$A^c \cap B^c$]= 1- z



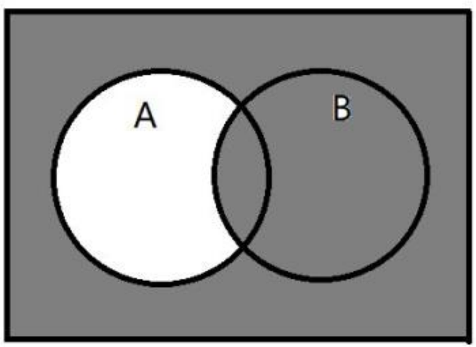
(c) P[$A^c \cup B^c$] =1 - x -y + z



(d) $P[A \cap B^c] = z - y$



(e) $P[A^c \cup B] = 1 - z + y$



7. (6 pts) The combination to a lock is given by three numbers from the set {0,1,...59}. Find the number of combinations possible. Ignore any mechanical limitations of combo locks. Good RPI students should know what those limitations are.
(Aside: A long time ago, RPI rekeyed the whole campus with a more secure lock. Shortly thereafter a memo was distributed that I would summarize as, "OK, you can, but don't you dare!")

Answer :
Number of combinations = 60 x 60 x 60 = 216000

9. (6 pts) Find a current policy issue where you think that probabilities are being misused, and say why, in 100 words. Full points will be awarded for a logical argument. I don't care what the issue is, or which side you take. Try not to pick something too too in ammatory; follow the Page 1 rule that an NSF lawyer taught me when I was there. (Would you be willing to see your answer on page 1 of tomorrow's paper?)

Answer :

Any reasonable answers will receive full credit.