8.1 $\quad \mu = 10 \quad \sigma_x^2 = 4 \quad n = 9$

ⓐ $P[\overline{X}_9 < 9] = P\left[\dfrac{\overline{X}_9 - \mu}{\sigma_x/\sqrt{n}} < \dfrac{9-\mu}{\sigma_x/\sqrt{n}}\right] = P\left[\dfrac{\overline{X}_9 - 10}{2/\sqrt{9}} < \dfrac{9-10}{2/3}\right]$

$= \cancel{1-}\, Q\left(-\frac{3}{2}\right) = 0.0668$

ⓑ $P[\min(X_1, \ldots X_9) > 8] = P[X_1 > 8]\, P[X_2 > 8] \ldots P[X_9 > 8]$

$= P[X_1 > 8]^9$

$= Q\left(\frac{8-10}{2}\right)^9 = Q(-1)^9$

$= 0.2112$

ⓒ $P[\max(X_1, \ldots, X_9) < 12] = P[X_1 < 12] \ldots P[X_9 < 12]$

$= \left(\cancel{1-}\, Q\left(\frac{12-10}{2}\right)\right)^9 = (1 - Q(1))^9$

$= 0.2112$

ⓓ $\overset{0.95}{=} P[|\overline{X}_n - 10| < 1] = P\left[\left|\dfrac{\overline{X}_n - 10}{2/\sqrt{n}}\right| < \dfrac{1}{2/\sqrt{n}}\right]$

$= P\left[-\dfrac{\sqrt{n}}{2} < \dfrac{\overline{X}_n - 10}{2/\sqrt{n}} < \dfrac{\sqrt{n}}{2}\right]$

$= P\left[-1.96 < \dfrac{\overline{X}_n - 10}{2\sqrt{n}} < 1.96\right]$

$\Rightarrow \sqrt{n} = 2(1.96) \qquad n = 4(1.96)^2 = 15.366 = 16$

(8.1) – continued –

Octave command to generate 100 samples of groups of 9

> normal_rnd (10, 4, 9, 100)

To find the sample mean of each group of 9:

> mean (normal_rnd (10, 4, 9, 100)

From sample of 100 we found:

$0.0[illegible] = \frac{[illegible]}{100}$ had values less than 9 vs. 0.0668 theory.

$0.19 = \frac{19}{100}$ had max of group < 12 vs. 0.2112

$0.18 = \frac{18}{100}$ had min of group > 8 vs. 0.2112

Max & MIN obtained using:

> max (normal_rnd (10, 4, 9, 100))

> min (normal_rnd (10, 4, 9, 100))

(8.2) X exponential $\mu = 50$ $n = 25$ $\sigma^2 = \frac{1}{\lambda^2} = \mu^2 = 50^2$

(a) $P\left[\,|\bar{X}_{25} - 50| < 1\,\right] = P\left[\,\left|\frac{\bar{X}_{25} - 50}{50/\sqrt{25}}\right| < \frac{1}{50/\sqrt{25}}\,\right]$

$= P\left[-\frac{1}{10} < \left|\frac{\bar{X}_{25} - 50}{10}\right| < \frac{1}{10}\right]$

$= 0.07966$

(b) $P[\max(X_1, \ldots, X_{25}) > 100] = 1 - P[\max(\ ) < 100]$

$= 1 - P[X_1 < 100]\,P[X_2 < 100] \ldots P[X_{25} < 100]$

$= 1 - (1 - e^{-100/50})^{25} = 1 - (1 - e^{-2})^{25}$

$= 0.9736$

(c) $P[\min(X_1 \ldots X_{25}) < 25] = 1 - P[\min(X_1 \ldots X_{25}) > 25]$

$= 1 - P[X_1 > 25]^{25} = 1 - (e^{-25/50})^{25}$

$= 1 - e^{-25/2} = 1 - 3.73 \times 10^{-6}$

(d) $0.90 = P\left[\,|\bar{X}_n - 50| < 5\,\right] = P\left[\,\left|\frac{\bar{X}_n - 50}{50/\sqrt{n}}\right| < \underbrace{\frac{5}{50/\sqrt{n}}}_{1.64}\,\right]$

$\frac{\sqrt{n}}{10} = 1.64$

$\sqrt{n} = 16.4$ $n = 269$

(e) Using approach in problem 8.1 (but generating exponential samples)

$0.08 = \frac{8}{100}$ samples were between 49 & 50 vs. 0.07966

$0.97 = \frac{97}{100}$ samples of max > 100 all samples of min < 25

8.49 $H_0 : \alpha = 30$ $\quad$ $n = 8$ measurements

$H_1 : \alpha > 30$ $\quad$ $\bar{X}_8 = 32 \Rightarrow \sum_{i=1}^{8} N_i = 256$

The experiments involves n measurements of a Poisson random variable. We take the sum of the total number of orders $N = \sum_{i=1}^{8} N_i$ (equivalent to taking the sample mean)

Accept H_0 if $N_T < T$ $\quad$ N Poisson with mean $n\alpha = 8\alpha$

Reject H_0 if $N_T \geq T$.

$$\alpha = 5\% = P[\text{Reject } H_0 \mid H_0] = P[N_T \geq T \mid H_0]$$

$$= \sum_{k=T}^{\infty} \frac{240^k}{k!} e^{-240} \qquad \bar{X}_8 = \frac{1}{8} N$$

$$\approx P\left[\frac{\bar{X}_8 - 30}{\sqrt{30}/\sqrt{8}} > \frac{T-30}{\sqrt{30}/\sqrt{8}} \right] = Q(1.64)$$

$$\Rightarrow T - 30 = \frac{1.64\sqrt{8}}{\sqrt{30}} + 30 = 30.847$$

$\bar{X}_8 = 32 > 30.847 \Rightarrow$ Reject H_0

$\alpha = 1\%$ $\quad$ $1\% = Q(2.326)$

$$\Rightarrow T = 30 + \frac{2.326\sqrt{8}}{\sqrt{30}} = 31.201$$

$\bar{X}_8 = 32 > 31.2 \Rightarrow$ Reject H_0

8.101

Any reasonable answers will receive full credit.