(c) $P_Y(j \mid X=k) = \dfrac{P[X=k \mid Y=j]P[Y=j]}{P[X=k]} = P_X(k \mid Y=j]$

Since $P[Y=j] = 1/3 = P[X=k]$

$\Rightarrow$ ML and MAP estimates the same

$\hat{Y}(-1) = 0$ or -1

$\hat{Y}(0) = 1$

$\hat{Y}(+1) = 0$ or 1

6.92 $X = X_1 + X_2 + \ldots + X_N$

$$\begin{aligned} m_a &= E[X] = Nm \\ \sigma_N^2 &= VAR[X] = N\ VAR[X_i] = N\sigma^2 \\ f_X(x) &= \frac{1}{\sqrt{2\pi}\sigma_N} \exp\left[-\frac{(x-m_a)^2}{2\sigma_N^2}\right] \end{aligned}$$

a)

$$\begin{aligned} p_{Loss} &= \int_T^\infty f_X(x)dx \\ &= \int_{m_a+t\sigma_N}^\infty \frac{1}{\sqrt{2\pi}\sigma_N} \exp\left(-\frac{(x-m_a)^2}{2\sigma_N^2}\right) dx \\ &= \int_{t\sigma_N}^\infty \frac{1}{\sqrt{2\pi}\sigma_N} \exp\left(-\frac{x^2}{2\sigma_N^2}\right) dx \\ &= \int_t^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx \\ &= Q(t) \end{aligned}$$

b)
$$\begin{aligned}
E[X_{Loss}] &= \int_T^\infty (x-T)f(x)dx \\
&= \int_T^\infty x\cdot f(x)dx - TQ(t) \\
&= \int_{m_a+t\sigma_N}^\infty x\frac{1}{\sqrt{2\pi}\sigma_N}\exp\left(-\frac{(x-m_a)^2}{2\sigma_N^2}\right)dx - TQ(t) \\
&= \int_{t\sigma_N}^\infty (y+m_a)\frac{1}{\sqrt{2\pi}\sigma_N}\exp\left(-\frac{y^2}{2\sigma_N^2}\right)dy - (m_a+t\sigma_N)Q(t) \\
&= \int_{t\sigma_N}^\infty y\frac{1}{\sqrt{2\pi}\sigma_N}\exp\left(-\frac{y^2}{2\sigma_N^2}\right)dy - t\sigma_N Q(t) \\
&= \int_{\sigma_N}^\infty \sigma_N u\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{u^2}{2}\right)du - t\sigma_N Q(t) \\
&= \sigma_N\int_{\sigma_N}^\infty \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{u^2}{2}\right)d\left(\frac{u^2}{2}\right) - t\sigma_N Q(t) \\
&= \frac{\sigma_N}{\sqrt{2\pi}}e^{-\frac{\sigma_N^2}{2}} - t\sigma_N Q(t)
\end{aligned}$$

c) fraction of bits lost $= \dfrac{\text{bits lost/33 ms}}{\text{bits produced/33 ms}} = \dfrac{E[X_{Loss}]}{m_a}$.

d)
$$\begin{aligned}
\text{Avg. \# bits allocated per source} &= \frac{m_a+t\sigma_N}{N} \\
&= \frac{Nm+t\sqrt{N}\sigma}{N} = m + t\frac{\sigma}{\sqrt{N}}
\end{aligned}$$

$$\begin{aligned}
\text{Avg. \# bits lost per source} &= \frac{\frac{\sigma_N}{\sqrt{2\pi}}e^{-\frac{\sigma_N^2}{2}} + t\sigma_N Q(t)}{N} \\
&= \frac{\sigma e^{-\sqrt{N}\sigma}}{\sqrt{2\pi N}} + \frac{tQ(t)\sigma}{\sqrt{N}}
\end{aligned}$$

Both quantities decrease with N.