3/28/19 — 1

X, Y: R.V.

$Z = \text{MAX}(X, Y)$.

EASIEST TO USE CDF.

$F_X(x) = P[X \le x]$. $F_Y(y) = P[Y \le y]$

$P_Z(Z \le z) = P_Z[\text{MAX}(X, Y) \le z]$

$= P_Z[X \le z \;\&\; Y \le z]$

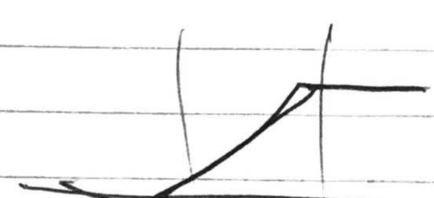
IF INDEPENDENT

$= P_Z[X \le z]\, P_Z[Y \le z]$

EX $X, Y : U[0, 1]$

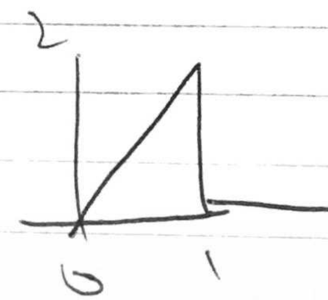
$f_X(x) = f_Y(y) = 1 \qquad 0 \le x \le 1$

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ x & 0 \le x \le 1 \\ 1 & x < 1 \end{cases}$$



$F_Z(z) = z^2$

$f_Z(z) = \frac{d}{dz} F(z) = 2z$



$W = MIN(X, Y)$

$F_W(w) = P[W \le w]$

$= P[MIN(X, Y) \le w]$

$= 1 - P[MIN(X, Y) > w]$

$= 1 - P[X > w \ \& \ Y > w]$

IF X, Y INDEPENDENT

$= 1 - P[X > w]\, P[Y > w]$

$F_W(w) = 1 - (1 - F_X(w))(1 - F_Y(w))$

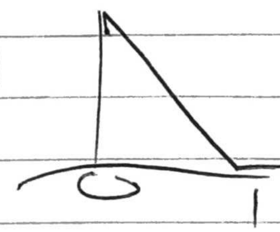
$X, Y = U[0, 1]$

$F_W(w) = 1 - (1-w)(1-w)$

$= 1 - 1 + 2w - w^2$

$= 2w - w^2$

$f_W(w) = 2 - 2w = 2(1-w)$



N VARS: $X_1 \ldots X_N$

→ YOU TAKE THIS COURSE N TIMES.
EACH GRADE IS $U[0,100]$

$Z = MAX(X_i)$.

Q: WHAT IS $E[Z]$, $f_Z(z)$ ETC?

$$F_{X_i}(x_i) = \frac{x_i}{100} \qquad 0 \le x_i \le 100$$

$$F_Z(z) = F_{X_i}(x_i)^N = \left(\frac{z}{100}\right)^N$$

$$\frac{d}{dz}F(z) = f(z) = \frac{N}{100}\left(\frac{z}{100}\right)^{N-1}$$

$$E[z] = \int_0^{100} z\, f(z)\, dz = \int N \left(\frac{z}{100}\right)^N dz$$

$$= \frac{N\ 100}{(N+1)} \left(\frac{z}{100}\right)^{N+1}\Bigg|_0^{100} = \frac{N}{N+1}\, 100$$

$N=1 \quad \mu = 50$
$N=2 \quad \mu = 67$
$N=10 \quad \mu = 90$