2/4/19 - 1

PROB (MORE THAN 10 THROWS TO GET 6 ON DIE)

$$\left(\frac{5}{6}\right)^{10}\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^{12}\left(\frac{1}{6}\right)+\dots$$

$$=\left(\frac{5}{6}\right)^{10}\left(\frac{1}{6}\right)\underbrace{\left[1+\frac{5}{6}+\left(\frac{5}{6}\right)^{2}+\dots\right]}$$

$$=\sum_{i=0}^{\infty}\left(\frac{5}{6}\right)^{i}$$

$$=\frac{1}{1-5/6}=6$$

$$=\left(\frac{5}{6}\right)^{10}\left(\frac{1}{6}\right)6=\left(\frac{5}{6}\right)^{10}$$

GEOMETRIC

1. FAIR COIN $p = 1/2$ $\quad q = 1-p = 1/2$

$P(k^{th} \text{ TOSS IS } 1^{ST} H) = q^{k-1}p = \frac{1}{2^k}$

X: R.V. FOR ~~# HEADS~~ # TOSSES TO 1^{ST} H

$P[X] = q^{k-1}p$

$E[X] = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + \ldots$

$= \sum_{k=1}^{\infty} k 2^{-k}$

$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \ldots$

HOW TO COMPUTE $\sum_{k=1}^{\infty} k 2^{-k}$

$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$

$$\frac{x}{(1-x)^2} = \sum k x^k$$

LET $x = \frac{1}{2}$

$$= \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2} = \sum k 2^{-k} = 2$$

EXPECTED # TOSSES TO 1^{ST} HEAD OF FAIR COIN.