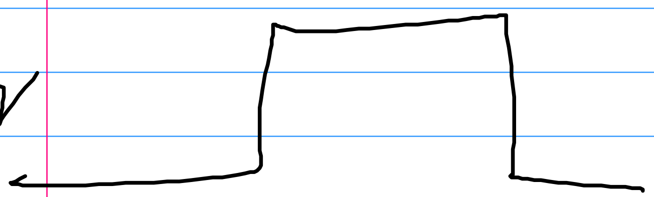
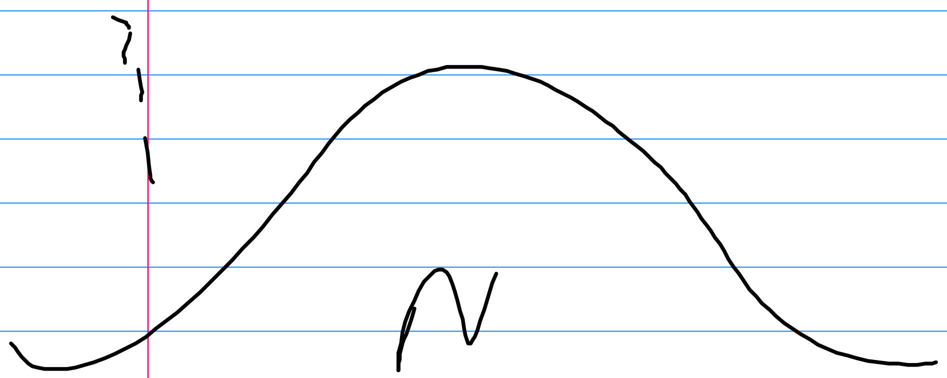
homework 8 q 1 start:

1v



v [0, 1]

2v



2

EX 3.3, p 204,

Note that the input and the noise are independent.
(In real world, who knows.)

in X: -1 w.p. $2/3$

1 $p = 1/3$

N = NORMAL $\mu = 0$ $\sigma = 1$

out $Y = X + N$

WANT $P[X = 1 \mid Y > 0]$

$P[X=1 \mid Y>0]\, P[Y>0] =$

$P[X=1 \;\&\; Y>0]$

$Y = X + N$
$= 1 + N$

$N > -1$

$= P[X=1 \;\&\; N > -1]$

$= P[X=1]\, P[N > -1]$

$= p(Q(-1))$

3

$P[X=1 \,\&\, Y>0] = p\,Q(-1)$

NEXT $P[Y>0]$

$P[Y>0] = P[Y>0 \,\&\, X=1] +$
$+ P[Y>0 \,\&\, X=-1]$

$P[Y>0 \,\&\, X=-1] =$

$P[X+N>0 \,\&\, X=-1]$

$= P[N>1 \,\&\, X=-1]$

$= P[N>1]\,P[X=-1]$

$= Q(1)\,(1-p)$

$P[Y>0] = P[Y>0 \,\&\, X=1] +$
$P[Y>0 \,\&\, X=-1]$

$= Q(-1)\,p + Q(1)(1-p)$

$5/6 \quad 1/3 \quad 1/6 \quad 2/3 \quad 4$

$= .4 \quad = .1$

WANT $P[X=1 \mid Y>0]$

$$= \frac{P[X=1 \wedge Y>0]}{P[Y>0]}$$

$$= \frac{.3}{.4} \approx .7$$

For conditional density, formula 5.45 use fig 5.6 on page 240 as an example.

note single variable f(x) = 1/6

x is 1st die, y is 2nd die.
p(x=4, y=4) = 2/42 from table.

what is p(x=4|y=4)?   p(x=4, y=4) / p(y=4) = 2/42 / (1/6) = 2/7

note that if dice were independent, it would be 1/6.

Using that table to show equation 5.48:

$$P[X=6] = \sum_{y=1}^{6} P[X=6 \mid Y=y]\, P[y]$$

$$P[i] = 1/6$$

$$P[X=6 \mid Y=6] = 2/7$$

$$P[X=1 \mid Y=6] = 1/7$$

2
3
4
5

$$P[X=6] = \frac{2}{7}\cdot\frac{1}{6} + 5\left(\frac{1}{7}\cdot\frac{1}{6}\right)$$

$$= \frac{1}{6}\left(\frac{2}{7} + 5/7\right) = 1/6$$

Example 5.33 on page 267

R.V. FOR # HITS IN SERVER

POISSON PARAM. βt

$$P[k] = e^{-\beta t}\frac{(\beta t)^k}{k!}$$

TIME TO SERVE A HIT

EXPONENTIAL R.V

$$P[x] = \alpha e^{-\alpha x}$$

CHECK

$$\int_0^\infty \alpha e^{-\alpha x}\,dx = \frac{\alpha}{-\alpha}e^{-\alpha x}\Big|_0^\infty = 1$$

We want the probability of N more hits during the service time for 1st hit.

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