4/5/18 p1

Ex 4.35 [illegible]

warmup problem. $Y=2X$ $dy/dx=2$ $f_Y(y) = 1/2$ $f_X(x) = f_X(y/2)/2$

e.g. $f_X(x) = 1$ for $0<=x<=1$ then $f_Y(y) = 1/2$ for $0<=y<=2$

new in 4.35 is that there are 2 values for x for each y.

$P(x0<x<x0+dx) = f_X(x0)\, dx$
$P(y0<y<y0+dy) = P(2x0<x<2x0+ dy)$. $dy = dy/dx\, dx$
$= f_Y(y0)\, dy$

$= P(2x0 < x < 2x0 + dy/dx\, dx)$

$= f(2x0)\, 2\, dx$

$P(y<y0) = F_Y(y0)$
$= P(x<y0/2) = F_X(y0/2)$

$F_Y(y) = F_X(y/2)$

back to 4.35. Since two values of x give each y, to find the probability density for y, we add the probabilities for the two x's.

Ex 4.36 p 180

$Y = \cos X$

$f_X(x) = \dfrac{1}{2\pi} \quad 0 < x < 2\pi$

$\displaystyle\int_0^{2\pi} f_X(x) = 1$

$$E(X) = \int_0^{2\pi} x f_X(x)\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x = \frac{1}{2\pi}\,\frac{x^2}{2}\Bigg|_0^{2\pi}$$

$$= \left(\frac{1}{2\pi}\right)\left(\frac{4\pi^2}{2}\right) = \pi$$

$Y = \cos X$

$$\frac{dy}{dx} = -\sin X = \sqrt{1-y^2}$$

$$f_Y(y) = \frac{1}{dy/dx} f_X(x) + \boxed{}$$

$$\frac{1}{2\pi\sqrt{1-y^2}} + \boxed{}$$

proving eqn 5.32.

$$VAR[x] = E[(x - E[x])^2]$$

*SHORTCUT $E[x] = \bar{x}$

$$VAR[x] = \overline{(x - \bar{x})^2}$$

$$= \overline{x^2 - 2x\bar{x} + \bar{x}^2}$$

$$= \overline{x^2} - \overline{2x\bar{x}} + \overline{\bar{x}^2}$$

$$= \overline{x^2} - 2\bar{x}^2 + \bar{x}^2$$

$$= \overline{x^2} - (\bar{x})^2$$

$$\sigma_x = \sqrt{VAR(x)}$$

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$$\overline{\left(\frac{x-\bar{x}}{\sigma_x} \pm \frac{y-\bar{y}}{\sigma_y}\right)^2} \geq 0$$

$$\overline{\left(\frac{x-\bar{x}}{\sigma_x}\right)^2} \pm 2\,\overline{\left(\frac{x-\bar{x}}{\sigma_x}\right)\left(\frac{y-\bar{y}}{\sigma_y}\right)} + \overline{\left(\frac{y-\bar{y}}{\sigma_y}\right)^2}$$

$$= 1 \qquad\qquad\qquad = 1$$

$$2 \geq \mp 2\,\overline{\left(\frac{x-\bar{x}}{\sigma_x}\right)\left(\frac{y-\bar{y}}{\sigma_y}\right)}$$

$$1 \geq \cancel{\pm} \frac{1}{\sigma_x \sigma_y}\,\overline{(x-\bar{x})(y-\bar{y})}$$

$$= \sigma_{xy}$$

$$|\sigma_{xy}| \leq 1$$

Ex 5.30 on p 263

p = prob that a defect that is somewhere is in region R.
given to you.

If there are a total of k defects, then what's prob of j being in R?

Bernoulli.

$$P_1(j \text{ in } R \mid k \text{ TOT}) = \binom{k}{j} p^j (1-p)^{k-j} \qquad k \ge j$$

$$P_2(k) = \frac{\alpha^k}{k!} e^{-\alpha}$$

P(k total defects of which j are in R) = $\binom{k}{j} p^j (1-p)^{k-j} \dfrac{\alpha^k}{k!} e^{-\alpha}$

P(j defects in R) = sum that over k

$$= \sum_{k=j}^{\infty} \cancel{\binom{k}{j}} p^j (1-p)^{k-j} \frac{\alpha^k}{k!} e^{-\alpha}$$

$$\frac{k!}{j!(k-j)!}$$

$$p(j) = \frac{p^j e^{-\alpha} \alpha^j}{j!} \sum_{k=j}^{\infty} \frac{(1-p)^{k-j}}{(k-j)!} \alpha^{k} \alpha^{-j} \text{ 6}$$

$$= \frac{(\alpha p)^j}{j!} e^{-\alpha} \sum_{k=j}^{\infty} \frac{(1-p)^{k-j}}{(k-j)!} \alpha^{k-j}$$

Let $i = k-j$

Now $\sum_{i=0}^{\infty}$

$$p(j) = \frac{(\alpha p)^j}{j!} e^{-\alpha} \underbrace{\sum_{i=0}^{\infty} \frac{(\alpha(1-p))^i}{i!}}_{e^{\alpha(1-p)}}$$

$$\boxed{\sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x}$$

$$p(j) = \frac{(\alpha p)^j}{j!} e^{-\cancel{\alpha} + \cancel{\alpha} - \alpha p}$$

$$P(j) = \frac{(\alpha p)^j}{j!} e^{-\alpha p}$$

That's Poisson with parameter alpha times p.

j is number of defects in region R
k is total number of defects on the chip, both in and out of R.