4/2/18 p1

Enrichment section
n=1,000,000   p=q=.5   m=np= 500,000   s=sqrt(npq) = 500

Example of normal approx.

P[more than 1,001,000 heads]   = 0

P[more than 501,000 heads] ?

501,000 is m+2s
That prob is 1- cdf for 2 in normal table: 0.023

My blog, section 3, q 2:

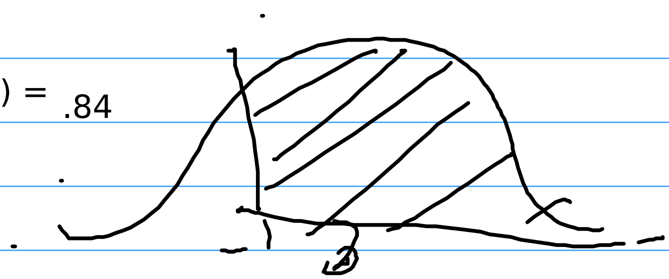
Expected # heads = 1,000,000 * .5005 = 500,500.

P[more than 500,000 heads]?

Here, m=500,500.   n=1,000,000 s=500.   (really, 499.99975)

500,000 = m-s.

P[more than 500,000] = 1-F_N(-1) = .84



section 3, q 3:

999,000 coins are fair.   1000 always heads.

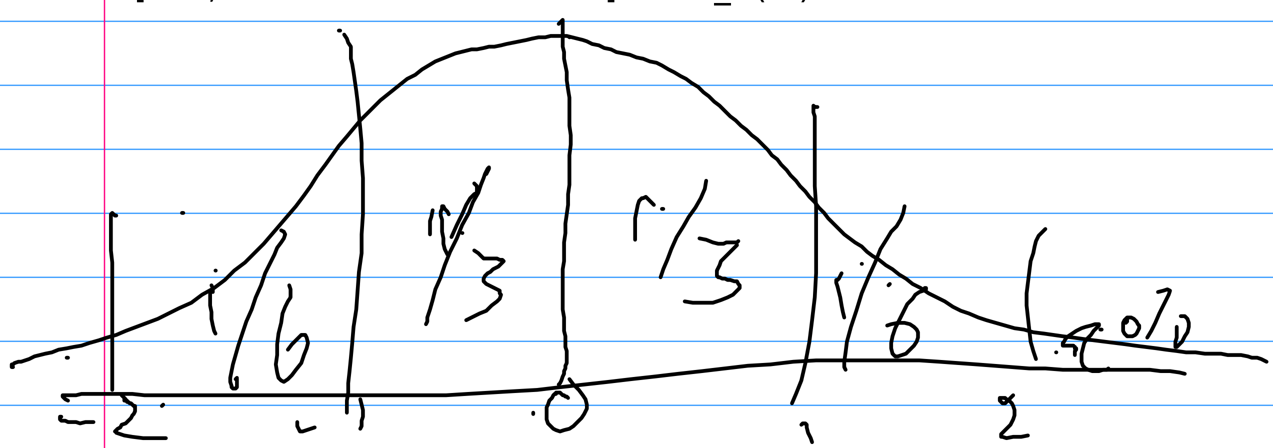
n=999,000.   m=499,500.   s=500.   <- ignoring 1000 fixed coins.

To get 500,000 total heads, we need 499,000 of the 999,000 fair coins to be heads.

499,000 = 499,500-500 = m-s.

Big F: cdf. _N: normal dist

P[500,000 or more total heads] = 1-F_N(-1) = .84.



Prob 5.17 on page 253.

$$f(x,y) = 2e^{-x}e^{-y} \qquad 2$$

$$0 \le x \le y$$

$$A = \int_0^{\cancel{x}} \int_0^{\min(x,1-x)} 2e^{-x-y}\,dy\,dx$$

$$A = 2\int_0^1 e^{-x} \underbrace{\int_0^{\min(x,1-x)} e^{-y}\,dy}\,dx$$

$$-e^{-y}\Big|_0^{\min(x,1-x)}$$

$$1 - e^{-\min(x,1-x)}$$

$$A = 2\int_0^1 e^{-x}\left(1 - e^{-\min(x,1-x)}\right)dx$$

$$A = 2\int_0^1 \left(e^{-x} - e^{-x-\min(x,1-x)}\right)^{3}_{4}$$

$$\int_0^1 e^{-x} = -e^{-x}\Big|_{0,\frac12}^{1} \quad 1 - e^{-1}$$

$$A = 2\left(1 - e^{-1}\right) - 2\int_0^{1/2} e^{-2x} - 2\int_{1/2}^{1} e^{-1}$$

$$\underbrace{\frac{e^{-2x}}{-2}\Big|_0^{1/2}} \qquad \underbrace{-e^{-1}}$$

$$e^{-2x}\Big|_0^1$$

$$1 - e^{-2}$$

$$A = 2 - 2e^{-1} + 1 - e^{-2} - e^{-1} = \boxed{3 - 3e^{-1} - e^{-2}}$$

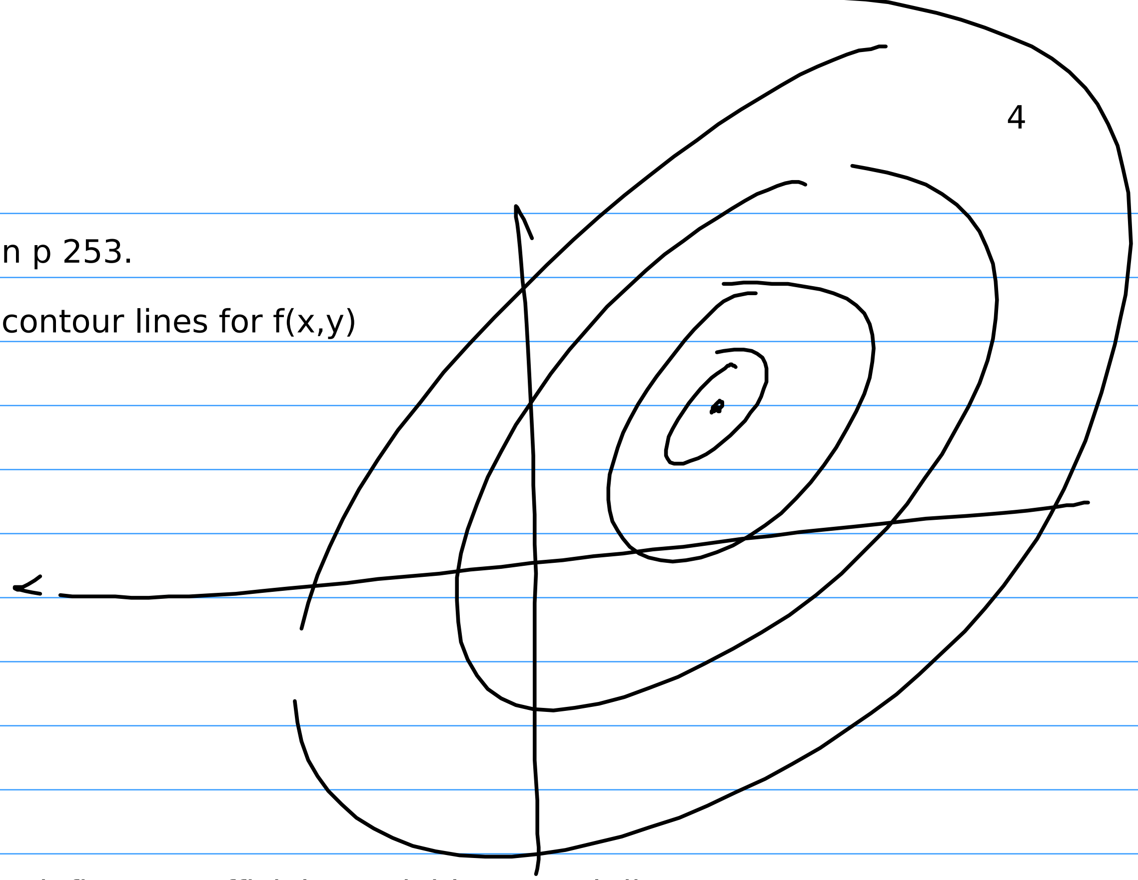
I disagree with book. Who is right?

A>1 => I'm wrong.

Student exercise: find my error.

Ex 5.18 on p 253.

contour lines for f(x,y)



Eqn 5.18 defines an official 2 variable normal dist.

ρ is correlation between x,y

-1<=rho<=1
0: no correlation
-1: perfect negative corr e.g. Y=-X. Y=-3X
1: perfect positive corr. e.g. Y=X, Y=5X

Variables could be dependent but uncorrelated since correlation is a linear property.

y=x^2

x,y not correlated over [-1,1]

E[function of 2 r.v.]

real world example: you want to know expected value of after-tax profit on numbers game.

power to move a ship is cubic in speed

ship's speed is r.v. What's expected fuel consumption?

COV[x,y] = E[(X-E[X])(Y-E[Y])]

$$= E\left[XY - XE[Y] - YE[X] + E[X]E[Y]\right]$$

$$= E[XY] - E[X]E[Y] \quad 5.30$$

Loss of precision in computation: 5.30 may be subtracting almost equal large numbers and so lose significant digits.
5.29 is numerically better.
Or, do 5.30 in double precision.

5.30: $\mathrm{cov} = \dfrac{\sum X_i Y_i}{N} - \left(\dfrac{\sum X_i}{N}\right)\left(\dfrac{\sum Y_i}{N}\right)$