3/22/18 P1

$$S = \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$$

PROVE IT

$$S = 1 + a + a^2 + a^3 + \cdots$$

$$= 1 + a(1 + a + a^2 + \cdots)$$

$$= 1 + aS$$

$$S(1-a) = 1$$

$$S = \frac{1}{1-a}$$

|a|<1 if this is to converge.

$$a = -\frac{1}{2} \qquad S = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$$

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} \cdots$$

2

$$\sum_{k=0}^{\infty} (1-p)p^k$$

$$= (1-p)\frac{1}{(1-p)} = 1$$

3

$$\sum k a^k = 0a^0 +$$

$$1a^1 +$$

$$a^2 + a^2$$

$$+ a^3 + a^3 + a^3$$

$$+ a^4 + a^4 + a^4 + a^4$$

$$1$$

$$\sum a^k + a\sum a^k + a^2 \sum a^k + \dots$$

$$= \frac{1}{1-a} + \frac{a}{1-a} + \frac{a^2}{1-a} + \dots$$

$$= \frac{1}{1-a}\left(1 + a + a^2 + \dots\right)$$

$$= \left(\frac{1}{1-a}\right)^2$$

(Close na součet)

$$\frac{a}{(1-a)^2}$$

$$f(k) = (1-p)p^k$$

$$E[f] = \sum k(1-p)p^k$$

$$= (1-p)\sum k p^k$$

$$= (1-p)\frac{p}{(1-p)^2}$$

$$= \frac{p}{1-p}$$

Ex 5.9 $\overline{5}$

$$f(N) = (1-p)p^N$$

$$0 \le N < \infty$$

$$N = qM + r \qquad 0 \le r < M$$

$$\sum_{k=0}^{M-1} a^k = \sum_{0}^{\infty} - \sum_{M}^{\infty}$$

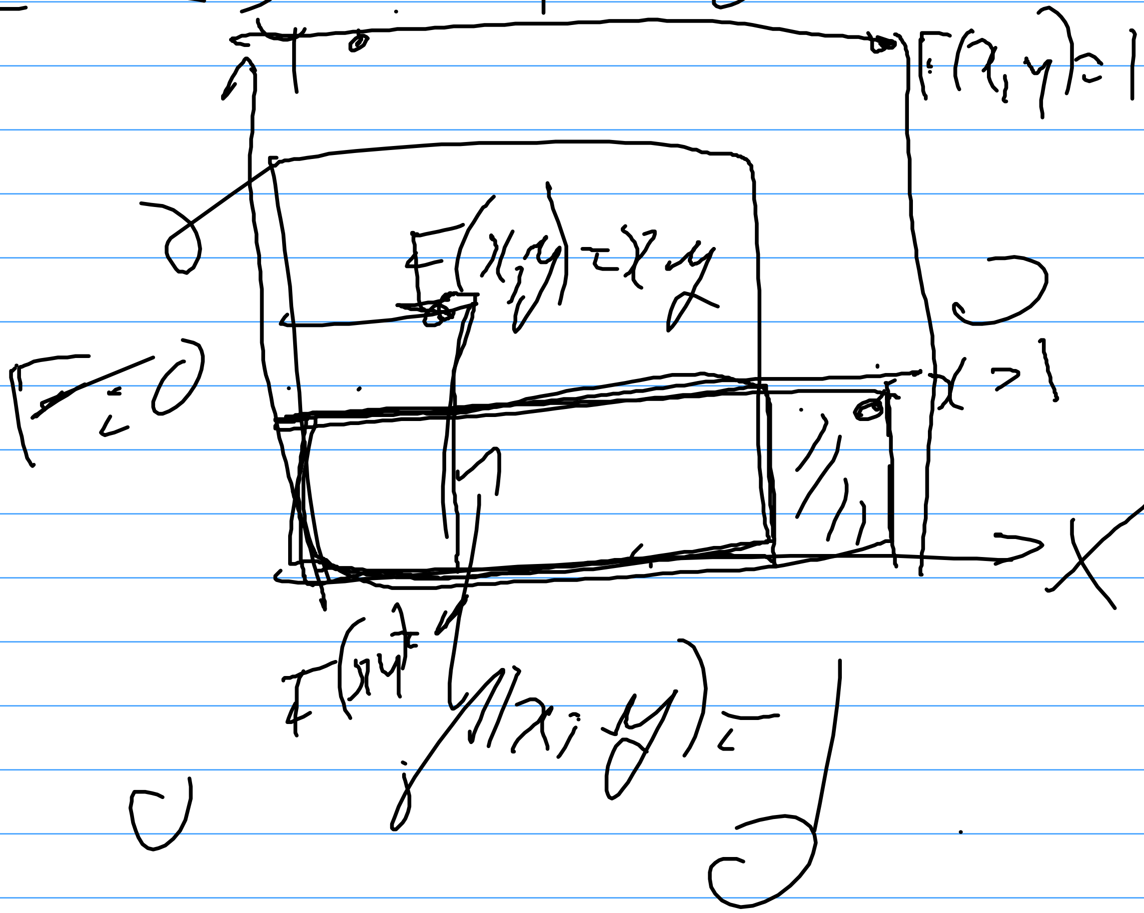
$$= \frac{1}{1-a} - a^M\left(\frac{1}{1-a}\right) = \frac{1-a^M}{1-a}$$

eg $a = 2 \qquad M = 4$

$$\sum_{0}^{3} 2^k = 1+2+4+8 = 15$$

$$\frac{1-2^4}{1-2} = \frac{-15}{-1} = 15$$

Ex 3.11 → G



EX 5.12

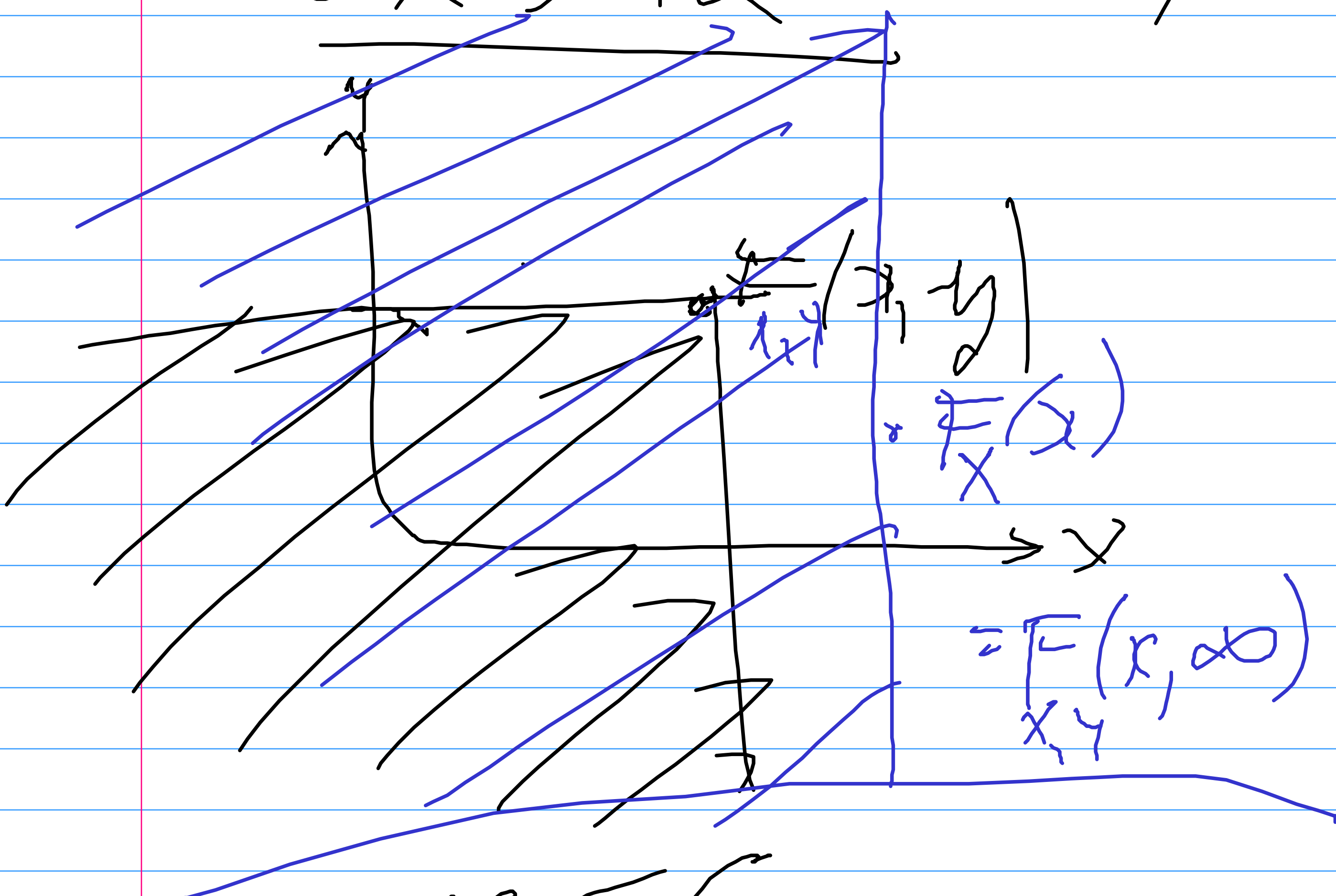


FIG 5.6

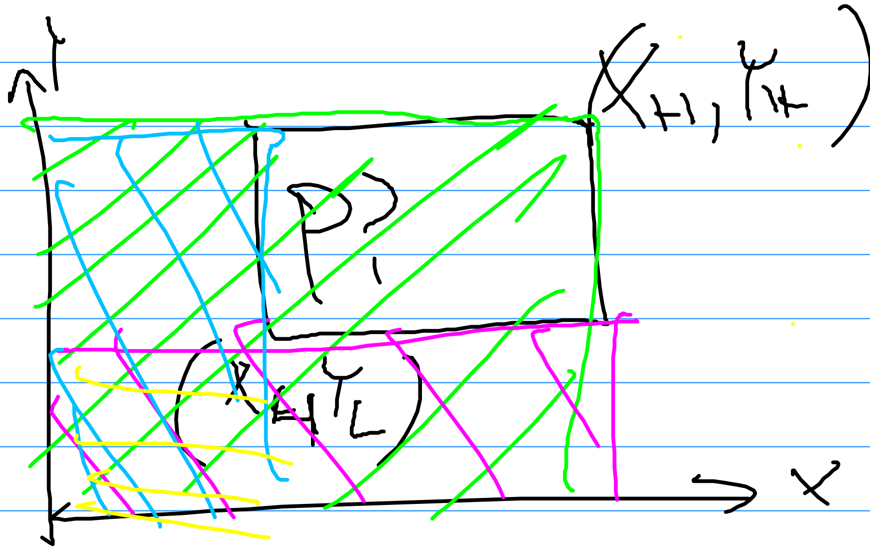
$$F(3,3) = \frac{12}{42}$$

$$F_X(3) = F_{XY}(3,\infty) = \frac{21}{42} = \frac{1}{2}$$

$$F_Y(2) = F_{XY}(\infty,2) = \frac{14}{42} = \frac{1}{3}$$

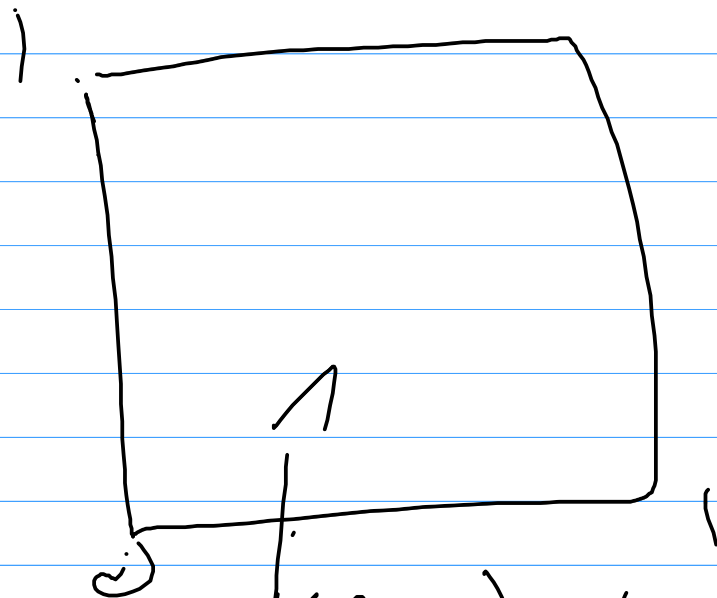
EX 4.13

maybe you need to find the probability of some rectangle



$$P(X_L \le X \le X_H,\ Y_L \le Y \le Y_H)$$
$$= F(X_H, Y_H) - F(X_H, Y_L) - F(X_L, Y_H) + F(X_L, Y_L)$$

Ex 5.15



$f(x,y) = 1$

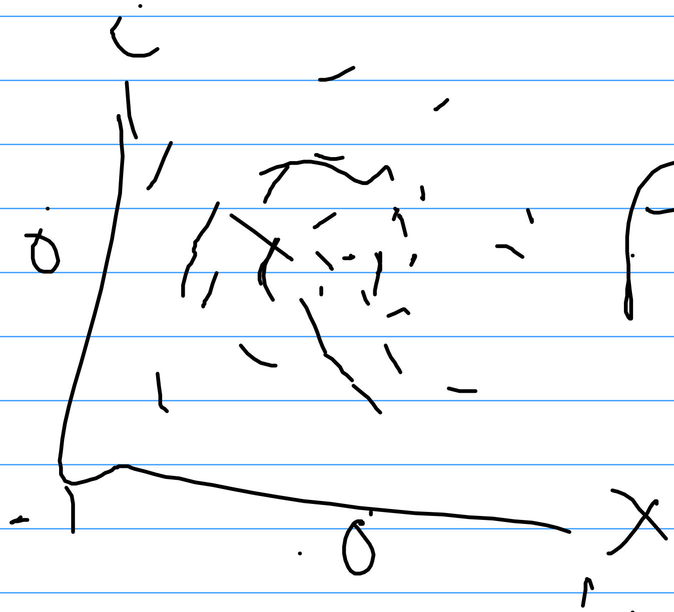
$F(x,y) = xy$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

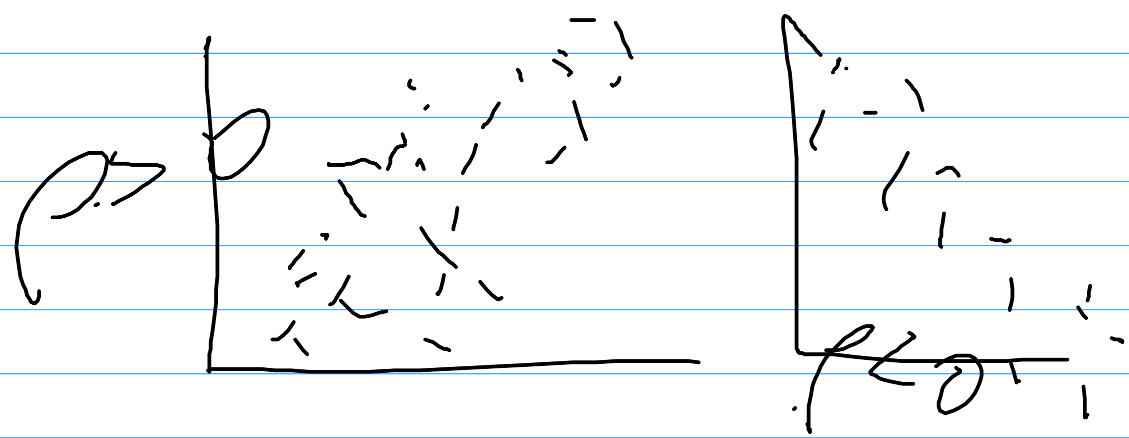
$= 1 \qquad 0 \le x \le 1$

$$F_X(x) = F(x,\infty)$$

EX 5.18



$\rho = 0$



Definitions of X and Y being independent

f_X(x) f_Y(y) = f_XY(x,y)

F_X(x) F_Y(y) = F_XY(x,y)